## Automating Scheduling

**Logistics**

- Final report for project due this Friday, 5/4/12
- Quiz 4 due this Monday, 5/7/12
- Poster session Thursday May 10 from 2-4pm
  - Distance students need to contact me to set up a skype time ASAP

**Today**

- Automating scheduling with the Farkas lemma (missed a step)
- Start Sparse Polyhedral Framework (SPF) for run-time reordering transformations



## Fourier-Motzkin and Farkas Questions (HW10)

**1D scheduling**

- Specify the data dependence relation with all inequality constraints. The result is a data dependence polyhedron.
- Want the function $\theta_S(\vec{j}) - \theta_R(\vec{i}) - 1$ to be non-negative over the dependence polyhedron
- Use Farkas lemma to set up a new set of constraints

$$\theta_R(\vec{i}) = \vec{v}^T\vec{i} + \vec{b} \qquad \theta_S(\vec{j}) - \theta_R(\vec{i}) - 1 = \lambda_0 + \vec{\lambda}^T(A \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{p} \\ 1 \end{bmatrix})$$
$$\theta_S(\vec{j}) = \vec{w}^T\vec{j} + \vec{c} \qquad \lambda_0 \geq 0 \text{ and } \vec{\lambda} \geq \vec{0}$$

- *Collect coefficients for each term on both sides of equality to create a set of affine equalities involving v, w, b, c, and lambdas*
- Use Fourier-Motzkin to project out all variables except for v, w, b, and c

## The process of determining set of legal schedules

**(1) Change all of the equality constraints in $D_{R\rightarrow S}$ to inequality constraints.**

$$D_{R\rightarrow S} = \{[\vec{i} \rightarrow \vec{j} \mid A' \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{p} \\ 1 \end{bmatrix} \geq \vec{0}\}$$

**(2) Use the Farkas lemma to create a set of constraints for the schedule.**

$$\theta_S(\vec{j}) - \theta_R(\vec{i}) - 1 = \lambda_0 + \vec{\lambda}^T (A \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{p} \\ 1 \end{bmatrix})$$

$$\lambda_0 \geq 0 \text{ and } \vec{\lambda} \geq \vec{0}$$

$$\theta_R(\vec{i}) = \vec{v}^T \vec{i} + \vec{b}$$

$$\theta_S(\vec{j}) = \vec{w}^T \vec{j} + \vec{c}$$

***(3) Collect coefficients for each term to create set of equalities.***

**(4) Solve for v, w, b, and c vector constraints by projecting out lambdas.**



## Example of using the Farkas lemma

**Original code (problem from HW10)**

```
do i = 0,N-1
  do j = 0,N-1
     A(i,j) = A(i-1,j-1)*.05
  enddo
enddo
```

**(1) Dependence polyhedron** $D_{I\rightarrow I} = \{[[i_1, j_1] \rightarrow [i_2, j_2] \mid \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{pmatrix} i_1 \\ j_1 \\ i_1 \\ j_2 \\ N \\ 1 \end{pmatrix} \geq \vec{0}\}$

**(2) Farkas lemma to set up constraints**

**(3)** ***Collect coefficients for each term to create set of equalities***

**(4) Project out lambdas to determine set of legal schedules**

# HW10 problem continued

$$\theta(i_2,j_2)-\theta(i_1,j_1)-1 \;=\; \lambda_0+\vec{\lambda}^T A' \begin{pmatrix} i_1 \\ j_1 \\ i_1 \\ j_2 \\ N \\ 1 \end{pmatrix}$$

$$\text{With } \theta(i,j) \;=\; a*i+b*j+c \qquad\qquad \lambda_x \geq 0,\ \forall 0 \leq x \leq 12$$

## (2) Farkas lemma to set up constraints

$$\text{a i}_2+bj_2+c-ai_1-bj_1-c-1=\lambda_0+\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \\ \lambda_7 \\ \lambda_8 \\ \lambda_9 \\ \lambda_{10} \\ \lambda_{11} \\ \lambda_{12} \end{bmatrix}^T \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 1 \\ -1 & 0 & 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 0 & -1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 & 1 & -1 \end{bmatrix} \begin{pmatrix} i_1 \\ j_1 \\ i_2 \\ j_2 \\ N \\ 1 \end{pmatrix}$$

## (3) *Collect coefficients for each term to create set of equalities*

$$\begin{aligned} -a &= \lambda_1-\lambda_2+\lambda_5-\lambda_9 \\ -b &= \lambda_3-\lambda_4+\lambda_6-\lambda_{10} \\ a &= -\lambda_1+\lambda_2+\lambda_7-\lambda_{11} \\ b &= -\lambda_3+\lambda_4+\lambda_8-\lambda_{12} \\ 0 &= \lambda_9+\lambda_{10}+\lambda_{11}+\lambda_{12} \\ -1 &= \lambda_0+\lambda_1-\lambda_2+\lambda_3-\lambda_4-\lambda_9-\lambda_{10}-\lambda_{11}-\lambda_{12} \\ & \lambda_x \geq 0,\ \forall 0 \leq x \leq 12 \end{aligned}$$

## (4) Project out lambdas to determine set of legal schedules

```
{[a,b,c]: 1 <= a+b}
```

Colorado State University

# Automating Run-Time Reordering Transformations with the Sparse Polyhedral Framework (SPF) and Arbitrary Task Graphs

Michelle Mills Strout

CS560 May 1, 2012

Somewhat modified from Imperial College talk given November 21, 2011

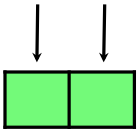


## The Big Picture Problem

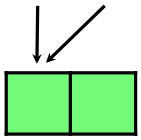
- Data movement is expensive …
  - in terms of execution time
  - in terms of power usage
- Data reordering and/or loop transformations can turn data reuse into data locality
- Research in the polyhedral model has led to significant automation for loop transformations that affect data locality
- However, sparse/irregular computations do not fit in the polyhedral model

# Goal: Turn Data Reuse into Data Locality

- *Spatial locality* occurs when memory locations mapped to the same cache-line are used before the cache line is evicted



- *Temporal locality* occurs when the *same memory location* is reused before its cache line is evicted



Colorado State University



# Run-time Reordering Transformations

Traverses index array →

Generates data reordering function $\sigma$ →

Reorder data and updates index array →

**Inspector**

```
for i=0,7
  ... r[i] ...
for j=0,7
  sigma[j] = ...

for j=0,7
  Z'[sigma[j]]=Z[j]
  r'[j]=sigma[r[j]]
```

