Loop Transformations, Dependences, and Parallelization

Announcements

- Quiz 1 is on RamCT and is due Friday night
- HW1 is due Wednesday February 8th

Today

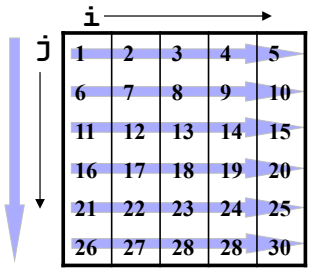
- Loop permutation
- Data dependences
- Loop parallelization
- Loop skewing

Loop Permutation for Improved Locality

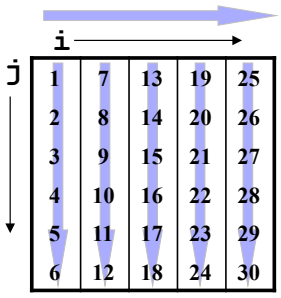
Sample code: Assume Fortran's Column Major Order array layout

```
do j = 1,6
  do i = 1,5
     A(j,i) = A(j,i)+1
  enddo
enddo
```

```
do i = 1,5
  do j = 1,6
    A(j,i) = A(j,i)+1
  enddo
enddo
```



poor cache locality



good cache locality

Loop Permutation Another Example

Idea

- Swap the order of two loops to increase parallelism, to improve spatial locality, or to enable other transformations
- Also known as **loop interchange**

Example

```
do i = 1,n
   do j = 1,n
      x = A(2,j)
   enddo
enddo
```

This access strides through a row of A

```
do j = 1,n
   do i = 1,n
      x = A(2,j)
   enddo
enddo
```

This code is invariant with respect to the inner loop, yielding better locality

Loop Permutation Legality

Sample code

```
do j = 1,6
 do i = 1,5
   A(j,i) = A(j,i)+1
 enddo
enddo
```

```
do i = 1,5
  do j = 1,6
   A(j,i) = A(j,i)+1
  enddo
enddo
```

Why is this legal?

- No loop-carried dependences, so we can arbitrarily change order of iteration execution
- Does the loop always have to have NO inter-iteration dependences for loop permutation to be legal?

Data Dependences

Recall

- A data dependence defines ordering relationship two between statements
- In executing statements, data dependences must be respected to preserve correctness

Example

```
s1  a := 5;
s2  b := a + 1;
s3  a := 6;
```

$\overset{?}{\equiv}$

```
s1  a := 5;
s3  a := 6;
s2  b := a + 1;
```


Dependences and Loops

Loop-independent dependences

```
do i = 1,100
   A(i) = B(i)+1
   C(i) = A(i)*2
enddo
```

Dependences within the same loop iteration

Loop-carried dependences

```
do i = 1,100
   A(i) = B(i)+1
   C(i) = A(i-1)*2
enddo
```

Dependences that cross loop iterations

Data Dependence Terminology

We say statement s_2 depends on s_1

- **True (flow) dependence**: s_1 writes memory that s_2 later reads
- **Anti-dependence**: s_1 reads memory that s_2 later writes
- **Output dependences**: s_1 writes memory that s_2 later writes
- **Input dependences**: s_1 reads memory that s_2 later reads

Notation: $s_1 \ \delta \ s_2$

- s_1 is called the **source** of the dependence
- s_2 is called the **sink** or **target**
- s_1 must be executed before s_2

Yet Another Loop Permutation Example

Consider another example

```
do i = 1,n
   do j = 1,n
      C(i,j) = C(i+1,j-1)
    enddo
enddo
```

```
do j = 1,n
   do i = 1,n
      C(i,j) = C(i+1,j-1)
    enddo
enddo
```

Before

(1,1) C(1,1) = C(2,0)
(1,2) C(1,2) = C(2,1)
. . .
(2,1) C(2,1) = C(3,0)

δ^a

After

(1,1) C(1,1) = C(2,0)
(2,1) C(2,1) = C(3,0)
. . .
(1,2) C(1,2) = C(2,1)

δ^f

Data Dependences and Loops

How do we identify dependences in loops?

```
do i = 1,5
   A(i) = A(i-1)+1
enddo
```

```
A(1) = A(0)+1
A(2) = A(1)+1
A(3) = A(2)+1
A(4) = A(3)+1
A(5) = A(4)+1
```

