Example from April 5th

4/12/12


```
    for (i=0; i<N; i++){
R:        X[i]= ...
    }

    for (j=0; j<N; j++) {
S:        .... X[j-1] ...
    }.
```

$[1,j,0] \rightarrow [0,j,1]$

Data Dependence Relation

$$D_{I_R \rightarrow I_S} = \{ [0,i,0] \rightarrow [1,j,0] \mid (j) = [1](i) + (1)$$

$$\wedge \begin{array}{c} M \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -1 \\ 0 & -1 & 1 & -1 \end{bmatrix} \end{array} \begin{pmatrix} i \\ j \\ N \\ 1 \end{pmatrix} \geq 0 \}$$

"one matrix format"

$$D_{I_R \rightarrow I_S} = \{ [0,i,0] \rightarrow [1,j,0] \mid \begin{bmatrix} 1 & -1 & 0 & +1 \\ -1 & 1 & 0 & -1 \\ & & & \\ & M & & \\ & & & \end{bmatrix} \begin{pmatrix} i \\ j \\ N \\ 1 \end{pmatrix} \geq 0$$

$j = i + 1$

$\boxed{j - i - 1 = 0}$

$j - i - 1 \geq 0$

$-j + i + 1 \geq 0$

Transform the data dependence by hand

$$D_{I_R \to I_S} = \{ [0,i,0] \to [1,j,0] \mid j = i+1 \wedge \begin{bmatrix} \quad \end{bmatrix} \begin{pmatrix} i \\ j \\ N \\ 1 \end{pmatrix} \geq 0 \}$$

$$T_{I_S \to I_S'} = \{ [1,q,0] \to [0,w,1] \mid w = q \}$$

$$T_{I_R' \to I_R} = \{ \underline{[0,t,0]} \to [0,v,0] \mid v = t \}$$

$$D_{I_R' \to I_S'} = \underline{T_{I_S \to I_S'} \circ (\underline{D_{I_R \to I_S} \circ T_{I_R' \to I_R}})}$$

$$D_{I_R \to I_S} \circ T_{I_R' \to I_R} = \{ [0,t,0] \to [1,j,0] \mid v = i \wedge j = i+1 \wedge v = t \wedge \begin{bmatrix} \quad \end{bmatrix} \begin{pmatrix} i \to t \\ j \\ N \\ 1 \end{pmatrix} \geq 0 \}$$

due to composition

$$\begin{matrix} v = i \\ j = i+1 \\ v = t \end{matrix} \Rightarrow \begin{matrix} t = i \\ j = i+1 \end{matrix} \Rightarrow \boxed{j = t+1}$$

$$T_{I_S \to I_S'} \circ (D_{I_R \to I_S} \circ T_{I_R' \to I_R}) = \{ [0,t,0] \to [0,w,1] \mid q = j,\ w = q,\ j = t+1 \wedge \begin{bmatrix} \quad \end{bmatrix} \begin{pmatrix} t \\ j \to w \\ N \\ 1 \end{pmatrix} \geq 0$$

$$\begin{matrix} q = j \\ w = q \\ j = t+1 \end{matrix} \Rightarrow \boxed{w = t+1}$$

$$D_{I_R' \to I_S'} = \{ [0,i,0] \to [0,j,1] \mid j = i+1 \wedge \begin{bmatrix} \quad \end{bmatrix} \begin{pmatrix} i \\ j \\ N \\ 1 \end{pmatrix} \geq 0 \}$$

Need $D_{I'_R \to I'_S} = T_{I_S \to I'_S} \circ D_{I_R \to I_S} \circ T_{I'_R \to I_R}$

$T_{I'_R \to I_R} = \{[0,i,0] \to [0,v,0] \mid (v) = [1](i) + (0)\}$

$T_{I_S \to I'_S} = \{[1,j,0] \to [0,w,1] \mid (w) = [1](j) + (0)\}$

General

$r1 = \{\vec{y} \to \vec{z} \mid \vec{z} = A\vec{y} + \vec{k}$

$\wedge\ M\begin{pmatrix}\vec{y}\\ \vec{z}\\ \vec{p}\\ 1\end{pmatrix} \geq 0\}$

$r2 = \{\vec{x} \to \vec{y} \mid \vec{y} = B\vec{x} + \vec{c}$

$\wedge\ W\begin{pmatrix}\vec{x}\\ \vec{y}\\ \vec{q}\\ 1\end{pmatrix} \geq 0\}$

Specific

$A = [1] \quad k = (1)$

$M = [\quad]$

$B = [1] \quad c = (0)$

$W = [0]$

$r = r1 \circ r2$

$= \{\vec{x} \to \vec{z} \mid \begin{bmatrix} AB & -I & (A\vec{c} + \vec{k}) \end{bmatrix}\begin{pmatrix}\vec{x}\\ \vec{z}\\ 1\end{pmatrix} = 0$

$\wedge\ M\begin{pmatrix}\vec{y}\\ \vec{z}\\ \vec{p}\\ 1\end{pmatrix} \geq 0$ with $\vec{y} = B\vec{x} + \vec{c}$

$M\begin{pmatrix}B\vec{x} + \vec{c}\\ \vec{z}\\ \vec{p}\\ 1\end{pmatrix}$

$\wedge\ W\begin{pmatrix}\vec{x}\\ \vec{y}\\ \vec{q}\\ 1\end{pmatrix} \geq 0$ with $\vec{y} = B\vec{x} + \vec{c}$

Performing a Substitution

$$M\begin{pmatrix}\vec{y}\\ \vec{z}\\ \vec{p}\\ 1\end{pmatrix} \geq 0 \qquad \vec{y} = B\vec{x} + \vec{c}$$

$$\begin{bmatrix} M_y & M_z & M_p & M_1 \end{bmatrix}\begin{pmatrix}\vec{y}\\ \vec{z}\\ \vec{p}\\ 1\end{pmatrix} \geq 0$$

$$M_y\vec{y} + M_z\vec{z} + M_p\vec{p} + M_1 \geq 0$$ substitute for $\vec{y}$

$$M_y B\vec{x} + M_z\vec{z} + M_p\vec{p} + (M_y\vec{c} + M_1) \geq 0$$

$$\begin{bmatrix} (M_y B) & M_z & M_p & (M_y\vec{c} + M_1) \end{bmatrix}\begin{pmatrix}\vec{x}\\ \vec{z}\\ \vec{p}\\ 1\end{pmatrix} \geq 0$$

$$W\begin{pmatrix}\vec{x}\\ \vec{y}\\ \vec{q}\\ 1\end{pmatrix} \geq 0 \qquad \vec{y} = B\vec{x} + \vec{c}$$

$$W_x\vec{x} + W_y\vec{y} + W_q\vec{q} + W_1 \geq 0$$ substitute for $\vec{y}$

$$W_x\vec{x} + W_y(B\vec{x} + \vec{c}) + W_q\vec{q} + W_1 \geq 0$$

$$\begin{bmatrix} (W_x + W_y B) & W_q & (W_y\vec{c} + W_1) \end{bmatrix}\begin{pmatrix}\vec{x}\\ \vec{q}\\ 1\end{pmatrix} \geq 0$$