Parallel Architecture

Announcements

- HW0 is due Friday night, thank you for those who have already submitted
- HW1 is due Wednesday night

Today

- Computing operational intensity
- Dwarves and Motifs
- Stencil computation demo
- Touchstone apps for 560 Spring 2012
  - Dynamic programming: Protein string matching
  - Sparse Linear Algebra: SpMV
  - Structured grids: stencil computations with implicit and explicit coefficients
  - N-body methods: n-body stars (HW3) and molecular dynamics
  - Dense Linear Algebra: Matrix matrix multiply, forward and backward substitution, and Cholesky (later)

Roofline Model (Will be using this in HW2)

Roofline: An Insightful Visual Performance Model for Multicore Architecture

- By Sam Williams, Andrew Waterman, and David Patterson
- ACM Communications, April 2009, Vol 52, No 4.

Operational Intensity: Operations per byte of DRAM traffic.

Roofline graph per machine

- FLOPS/sec versus operational intensity
- Horizontal line for the peak floating point performance (compute bound)
- Diagonal line for the measured peak memory performance (memory bound)

Placing ceilings to represent how performance optimizations can help

- Improve ILP and apply SIMD (computation bound)
- Balance floating point operation mix (computation bound)
- Unit stride accesses (memory bound)
- Memory affinity (memory bound)

Operational Intensity for SpMV

Sparse Matrix Vector Product $y = Ax$

Operational Intensity is flops/(byte of memory traffic)

Computed using coordinate storage (COO)

```
for (i=0; i<N; i++) { Y[i] = 0; }
for (p=0; p<NNZ; p++) {
  Y[row[p]] += val[p]*X[col[p]];
}
```

Computed using more common compressed sparse row (CSR)

<demo CSR>

```
for (i=0; i<N; i++) {
  y = 0;
  for (p=rowptr[i]; p<rowptr[i+1]; p++) {
    y += val[p]*X[col[p]];
  }
  Y[i] = y;
}
```


The Berkeley View

Conventional Wisdom and their replacements <read and discuss in class>

13 Dwarfs/Motifs

- Dense linear algebra, dense matrices and vectors, matrix-matrix, matrix vector
- Sparse linear algebra, explicitly store only non-zeros, explicit storage of indices
- Spectral methods, FFT, specific pattern of data permutation, all to all communication
- N-body methods, interactions between discreet points, various algorithms
- Structured grids, regular grid, neighbor relationships implicit in multi-dim array structure
- Unstructured grids, irregular grid, connectivity of grid must be explicit
- Monte carlo, repeated random trials with some final summary, map-reduce
- Combinational Logic, logical functions with stored state
- Graph traversal, e.g. quicksort
- Dynamic programming, filling a table with solutions to subproblems to build final solution
- Backtrack and Branch and Bound, recursive division of feasible solution space with pruning
- Construct Graphs, e.g., Hidden Markov Models and Bayesian networks
- Finite State Machine, serial with single state at one time and transitions to other states

1D Stencil Computation

Stencil Computations

- Computations operate over some mesh or grid
- Computation is modifying the value of something over time or as part of a relaxation to find steady state
- Each computation has some nearest neighbor data dependence pattern
- The coefficients multiplied by neighbor can be constant or variable

1D Stencil Computation version 1 <demo in class>

```
// assume A[0,i] initialized to some values
for (t=1; t<(T+1); t++) {
  for (i=1; i<(N-1); i++) {
    A[t,i] = 1/3 * (A[t-1,i-1] + A[t-1,i] + A[t-1,i+1];
  }
}
```


1D Stencil Computation (take 2)

