Summary of Chap 4 - Regular Properties

September 29, 2011

Automata on Finite Words

- Deterministic and Non-deterministic Finite Automata (DFA, NFA)

$\Rightarrow$ A word ω is *recognized by* a NFA (or a DFA) if it admits (at least) one run which ends on an acceptance state.

- The *language recognized by* $\mathcal{A}$, $\mathcal{L}(\mathcal{A})$, is the set of recognized words.

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Theorem (Equivalence DFA and NFA)

The sets of language recognized by DFA and NFA are the same. Moreover, it is possible to transform a NFA into a DFA which recognizes the same language.

- This set of languages is called *regular languages* (and correspond to: $\mathcal{L} = a \in \Sigma \mid \mathcal{L}_1.\mathcal{L}_2 \mid \mathcal{L}_1 + \mathcal{L}_2 \mid \mathcal{L}^*$)

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- $\mathcal{L}$ is *ω-regular language* iff $\mathcal{L} = \mathcal{L}_\omega(E_1.F_1^\omega + \cdots + E_n.F_n^\omega)$
- A *ω-regular property* P over AP is a LT property over AP, such that P is a ω-regular language over 2^{AP}.

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- Non-deterministic Büchi Automata (NBA)

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- Variants: Deterministic/Generalized Büchi Automata (DBA/GNBA).

Theorem (Equivalences)

- *The languages recognized by NBA and GNBA are ω-regular languages.*
- *DBA are less expressive than NBA. (cf: $(A + B)^* B^\omega$).*

Model checking 1

- **Regular safety properties:**
  - Does TS verifies the regular property P?
  - We build $\mathcal{A}$ NFA recognizing the bad prefixes of P.

  $\rightarrow$ Then, we build $(TS \otimes \mathcal{A})$:
  $TS \models P \Leftrightarrow trace_{fin}(TS) \cap \mathcal{L}(\mathcal{A}) = \emptyset \Leftrightarrow TS \otimes \mathcal{A} \models P_{inv(\mathcal{A})}$
  (with $P_{inv(\mathcal{A})} = \neg F$ an invariant property).

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  (with $P_{inv(\mathcal{A})} = \neg F$ an invariant property).

- **How to check that $TS \models P_{inv(\Phi)}$?**

  $\Leftrightarrow$ Does it exist a reachable $s \in TS$ which violates Φ?
  - We can check this by using a DFS algorithm on the graph TS.

Model checking 2

- ω-**regular safety properties:**
  - Does TS verifies the ω-regular property P?
  - We build $\mathcal{A}$ NBA with $\mathcal{L}(\mathcal{A}) = \bar{P}$

  $\rightarrow$ Then, we build $(TS \otimes \mathcal{A})$:
  $TS \models P \Leftrightarrow trace(TS) \cap \mathcal{L}_\omega(\mathcal{A}) = \emptyset \Leftrightarrow TS \otimes \mathcal{A} \models P_{pers(\mathcal{A})}$
  (with $P_{pers(\mathcal{A})}$ = "eventually forever $\neg F$", a persistence property).

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- **ω-regular safety properties:**
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  (with $P_{pers(\mathcal{A})}$ = "eventually forever $\neg F$", a persistence property).

- **How to check that $TS \models P_{pers(\Phi)}$?**

  $\Leftrightarrow$ Does it exist a reachable $s \in TS$:
    - which violates Φ,
    - and which is inside a cycle of TS ?
  - We check this by using a combination of DFS algorithm on TS.