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```
for (i=0; i<N; i++) {
R:    X[i] = ...
}
for (j=0; j<N; j++) {
S:    ... X[j-1] ...
```

Goal: apply fusion

$\boxed{\text{Iteration Space for } R \,\&\, S}$

$-i+N \geq 0$

$N \geq i$

$$I_R = \{[0,i,0] \mid 0 \leq i < N\}$$

$$= \{[0,i,0] \mid \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} \begin{pmatrix} i \\ N \\ 1 \end{pmatrix} \geq 0 \}$$

$$I_S = \{[1,j,0] \mid \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} \begin{pmatrix} j \\ N \\ 1 \end{pmatrix} \geq 0 \}$$

$\boxed{\text{Data Dependence between } R \,\&\, S}$

$$D_{I_R \to I_S} = \{[\underline{0},i,0] \to [\underline{1},j,0] \mid 0 \leq i,j < N \wedge i = j-1 \wedge \text{prec already sat}\}$$

$\boxed{\text{Access Functions}}$

$$F_{I_R \to X} = \{[0,i,0] \to [i]\}$$

$$= \{[0,i,0] \to [v] \mid \begin{bmatrix} 1 & -1 & 0 & 0 \end{bmatrix} \begin{pmatrix} i \\ v \\ N \\ 1 \end{pmatrix} = 0$$

$$F_{I_S \to X} = \{[1,j,0] \to [\overset{j-1}{v}] \mid v = j-1\}$$

Fusion Transformation

$$T_{I_R \to I'_R} = \{[0, i, 0] \to [0, i, 0]\}$$

$$T_{I_S \to I'_S} = \{[1, j, 0] \to [0, j, 1]\}$$

Transform the Iteration Space

$$I'_R = T_{I_R \to I'_R}(I_R)$$

$$\vec{x} = A^{-1}y - \vec{k}$$

Recall for $S = r1(S2)$ with $r1 = \{\vec{x} \to \vec{y} \mid \vec{y} = A\vec{x} + \vec{k}\}$

$$S2 = \{\vec{x} \mid Q\vec{x} \geq (\vec{q} + B\vec{p})\}$$

$$S = \{\vec{y} \mid QA^{-1}\vec{y} - Q\vec{k}) \geq (\vec{q} + B\vec{p})\}$$

$$S = \{\vec{y} \mid \begin{bmatrix} QA^{-1} & -B & -(Q\vec{k} + \vec{q}) \end{bmatrix} \begin{pmatrix} \vec{y} \\ \vec{p} \\ 1 \end{pmatrix} \geq 0\}$$

$$S2 = I_R = \{[0, i, 0] \mid \begin{array}{ccc} Q & -B & -\vec{q} \\ \end{array} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} \begin{pmatrix} i \\ N \\ 1 \end{pmatrix} \geq 0\}$$

$$r1 = T_{I_R \to I'_R} = \{[0, i, 0] \to [0, v, 0] \mid (v) = \overset{A}{[1]}(i) + \overset{K}{(0)}$$

$$T_{I_R \to I'_R}(I_R) = \{[0, v, 0] \mid \begin{array}{ccc} QA^{-1} & -B & -(Q\vec{k} + \vec{q}) \end{array} \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & -1 \end{bmatrix} \begin{pmatrix} v \\ N \\ 1 \end{pmatrix}$$

Transform the Data Dependences

$$D_{I_R \to I_s} = \{ [0, i, 0] \to [1, j, 0] \mid (j) = \underset{A}{[1]}(i) + \underset{\vec{k}}{(1)}$$

$$\wedge \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & -1 \\ 0 & -1 & 1 & -1 \end{bmatrix} \begin{pmatrix} i \\ j \\ N \\ 1 \end{pmatrix} \geq 0$$

$$D_{I'_R \to I'_s} = T_{I_s \to I'_s} \circ D_{I_R \to I_s} \circ T_{I'_R \to I_R}$$

Recall $r = r1 \circ r2$ with $r1 = \{ \vec{y} \to \vec{z} \mid \vec{z} = A\vec{y} + k \wedge M \begin{pmatrix} \vec{y} \\ \vec{z} \\ \vec{p} \\ 1 \end{pmatrix} \geq 0 \}$

$r2 = \{ \vec{x} \to \vec{y} \mid \vec{y} = B\vec{x} + \vec{c} \wedge W \begin{pmatrix} \vec{x} \\ \vec{y} \\ \vec{q} \\ 1 \end{pmatrix} \geq 0 \}$

$$r = \{ \vec{x} \to \vec{z} \mid \exists \vec{y} : \vec{y} = B\vec{x} + \vec{c} \wedge \vec{z} = A\vec{y} + \vec{k} \wedge \text{other constraints} \}$$

$$\mid \vec{z} = AB\vec{x} + A\vec{c} + \vec{k} \wedge \text{other}[\vec{y} \setminus B\vec{x} + \vec{c}] \}$$

$$= \{ \vec{x} \to \vec{z} \mid [AB \quad -1 \quad (A\vec{c} + \vec{k})] \begin{pmatrix} \vec{x} \\ \vec{z} \\ 1 \end{pmatrix} = 0 \}$$

$$[\quad](\quad) \geq 0$$

$$D_{I_R \to I'_R} \circ T_{I'_R \to I_R} \qquad T_{I'_R \to I_R} = \{ [0, v, 0] \to [0, i, 0] \mid (i) = \underset{B}{[1]}(v) + \underset{\vec{c}}{(0)} \}$$

$$= \{ [0, v, 0] \to [1, j, 0] \mid \begin{array}{ccc} AB & & (A\vec{c} + \vec{k}) \\ \end{array} \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{pmatrix} v \\ j \\ 1 \end{pmatrix} = 0$$

$$v = j - 1$$

$$D_{I'_R \to I'_s} = \{ [0, i, 0] \to [0, j, 1] \mid i = j - 1 \}$$