**Original Code**

```
for i=0,7
  Y[i] = Z[r[i]]
```

**Executor**

```
for i=0,7
  Y[i] = Z'[r'[i]]
```

## Example Inspector/Executor Strategies

- Gather/scatter parallelization [Saltz et al. 94]
- Cache blocking [Im &Yelick 98]
- Irregular cache blocking [Douglas & Rude 00]
- Full sparse tiling (ICCS 2001)
- Communication avoiding [Demmel et al 08]
- Run-time data and iteration permutation [Chen and Kennedy 99, Mitchell 99, …]
- ***Compositions*** of the above (PLDI 2003)

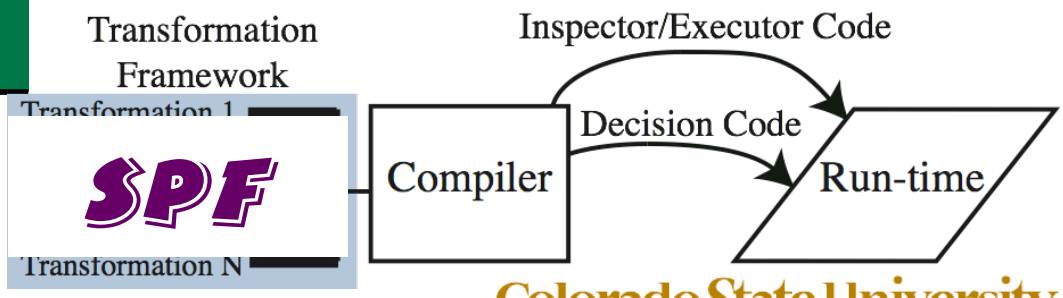


## Inspector/Executor Strategies show great promise **BUT**…

- Only a couple have been automated
- There is library support for some I/E strategies, but specializing the library for the given sparse data structures is non-trivial
- *How can we automate or semi-automate the application of I/E strategies?*

# Run-time Reordering Transformations

- Challenge: unable to effectively reorder data and computation at compile-time in irregular applications
- Approach: run-time reordering transformations
- Vision:






# Sparse Polyhedral Framework (SPF)

- Adds uninterpreted functions to the polyhedral framework
  - Polyhedral model includes affine inequality constraints to represent iteration spaces.
  - SPF adds constraints such as x=f(y), where f is a function and its input domain and output range are polyhedra.
  - [Pugh & Wonnacott 94] used for data dependence analysis. SPF uses to represent transformations.
- Code generation for SPF results in inspector and executor code.

## Run-time Reordering Transformations

Traverses index array →

Generates data reordering function $\sigma$ →

Reorder data and updates index array →

**Inspector**

```
for i=0,7
  ... r[i] ...
for j=0,7
  sigma[j] = ...

for j=0,7
  Z'[sigma[j]]=Z[j]
  r'[j]=sigma[r[j]]
```

**Original Code**

```
for i=0,7
  Y[i] = Z[r[i]]
```

**Executor**

```
for i=0,7
  Y[i] = Z'[r'[i]]
```

9

## Computation Specification in SPF

**Original Code**

```
for i=0,7
  Y[i] = Z[r[i]]
```

- Each data array has a data space

$Y_0 = \{[y] \mid 0 \leq y \leq 7\}$ $Z_0 = \{[z] \mid 0 \leq z \leq 7\}$

- Each index array is represented with and uninterpreted function $r()$

  and has a domain and range

$$\{[v] \rightarrow [w] \mid 0 \leq v, w \leq 7\}$$

Colorado State University

# Computation Specification in SPF cont …

**Original Code**

```
for i=0,7
  Y[i] = Z[r[i]]
```

- Each statement represented with …
  - An iteration space set $S0 = \{[i] \mid 0 \leq i \leq 7\}$
  - Scheduling function $S_{S0 \to I_0} = \{[i] \to [0, i, 0]\}$
  - Access functions for each data array reference

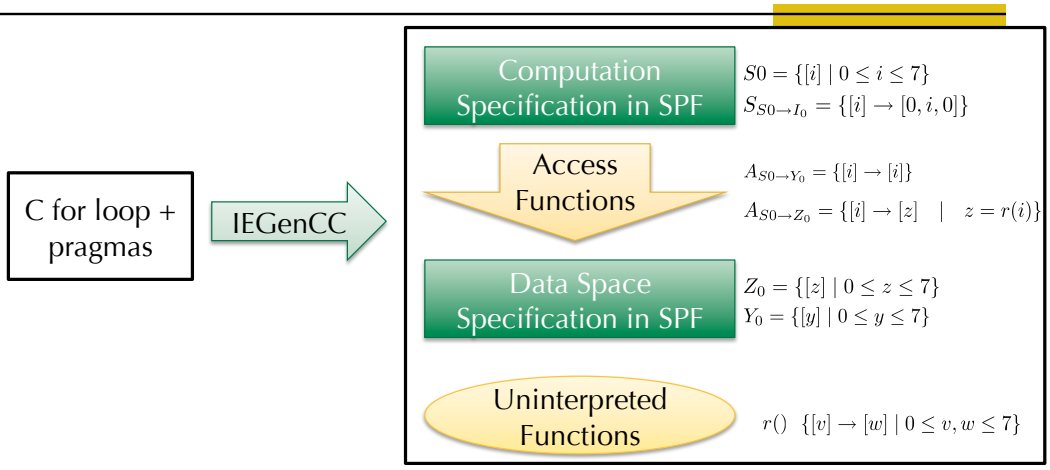
$$A_{S0 \to Y_0} = \{[i] \to [i]\}$$

$$A_{S0 \to Z_0} = \{[i] \to [z] \quad | \quad z = r(i)\}$$

Colorado State University



# Computation Specification in SPF




IEGenCC tool helps generate the SPF specification of algorithm.