1D Stencil Computation, version 2 <demo in class>

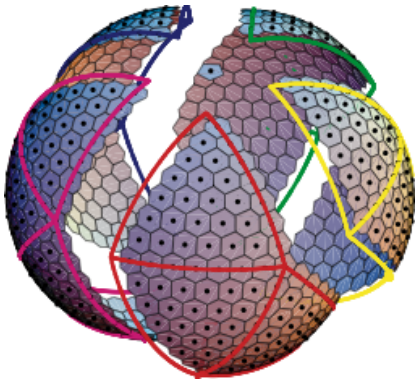
```
// assume A[i] initialized to some values
for (t=0; t<T; t++) {
  for (i=1; i<(N-1); i++) {
    A[i] = 1/3 * (A[i-1] + A[i] + A[i+1];
  }
}
```

Analysis

- Are version 1 and version 2 computing the same thing?
- What is the operational intensity of version 1 versus version 2?
- What parallelism is there in version 1 versus version 2?

Jacobi in SWM code (Stencil Computation with Explicit Weights)

Source: David Randall's research group



```
do ksdm=1,nsdm
   do j=npad+1,ny-npad
      do i=npad+1,nx-npad
         work(i,j,:) =                                                  &
             rw7(i, j, ksdm) * ( + rhs( i, j, :, ksdm)                  &
                - l_weights( 1, i, j, ksdm) * xout(i-1, j  , :, ksdm)&
                - l_weights( 2, i, j, ksdm) * xout(i-1, j-1, :, ksdm)&
                - l_weights( 3, i, j, ksdm) * xout(i  , j-1, :, ksdm)&
                - l_weights( 4, i, j, ksdm) * xout(i+1, j  , :, ksdm)&
                - l_weights( 5, i, j, ksdm) * xout(i+1, j+1, :, ksdm)&
                - l_weights( 6, i, j, ksdm) * xout(i  , j+1, :, ksdm))
      enddo
   enddo
enddo
```


Forward Substitution (Dense Matrix)

Given an NxN lower triangular matrix with unit diagonals and a n-vector b solve for the vector x in $Lx = b$

$$b_i = \sum_{j=1}^{N} L_{i,j} x j$$

How do we solve for x?

How do we turn this into a loop program?

Moldyn <draw iteration space>

```
for (tstep=0;tstep<=n_tstep-1;tstep++) {
   …
   for (i=0;i<=n_moles-1;i++) {
     x(i) = x(i) + vhx(i) + fx(i);
     …
     if ( x(i) < 0.0 )  x(i) = x(i) + side ; …
     if ( x(i) > side ) x(i) = x(i) - side ; …

     vhx(i) = vhx(i) + fx(i); …
     fx(i)  = 0.0; …
   }
   for (ii=0;ii<=n_inter-1;ii++) {
     i = inter1(ii);  j = inter2(ii);
     fx(i) += … x(i)… x(j)…
     fx(j) += … x(i)… x(j)…
   }
   for (i=0;i<=n_moles-1;i++) {
     …
     vhx(i) = … fx(i) …; …
   }
}
```


Concepts

Computing operational intensity

Berkeley dwarves/motifs

- What they are and examples
- Their parallel performance properties
- Explicit versus implicit storage of indices, graph connectivity, etc.

Stencil computations

- Nearest neighbor data dependences

Touchstone apps for the class

- The Berkeley dwarf/motif categories they represent.
- Data reuse within the touchstone apps
- Parallelism within the touchstone apps