Simple view

- Imagine that all loops are fully unrolled
- Examine data dependences as before

Problems

- Impractical and often impossible
- Lose loop structure

Iteration Spaces

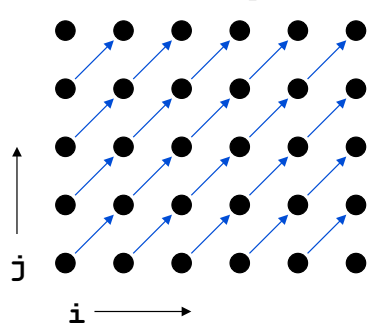
Idea

- Explicitly represent the iterations of a loop nest

Example

```
do i = 1,6
  do j = 1,5
     A(i,j) = A(i-1,j-1)+1
  enddo
enddo
```

Iteration Space



Iteration Space

- A set of tuples that represents the iterations of a loop
- Can visualize the dependences in an iteration space

Distance Vectors

Idea

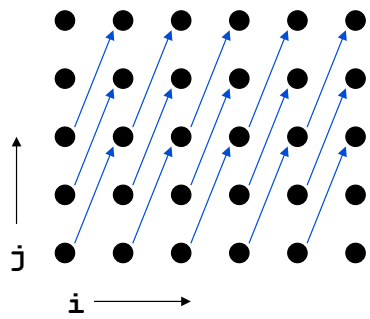
- Concisely describe dependence relationships between iterations of an iteration space
- For each dimension of an iteration space, the distance is the number of iterations between accesses to the same memory location

Definition

- $\mathbf{v} = \mathbf{i}^T - \mathbf{i}^S$

Example

```
do i = 1,6
   do j = 1,5
      A(i,j) = A(i-1,j-2)+1
   enddo
enddo
```



outer loop

Distance Vector: (1,2)

inner loop

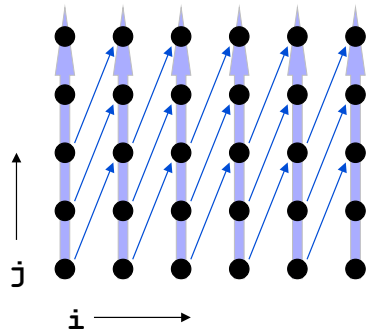
Distance Vectors and Loop Transformations

Idea

- Any transformation we perform on the loop must respect the dependences

Example

```
do i = 1,6
   do j = 1,5
      A(i,j) = A(i-1,j-2)+1
   enddo
enddo
```



Can we permute the `i` and `j` loops?

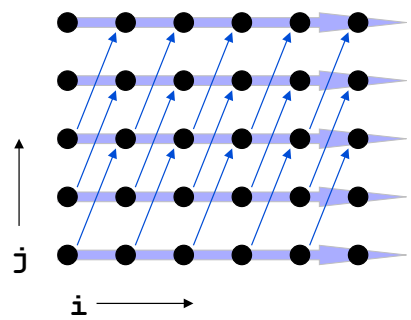
Distance Vectors and Loop Transformations

Idea

- Any transformation we perform on the loop must respect the dependences

Example

```
do j = 1,5
  do i = 1,6
     A(i,j) = A(i-1,j-2)+1
  enddo
enddo
```



Can we permute the `i` and `j` loops?

- Yes

Distance Vectors: Legality

Definition

- A dependence vector, *v*, is **lexicographically nonnegative** when the left-most entry in *v* is positive or all elements of *v* are zero
  Yes: (0,0,0), (0,1), (0,2,-2)
  No: (-1), (0,-2), (0,-1,1)

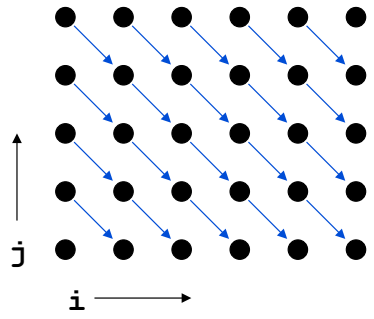
- A dependence vector is **legal** when it is lexicographically nonnegative (assuming that indices increase as we iterate)

Why are lexicographically negative distance vectors illegal?