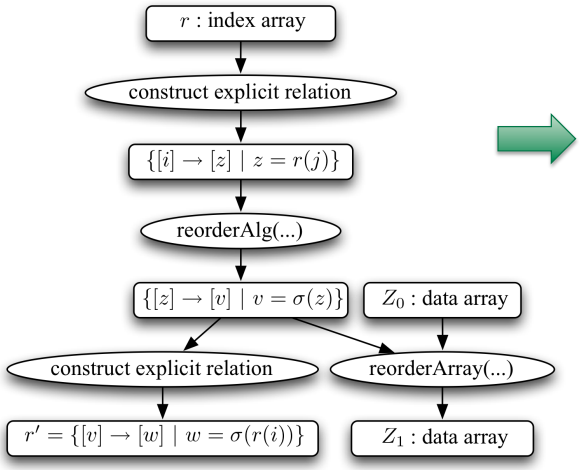
Colorado State University

# Transformation Specification in SPF and Inspector/Executer Generator (IEGen)

**Data Reordering Transformation Relation**

$$R_{Z_0 \to Z_1} = \{[z] \to [z'] \quad | \quad z' = \sigma(z)\}$$

**Inspector Dependence Graph (IDG)**




**Inspector**

```
for i=0,7
  ... r[i] ...
for j=0,7
  sigma[j] = ...

for j=0,7
  Z'[sigma[j]]=Z[j]
  r'[j]=sigma[r[j]]
```



# Computation Specification for Transformed Code / Executor

**Executor**

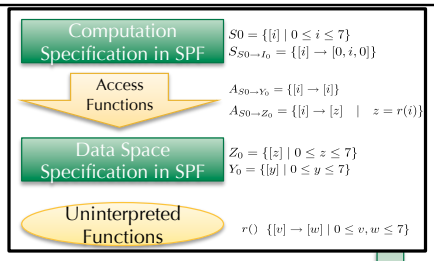
```
for i=0,7
  Y[i] = Z'[r'[i]]
```

- Each statement represented with …
  - An iteration space set $S0 = \{[i] \mid 0 \leq i \leq 7\}$
  - Scheduling function $S_{S0 \to I_0} = \{[i] \to [0, i, 0]\}$
  - Access functions for each array reference

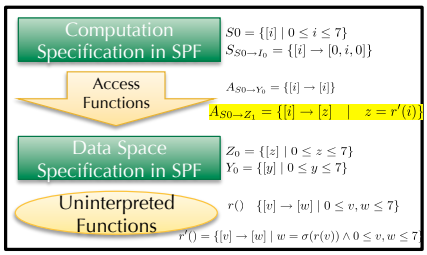
$$A_{S0 \to Y_0} = \{[i] \to [i]\}$$

$$A_{S0 \to Z_1} = \{[i] \to [z'] \quad | \quad z' = \sigma(r(i)) = r'(i)\}$$

# Implementation Details: Transformations on Computation and Data Spaces




Reordering transformations modify the computation specification




Data Reordering followed by Pointer Update

$$R_{Z_0 \to Z_1} = \{[z] \to [z'] \mid z' = \sigma(z)\}$$

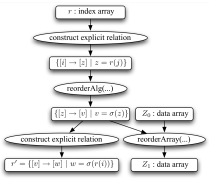
$$r'() = \{[v] \to [w] \mid w = \sigma(r(v)) \wedge 0 \le v, w \le 7\}$$

Colorado State University

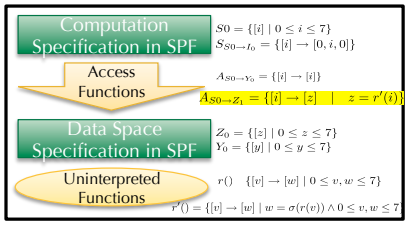


# Inspector and Executor Code Gen

**Inspector Dependence Graph (IDG)**




**MapIR**




IEGen

**Inspector**

```
for i=0,7
  ... r[i] ...
for j=0,7
  sigma[j] = ...

for j=0,7
  Z'[sigma[j]]=Z[j]
  r'[j]=sigma[r[j]]
```

**Executor**

```
for i=0,7
  Y[i] = Z'[r'[i]]
```

- Inspector generated from Inspector Dependence Graph created during transformation process
- Executor generated from transformed algorithm specification

Colorado State University

## Computation and Transformation Specification in SPF

- Data and index array specifications
- Each statement represented with …
  - An iteration space set
  - A **schedule mapping** to full iteration space
  - **Access functions** for each data array reference
- **Data dependences relations** between iterations in full iteration space
- **Transformation specification** is a sequence of data and iteration reorderings represented as integer tuple **relations**



## Key Insights in SPF and IEGen

- The inspectors ***traverse*** the access relations and/or the data dependences
- We can ***express*** how the access relations and data dependences will change
- Subsequent inspectors ***traverse the new*** data mappings and data dependences
- Use polyhedral code generator (Cloog) for outer loops and deal with sparsity in inner loops and access relations

# Another Example (MOLDYN)

```
for s=1,T
  for i=1,n
    ... = ...Z[i]
  endfor

  for j=1,m
    Z[l[j]] = ...
    Z[r[j]] = ...
  endfor

  for k=1,n
    Z[k] += ...
  endfor
endfor
```

Access Relation for i loop

$$A_{I_0 \to Z_0} = \{[i] \to [i]\}$$

Access Relation for j loop

$$A_{J_0 \to Z_0} = \{[j] \to [i] | i = l(j) \vee i = r(j)\}$$

Data Dependences between i and j loop

$$D_{I_0 \to J_0} = \{[i] \to [j] | (i = l(j)) \vee (i = r(j))\}$$

Colorado State University 19

# Data Permutation Reordering

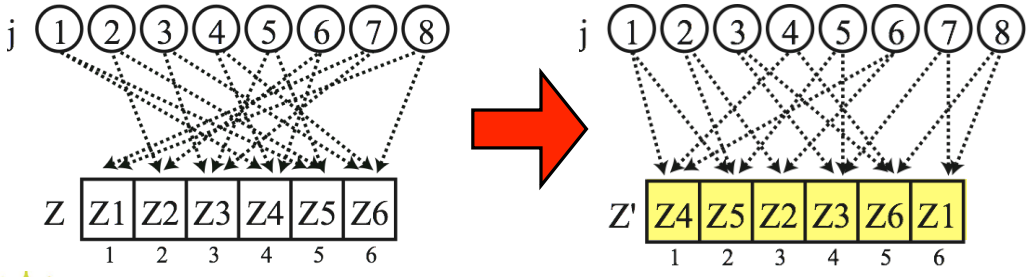
(Equations are the compile-time abstraction)

$$R_{Z_0 \to Z_1} = T_{I_0 \to I_1} = \{[i] \to [\sigma(i)]\}$$

CPACK reordering heuristic [Ding & Kennedy 99]