Next Time

Reading

- Advanced Compiler Optimizations for Supercomputers by Padua and Wolfe

Homework

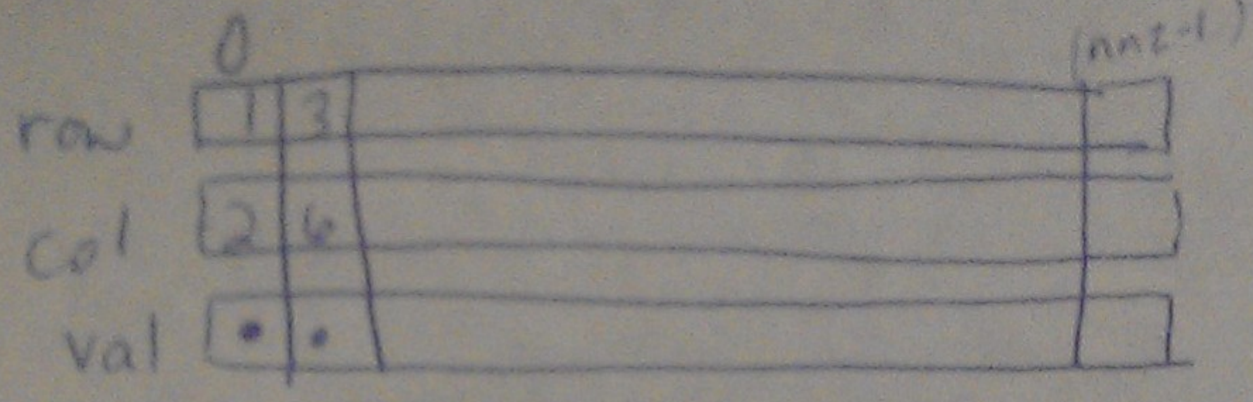
- HW0 is due Friday 1/27/12
- HW1 is due Wednesday 2/1/12

Lecture

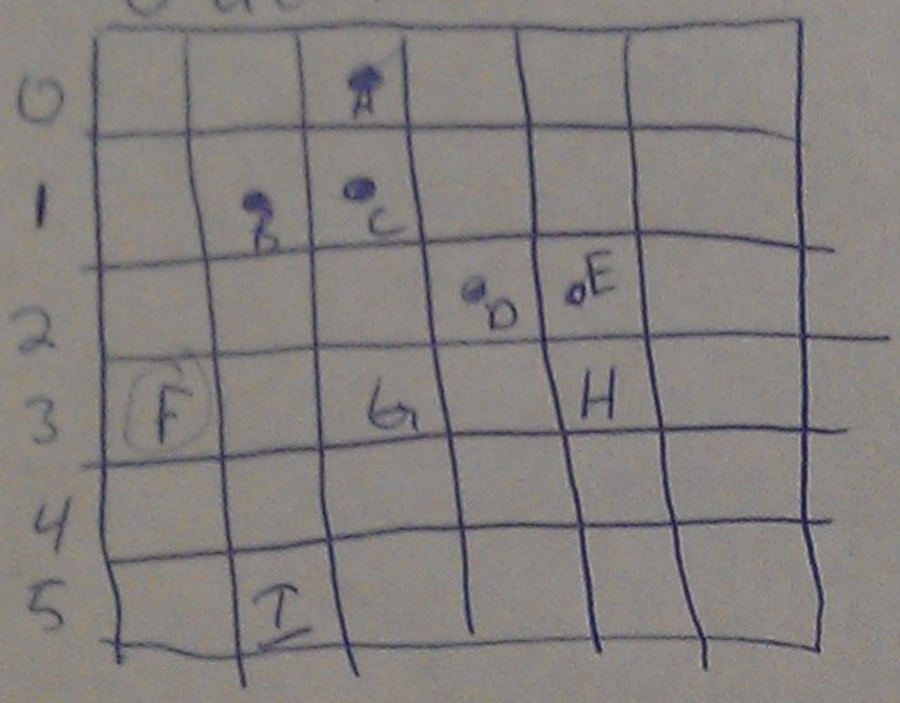
- Parallelization and Performance Optimization of Applications