Example where permutation is not legal

Sample code

```
do i = 1,6
  do j = 1,5
     A(i,j) = A(i-1,j+1)+1
  enddo
enddo
```



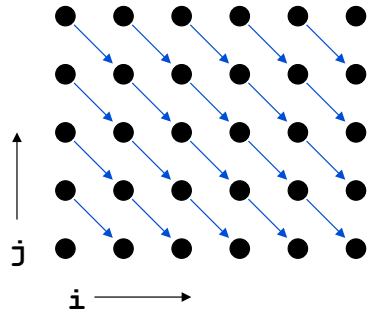
Kind of dependence: Flow

Distance vector: (1, −1)

Exercise

Sample code

```
do j = 1,5
  do i = 1,6
     A(i,j) = A(i-1,j+1)+1
  enddo
enddo
```



Kind of dependence: Anti

Distance vector: (1, -1)

Loop-Carried Dependences

Definition

- A dependence $D=(d_1,...d_n)$ is **carried** at loop level i if d_i is the first nonzero element of D

Example

```
do i = 1,6
   do j = 1,6
      A(i,j) = B(i-1,j)+1
      B(i,j) = A(i,j-1)*2
   enddo
enddo
```

Distance vectors: (0,1) for accesses to **A**
(1,0) for accesses to **B**

Loop-carried dependences

- The j loop carries dependence due to **A**
- The i loop carries dependence due to **B**

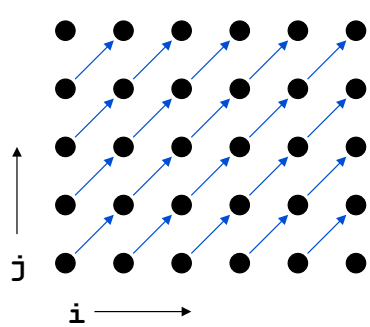
Direction Vector

Definition

- A direction vector serves the same purpose as a distance vector when less precision is required or available
- Element i of a direction vector is <, >, or = based on whether the source of the dependence precedes, follows or is in the same iteration as the target in loop i

Example

```
do i = 1,6
   do j = 1,5
      A(i,j) = A(i-1,j-1)+1
   enddo
enddo
```



Direction vector: **(<,<)**
Distance vector: (1,1)

Legality of Loop Permutation

Case analysis of the direction vectors

(=,=)

The dependence is loop independent, so it is unaffected by permutation

(=,<)

The dependence is carried by the j loop.

After permutation the dependence will be (<,=), so the dependence will still be carried by the j loop, so the dependence relations do not change.

(<,=)

The dependence is carried by the i loop.

After permutation the dependence will be (=,<), so the dependence will still be carried by the i loop, so the dependence relations do not change.

Legality of Loop Permutation (cont)

Case analysis of the direction vectors (cont.)

(<,<)

The dependence distance is positive in both dimensions.

After permutation it will still be positive in both dimensions, so the dependence relations do not change.

(<,>)

The dependence is carried by the outer loop.

After interchange the dependence will be (>,<), which changes the dependences and results in an illegal direction vector, so interchange is illegal.

(>,*) (=,>)

Such direction vectors are not possible for the original loop.

Loop Permutation Example

Consider the (<,>) case

```
do i = 1,n
   do j = 1,n
      C(i,j) = C(i+1,j-1)
   enddo
enddo
```

```
do j = 1,n
   do i = 1,n
      C(i,j) = C(i+1,j-1)
   enddo
enddo
```

Before

(1,1) C(1,1) = C(2,0)
(1,2) C(1,2) = C(2,1)
. . .
(2,1) C(2,1) = C(3,0)

δ^a

After

(1,1) C(1,1) = C(2,0)
(2,1) C(2,1) = C(3,0)
. . .
(1,2) C(1,2) = C(2,1)

δ^f

Loop-Carried Dependences

Definition

– A dependence $D=(d_1,...d_n)$ is **carried** at loop level i if d_i is the first nonzero element of D

Example

```
do i = 1,6
   do j = 1,6
      A(i,j) = B(i-1,j)+1
      B(i,j) = A(i,j-1)*2
   enddo
enddo
```

Distance vectors: (0,1) for accesses to **A**
(1,0) for accesses to **B**

Loop-carried dependences

– The j loop carries dependence due to **A**
– The i loop carries dependence due to **B**

Parallelization

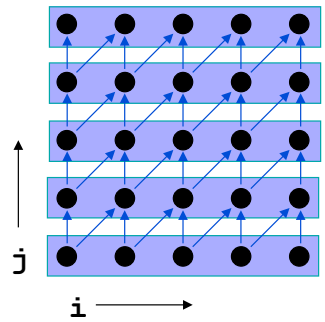
Idea

– Each iteration of a loop may be executed in parallel **if that loop carries no dependences**