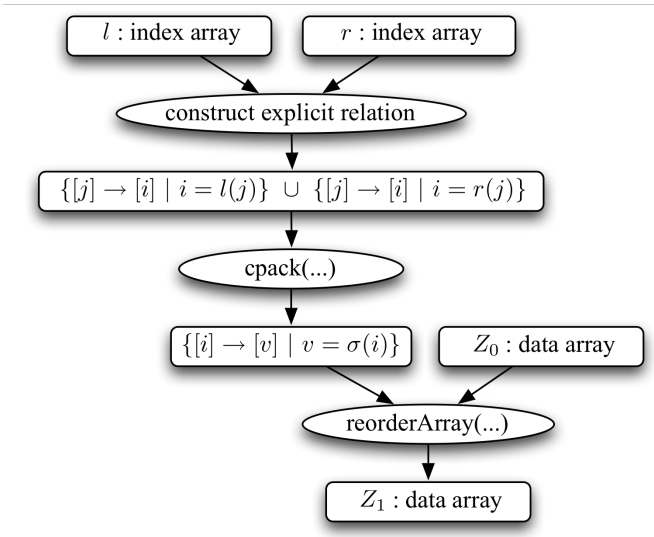
$$A_{J_0 \to Z_0} = \{[j] \to [i] | i = l(j) \vee i = r(j)\}$$

$$A_{J_0 \to Z_1} = \{[j] \to [i] \mid i = \sigma(l(j)) \vee i = \sigma(r(j))\}$$




Colorado State University 20

# Effect of Data Reordering on Inspector Dependence Graph (IDG)



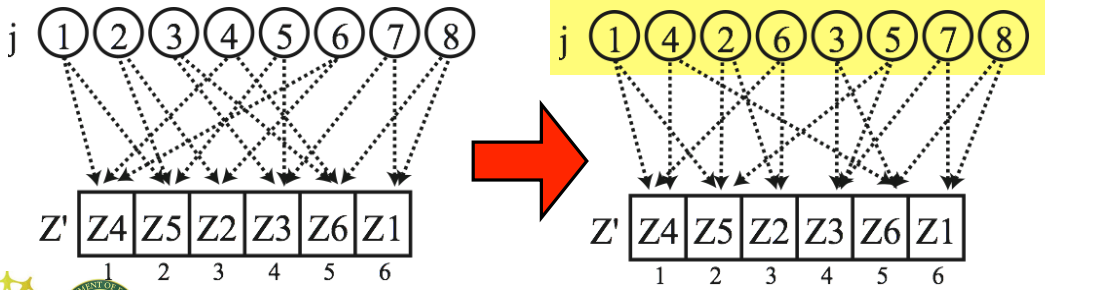



# Iteration Permutation Reordering

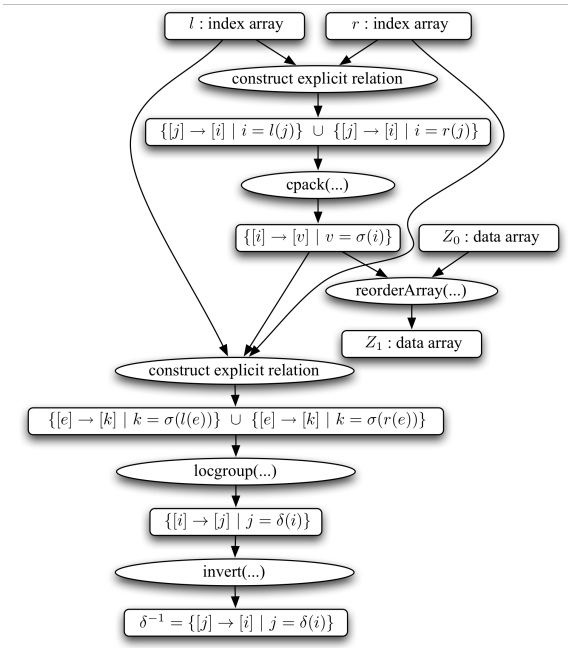
$$T_{J_0 \rightarrow J_1} = \{[j] \rightarrow [x] \quad | \quad x = \delta(j)\}$$

$$A_{J_0 \rightarrow Z_1} = \{[j] \rightarrow [i] \quad | \quad i = \sigma(l(j)) \vee i = \sigma(r(j))\}$$

$$A_{J_1 \rightarrow Z_1} = \{[j] \rightarrow [i] \quad | \quad i = \sigma(l(\delta^{-1}(j))) \vee i = \sigma(r(\delta^{-1}(j)))\}$$




# IDG After Iteration Permutation

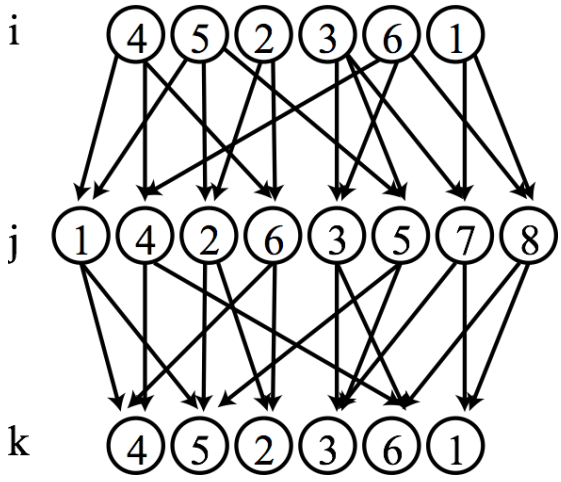





# Dependences Between Loops after other transformations

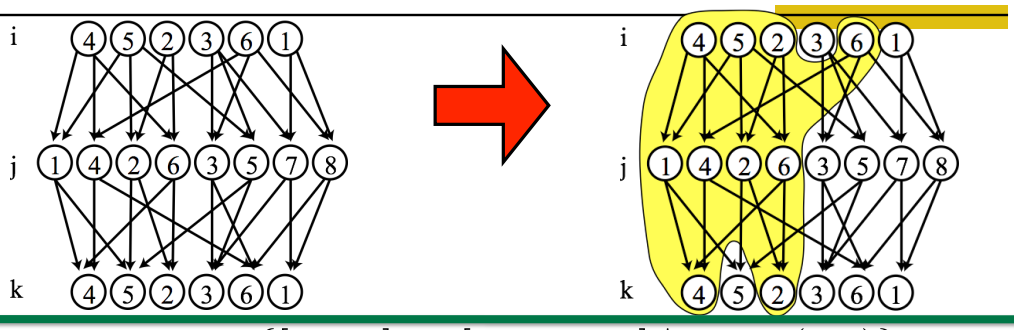
```
for s=1,T
  for i=1,n
    ... = ...Z[i]
  endfor

  for j=1,m
    Z[l[j]] = ...
    Z[r[j]] = ...
  endfor

  for k=1,n
    ... += Z[k]
  endfor
endfor
```

$$D_{I_1 \rightarrow J_1} = \{[0,i] \rightarrow [1,j] \quad | \quad i = \sigma(l(\delta^{-1}(j))) \vee i = \sigma(r(\delta^{-1}(j)))\}$$