Test

COO coordinate Storage



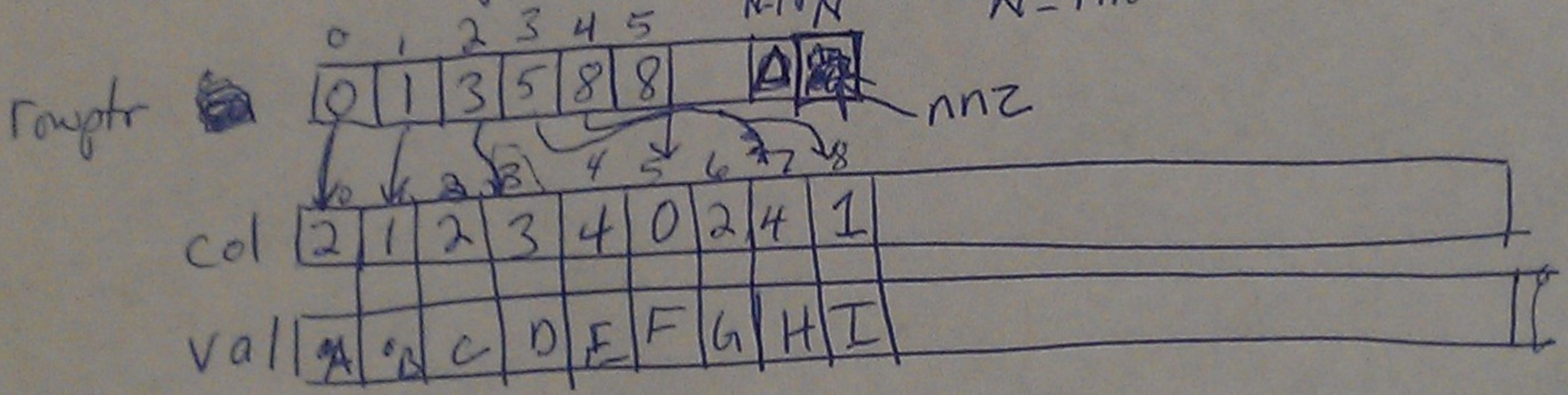
dense in 2D array



nnz < 1% (nrow * ncol)

CSR compressed sparse row

N = nrows



for (p=0 to nnz-1

~~y[r] = x[~~
y[row[p]] = y[row[p]] + x[col[p]]*val[p]
~~y[col~~

2 flops

loads: row[p], col[p], y[ ], val[ ], x[ ]

32

assumptions: i in reg, row[p] loaded once
all others go to memory

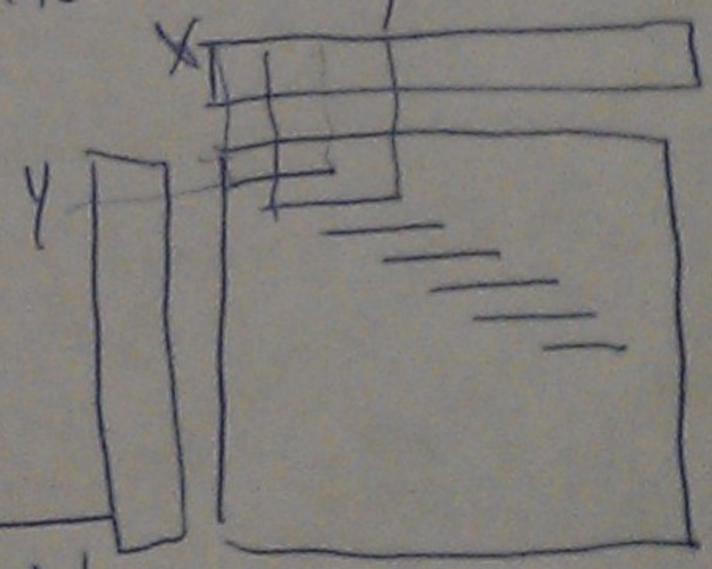
operational intensity = $\frac{2}{32}$ = .0625

arithmetic

.17 to .25 ?