Example (different from last slide)

```
do j = 1,5
   do i = 1,6
      A(i,j) = B(i-1,j-1)+1
      B(i,j) = A(i,j-1)*2
   enddo
enddo
```

Iteration Space



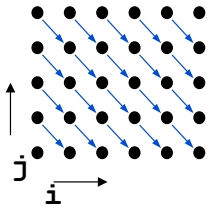
Parallelize `i` loop? Distance Vectors:

(1,0) for **`A`** (flow)
(1,1) for **`B`** (flow)

Loop Skewing

Original code

```
do i = 1,6
 do j = 1,5
    A(i,j) = A(i-1,j+1)+1
 enddo
enddo
```

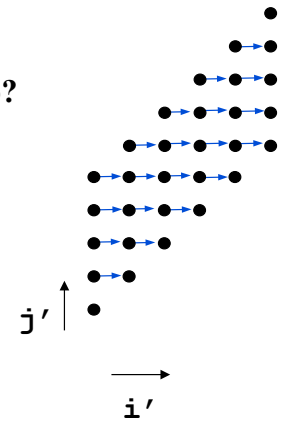


Distance vector: (1, -1)

Can we permute or parallelize the original loop?

What about after skewing?

```
do i' = 1,6
  do j' = i',i'+5-1
    A(i',j'-i'+1)=A(i'-1,j'-i'+2)+1
  enddo
enddo
```



Protein String Matching Example (smithWaterman.c)

```
for (i=1;i<=a[0];i++) {
  for (j=1;j<=b[0];j++) {
    diag    = h[i-1][j-1] + sim[a[i]][b[j]];
    down    = h[i-1][j] + DELTA;
    right   = h[i][j-1] + DELTA;
    max=MAX3(diag,down,right);
    if (max <= 0)  {
      h[i][j]=0; xTraceback[i][j]=-1; yTraceback[i][j]=-1;
    } else if (max == diag) {
      h[i][j]=diag; xTraceback[i][j]=i-1; yTraceback[i][j]=j-1;
    } else if (max == down) {
      h[i][j]=down; xTraceback[i][j]=i-1; yTraceback[i][j]=j;
    } else  {
      h[i][j]=right; xTraceback[i][j]=i; yTraceback[i][j]=j-1;
    }
    if (max > Max){
       Max=max; xMax=i; yMax=j;
    }
 }}  // end for loops
```

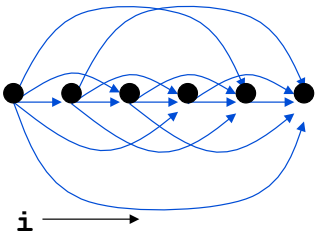

Loop-Carried, Storage-Related Dependences

Problem

- Loop-carried dependences inhibit parallelism
- Scalar references result in loop-carried dependences

Example

```
do i = 1,6
t = A(i) + B(i)
    C(i) = t + 1/t
enddo
```



Can this loop be parallelized? No.

What kind of dependences are these? Anti dependences.

Convention for these slides: Arrays start with upper case letters, scalars do not

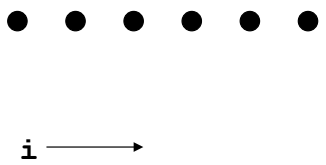
Removing False Dependences with Scalar Expansion

Idea

– Eliminate false dependences by introducing extra storage

Example

```
do i = 1,6
    T(i) = A(i) + B(i)
    C(i) = T(i) + 1/T(i)
enddo
t = T[6]
```



Can *this* loop be parallelized?

Disadvantages?

Scalar Expansion Details

Restrictions

– The loop must be a **countable** loop

i.e. The loop trip count must be independent of the body of the loop

– The expanded scalar must have no upward exposed uses in the loop

```
do i = 1,6
    print(t)
    t = A(i) + B(i)
    C(i) = t + 1/t
enddo
```

– Nested loops may require much more storage

– When the scalar is live after the loop, we must move the correct array value into the scalar

Concepts

Loop Transformations

- Loop permutation and when it is applicable
- Data dependences including distance vectors, loop carried dependences, and direction vectors
- Loop parallelization
  - Loop carried dependences
  - When loop parallelization is directly applicable
  - When it is not directly applicable
  - Scalar expansion

Next Time

Reading

- Read: Automatic Parallelization in the Polytope Model by Feautrier

Homework

- Quiz 1 is due tomorrow night
- HW1 is due Wednesday 2/8/12

Lecture

- Automating Parallelization and Performance Optimization of Applications