Colorado State University

# Full Sparse Tiling (FST)




$$T_{F_1 \to F_2} = \{[s,0,i] \to [s,0,t,0,i] \mid t = \Theta(0,i)\}$$
$$\cup \{[s,1,j] \to [s,0,t,1,j] \mid t = \Theta(1,j)\} \cdots$$

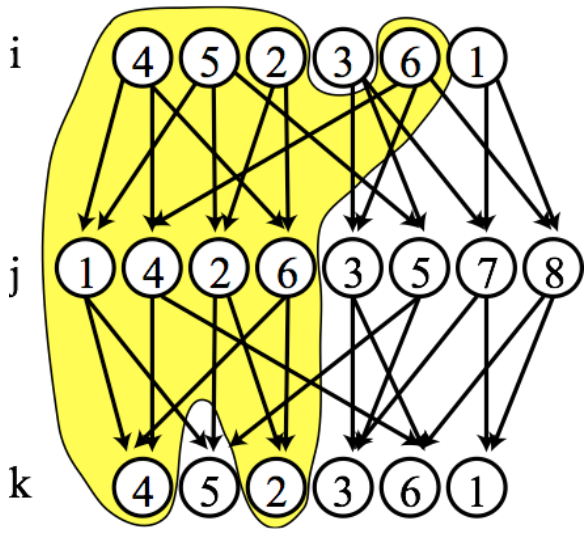
$$F_1 = \{[s,0,i]\} \cup \{[s,1,j]\} \cup \{[s,2,k]\}$$

$$F_2 = \{[s,0,t,0,i] \mid t = \Theta(0,i)\} \cup \{[s,0,t,1,j] \mid t = \Theta(1,j)\} \cdots$$

Colorado State University 25

# Executor After Full Sparse Tiling

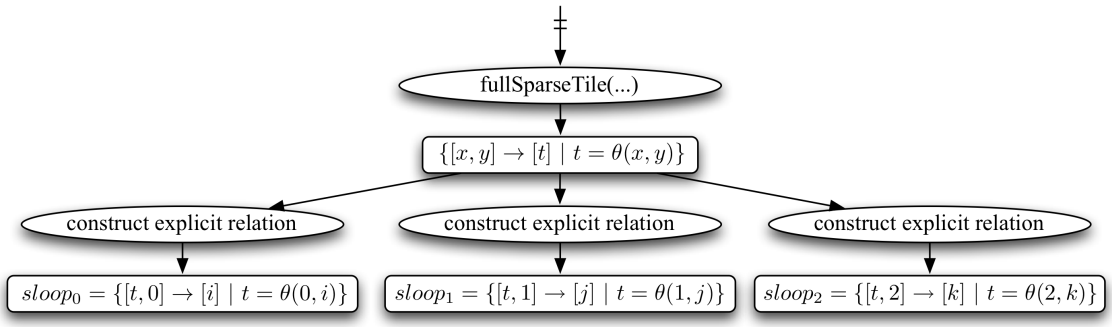
```
for s=1,T
 for t=0,nt
  for i in sloop0(t)
    ... = ...Z[i]
  endfor

  for j in sloop1(t)
    Z[l''[j]] = ...
    Z[r''[j]] = ...
  endfor

  for k in sloop2(t)
    ... += Z[k]
  endfor
 endfor
endfor
```




Colorado State University 26

# Handling Those Inner Sparse Loop

**Inspector Dependence Graph (IDG)**

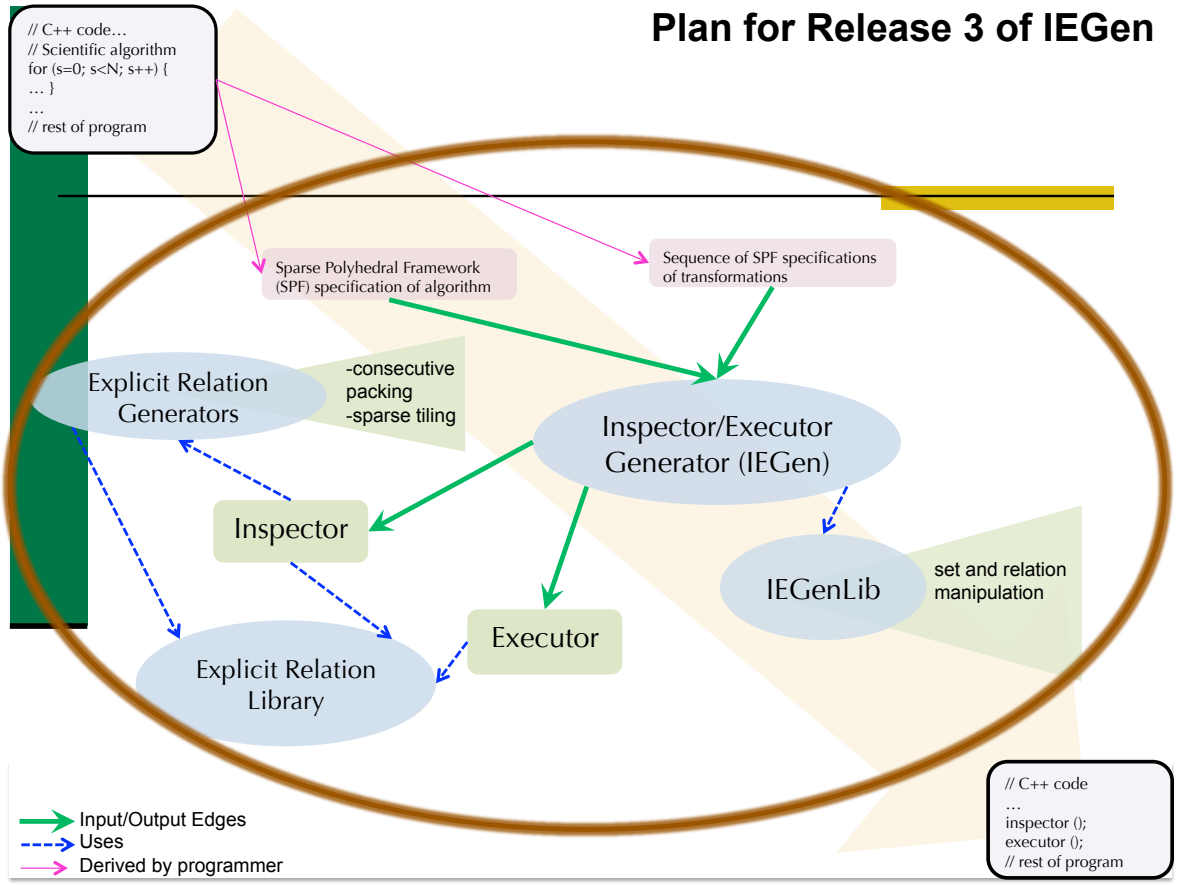





# Inspector/Executor Generator (IEGen) Summary

- The original computation is specified with sets, scheduling functions, access functions, and dependences
- Transformations are specified in terms of a transformation relations
- Uninterpreted functions are associated with input domains, output ranges, run-time routines that will generate them, and symbolic relations that represent input to those routines
- IEGen builds a MapIR to represent the executor and an Inspector Dependence Graph (IDG) to represent the inspector
- After all transformations have been applied at compile time the inspector and executor functions are generated