---

assumption: everything brought into memory once



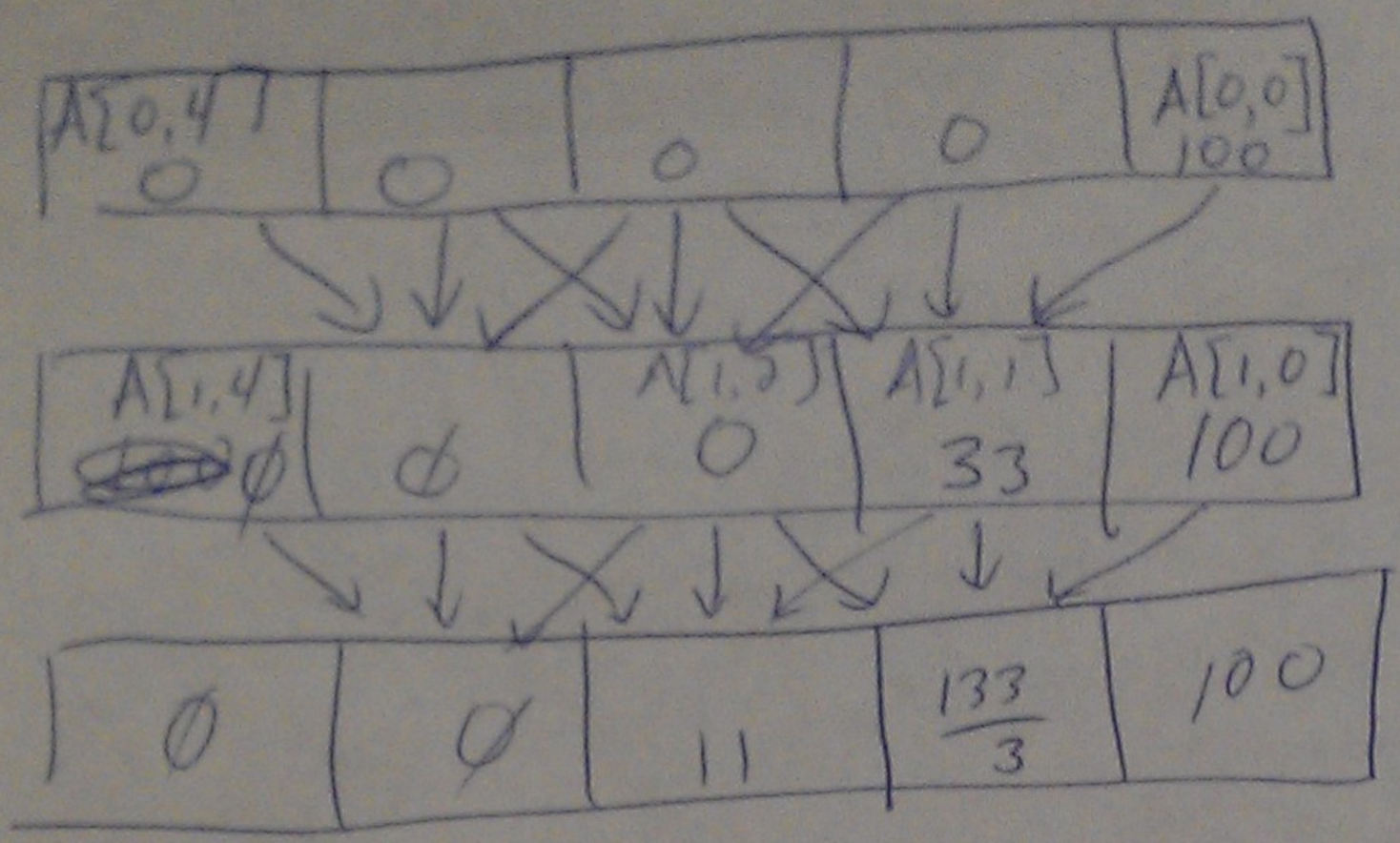
bytes = nnz*(16) + 2N(8)