## Plan for Release 3 of IEGen

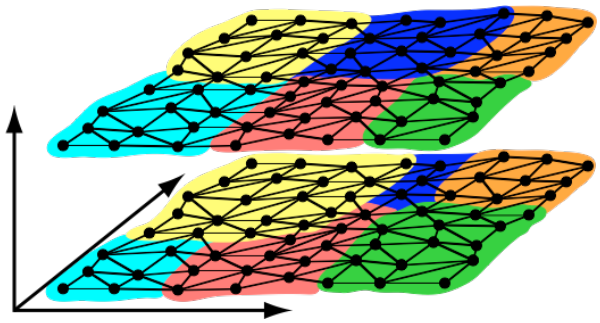



# Putting All the Pieces Together

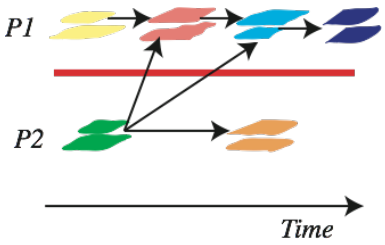
- Subcomputations with indirect memory accesses will identified with pragmas or an IDE
- Sparse Polyhedral Framework (SPF) enables the specification of computations and run-time reordering transformations
- Transformation writers will provide run-time libraries to compute sparse tilings, etc.
- The Inspector/Executor Generator (IEGen) will generate the inspector and executor that implement a specified sequence of transformations

# Parallelization using Inspector/Executor Strategies


Break computation that sweeps over mesh/sparse matrix into chunks/sparse tiles



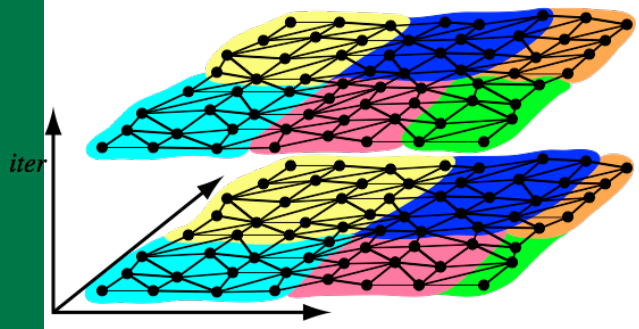
Full Sparse Tiled Iteration Space

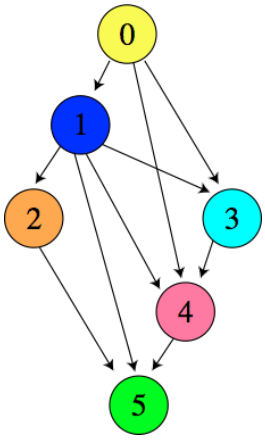



Task Graph



# Parallelism in Full Sparse Tiled Jacobi

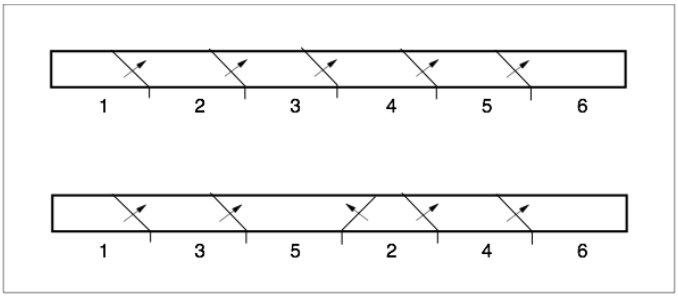



Average parallelism = (# tiles) / (# tiles in critical path)
= 6 / 5 = 1.2

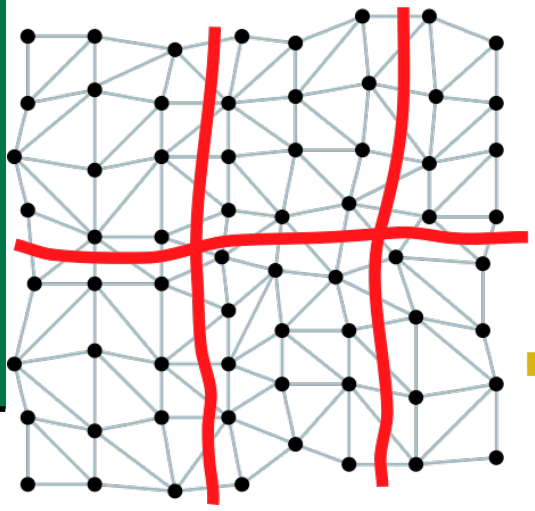
## How can we increase parallelism between tiles?

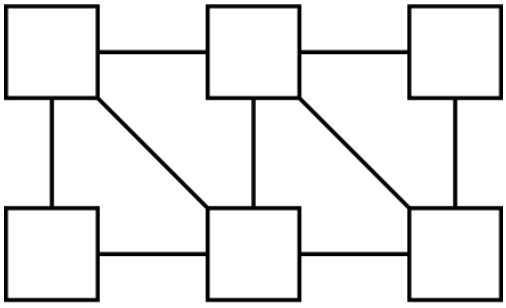
- Order that tile growth is performed matters
- Best is to first grow tiles whose seed partitions are not adjacent




Colorado State University

## Improving Average Parallelism Using Coloring

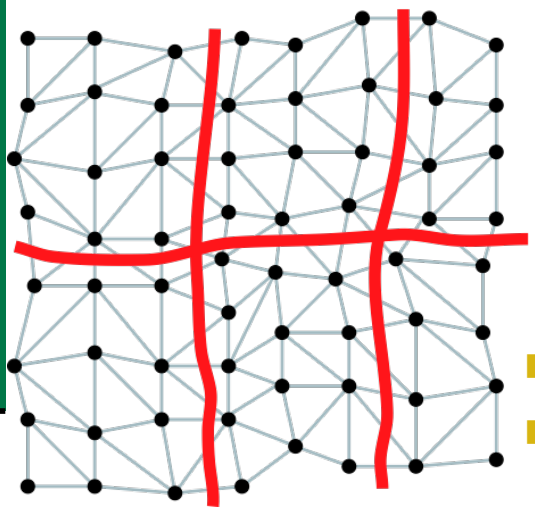


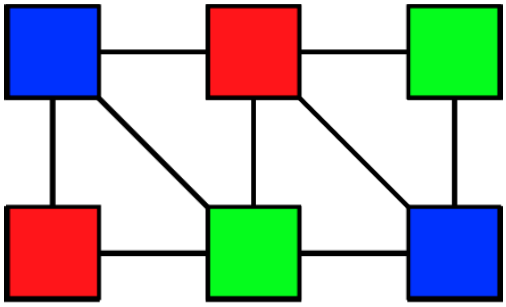


- Create a partition graph

Colorado State University

# Improving Average Parallelism Using Coloring

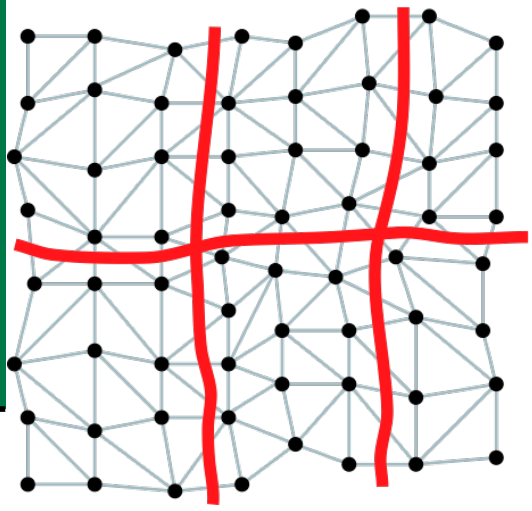


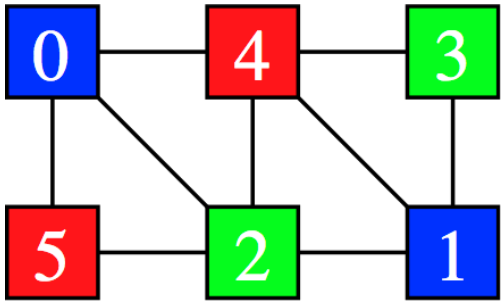


- Create a partition graph
- Color the partition graph

Colorado State University

# Improving Average Parallelism Using Coloring

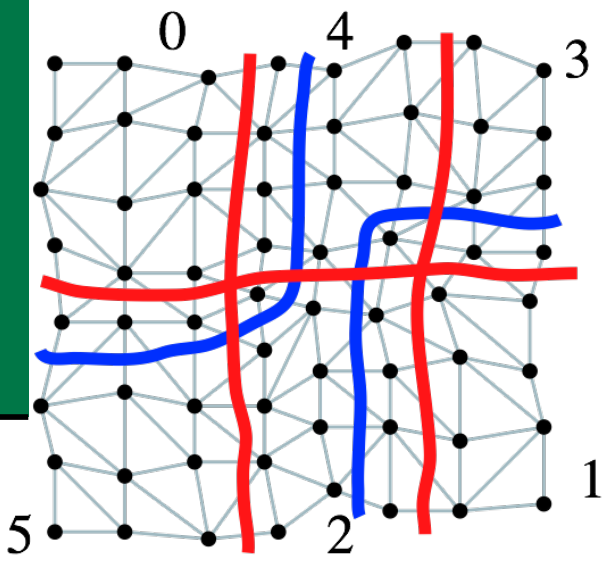





- Create a partition graph
- Color the partition graph
- Renumber partitions consecutively by color

Colorado State University

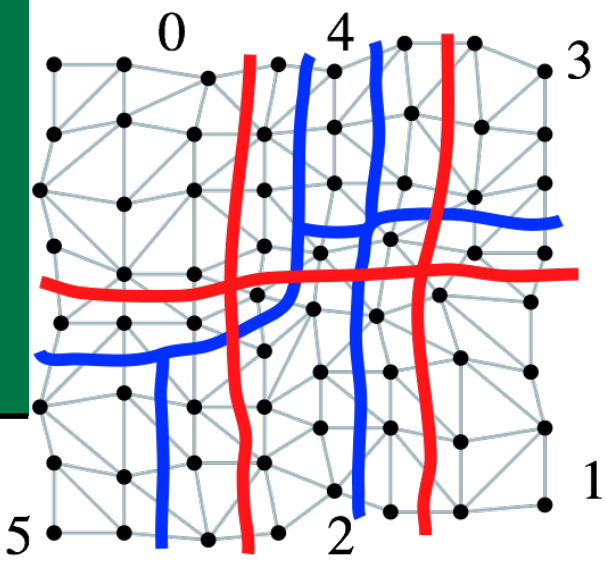
## Grow Using New Partition Order




- Renumber the seed partition cells based on coloring
- Grow tiles using new ordering
- Notice that tiles 0 and 1 may be executed in parallel

Colorado State University

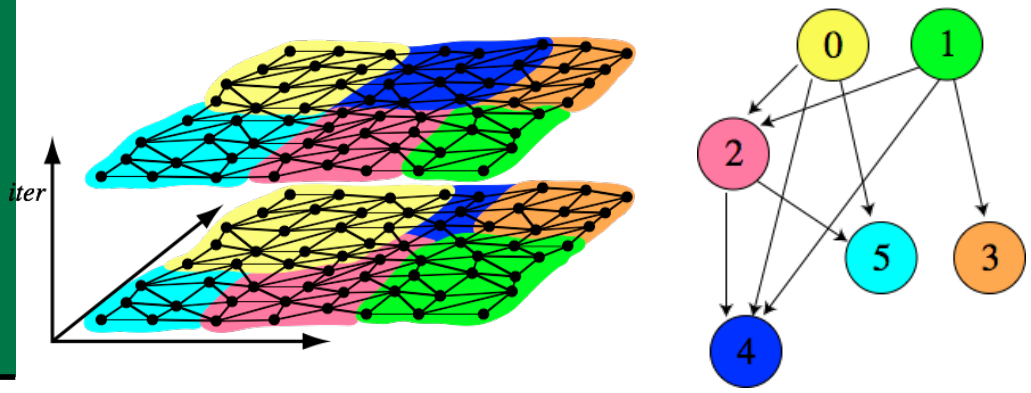
## Re-grow Using New Partition Order




- Renumber the seed partition cells based on coloring
- Grow tiles using new ordering
- Notice that tiles 0 and 1 may be executed in parallel
- Tiles 4 and 5 may also be executed in parallel