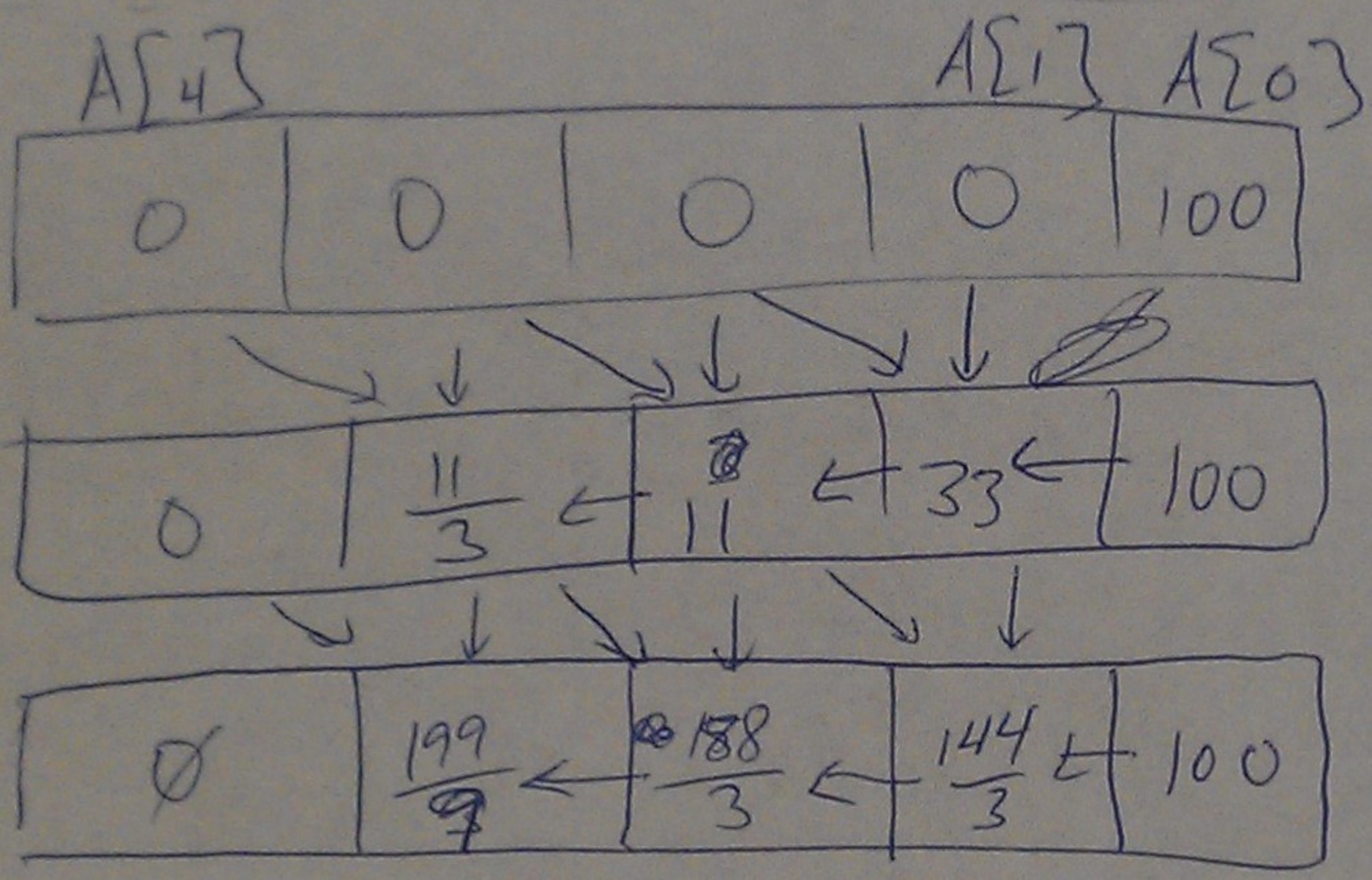
flops = 2 nnz

op intensity = $\frac{2\ nnz}{16\ nnz + 16\ N}$

arith

.125


A[0,4]
0
A[0,0]
100
A[1,4]
A[1,3]
0
A[1,1]
33
A[1,0]
100
0
11
133/3
100


A[4]
A[1]
A[0]
0
100
11/3
11
33
199/7
188/3
144/3
100