Colorado State University

# Average Parallelism is Improved



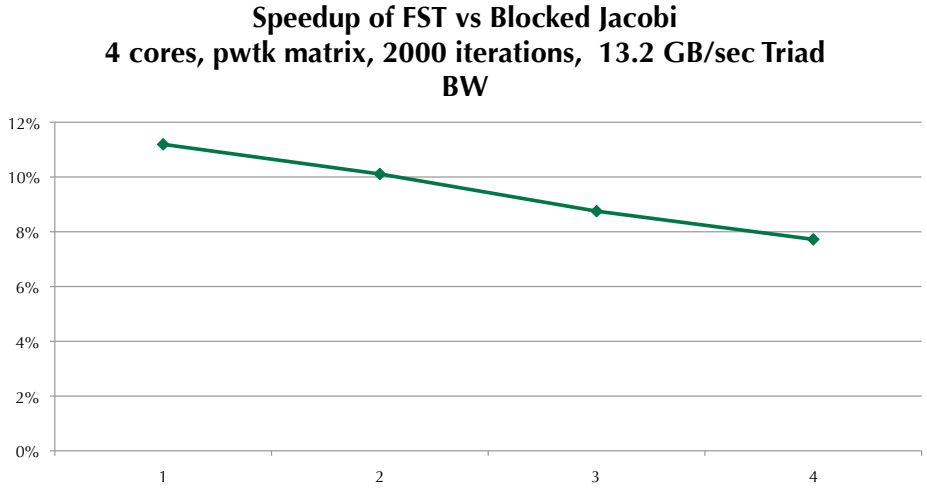

Average parallelism = (# tiles) / (# tiles in critical path)
= 6 / 3 = 2

Colorado State University

# Performance Evaluation

- Comparing OpenMP with blocking and dynamic scheduling versus full sparse tiling
  - OpenMP is simpler to express in original code, less overhead
  - FST has better temporal locality and asynchronous parallelism
- Comparing various programming models for specification and execution of arbitrary task graphs
  - TBB – Threading Building Blocks
  - Pthreads
  - OpenMP Tasks and OpenMP frontiers in task graph
  - CnC – Concurrent Collections

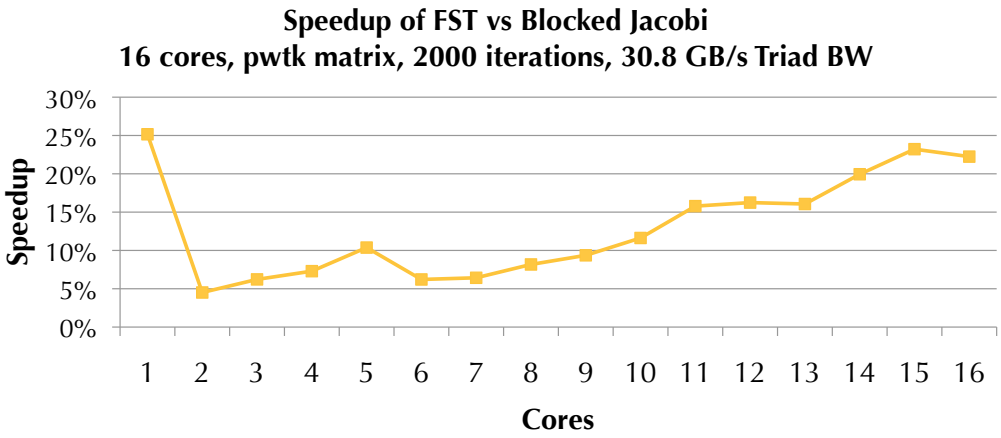
# Full Sparse Tiling Helps When Have Lower Available Bandwidth




Colorado State University

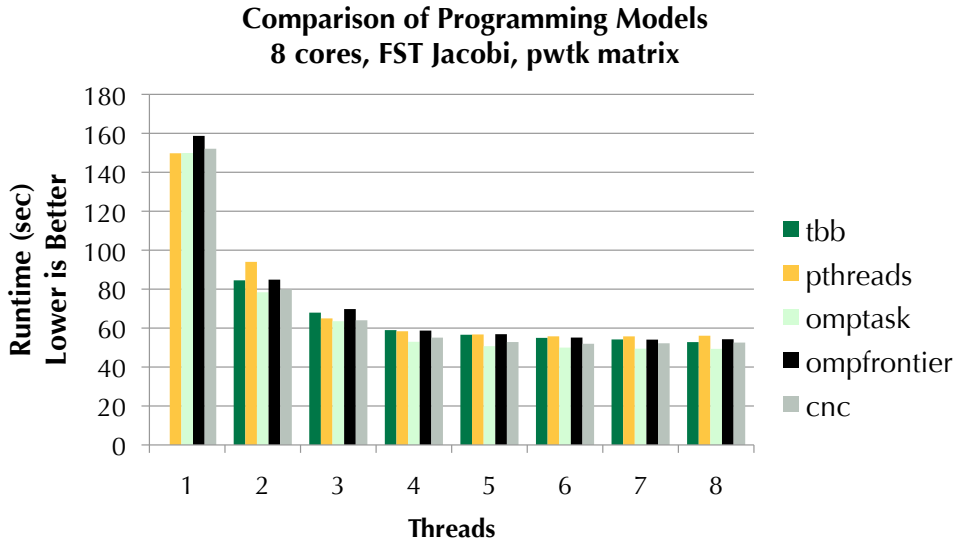


# Full Sparse Tiling Helps When Have Lower Available Bandwidth




Colorado State University

# Expressing Arbitrary Graphs in Various Programming Models






# Conclusions

- Sparse Polyhedral Framework (SPF) provides abstractions needed to automate performance transformation of irregular/sparse apps
- Inspector/executor code generator (IEGen) will provide an approach for semi-automating the application of inspector/executor strategies
- Sparse tiling is an inspector/executor strategy that …
  - turns data reuse into data locality
  - results in arbitrary task graphs at runtime
  - enables putting off the point at which bandwidth bound computations quit scaling

# Contributors

- Original SPF concept developed in collaboration with Larry Carter and Jeanne Ferrante
- Chris Krieger – Task graph programming model and possibly composition of prog models
- Geri George – Project planning and management
- Alum: Alan LaMielle – IEGen prototype
- Alum: Jon Roelofs – IEGenCC tool prototype