Homework 4 Solution

March 4, 2009

Question 1-7 and 9: 10 points each. Question 8 a) and b): 5 points each. Question 8 c) and d): 10 points each.

1. (a) YES. $X^* = XYWZ$
   (b) No. $(XW)^* = XW$, Z is not in there.

2. $r$ satisfies $AD \rightarrow B$, $C \rightarrow DE$, $CD \rightarrow A$, $AE \rightarrow B$. But $r$ does not satisfy $A \rightarrow B$ since the first two tuples of $r$ have the same values for $A$, but different values for $B$.

3. (a) $A^* = ABEC$
   (b) $(AE)^* = ABEC$
   (c) $(ADE)^* = ABCDEI$

4. To see that $F$ and $G$ are equivalent, we need to verify that every FD in $F$ is in $G^*$, and vice versa. We first check if $F \subseteq G^*$: (i) $A^* = ACD$, so $A \rightarrow C$ is in $G^*$. (ii) $AC_G^* = ACD$, so $AC \rightarrow D$ is in $G^*$. (iii) $(E)_G^* = ACDEH$, so $E \rightarrow AD$ is in $G^*$.
Next we verify if $G \subseteq F^*$: (i) $(A)_F^* = ACD$, so $A \rightarrow CD$ is in $F^*$. (ii) $(E)_F^* = ACDE$, so $E \rightarrow AHE$ is not in $F^*$.
Thus $F$ and $G$ and NOT equivalent.

5. (a) Let $F = \{A \rightarrow BC, B \rightarrow C, AB \rightarrow D\}$. We need to obtain an equivalent set of FDs that satisfies the three properties of a minimal cover.
   • Right side of each FD in $F$ must ne a single attribute: so we replace $F$ by $F_1 = \{A \rightarrow B, A \rightarrow C, B \rightarrow C, AB \rightarrow D\}$.
   • No extraneous attributes on the left side. We first check if $A$ can be deleted from $AB \rightarrow D$. We can do so if $B \rightarrow D$ follows from $F_1$. Since $(B)_F^* = BC$, the answer is NO.
   We next check if $B$ can be deleted from $AB \rightarrow D$. We can do if $A \rightarrow D$ follows from $F_1$. Since $(A)_F^* = ABCD$, the answer is YES. Let $F_2 = \{A \rightarrow BC, B \rightarrow C, A \rightarrow D\}$.
   • No redundant FDs: $A \rightarrow C$ can be deleted from $F_2$. Minimal cover = \{A $\rightarrow B$, B $\rightarrow C$, A $\rightarrow D$\}.
(b) Let \( F = \{ A \to C, AB \to C, C \to DI, EC \to AB, EI \to C \} \). We need to obtain an equivalent set of FDs that satisfies the three properties of a minimal cover.

- We replace \( F \) by \( F_1 = \{ A \to C, AB \to C, C \to D, C \to I, EC \to A, EC \to B, EI \to C \} \).
- No extraneous attributes on left side: it can be checked that \( B \) can be deleted from \( AB \to C \). Let \( F_2 = \{ A \to C, C \to D, C \to I, EC \to A, EC \to B, EI \to C \} \).
- No redundant FDs: None of the FDs are redundant.

Minimal Cover is \( \{ A \to C, C \to D, C \to I, EC \to A, EC \to B, EI \to C \} \).

6. (a) \( \rho \) is loss since \( AB \cap BCD = B \), and neither \( B \to AB \) not \( B \to BCD \) is true.

(b) The initial table:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>a2</td>
<td>a3</td>
<td>b1</td>
<td>b2</td>
<td>b3</td>
</tr>
<tr>
<td>a1</td>
<td>b4</td>
<td>b5</td>
<td>a4</td>
<td>b6</td>
<td>a6</td>
</tr>
<tr>
<td>b7</td>
<td>a2</td>
<td>b8</td>
<td>a4</td>
<td>a5</td>
<td>a6</td>
</tr>
<tr>
<td>b9</td>
<td>b10</td>
<td>a3</td>
<td>a4</td>
<td>a5</td>
<td>a6</td>
</tr>
</tbody>
</table>

By applying the three FDs, we obtain a tableau that has one row consisting entirely of a’s. Hence \( \rho \) is lossless.

7. • No change if applying step 1.
• We can check that \( F \) does not have any extraneous FDs.
• For step 3, we need only consider \( AB \to C \). We can see that \( B \) is redundant by considering \( (A)_C \). Since \( (A)_C = ABC \), \( B \) is redundant. Thus, we replace \( AB \to C \) by \( B \to C \) to get the minimal cover \( \{ A \to C, C \to D, A \to B \} \). Unfortunately, this is not a minimal cover since the FD \( A \to B \) is now extraneous.

8. (a) \( IS \) is a candidate key since \( (IS)^+ = IBO \) and \( S^+ = SD \).
(b) \( IS \) is the only candidate key since neither \( I \) nor \( S \) appear in the right hand side if any FD. Therefore any candidate key will have to contain both \( I \) and \( S \). But since \( IS \) forms a candidate key, it is the only candidate key.
(c) One possible decomposition is obtained as follows:
(d) We first find the minimal cover \( F = \{ S \to D, I \to B, IS \to Q, B \to O \} \). It turns that \( F \) is minimal. Thus \{SD, IB, ISQ, BO\} is required decomposition. Notice \( ISQ \to BOISQD \), hence the decomposition has lossless property.

9. Let’s rewrite the relation scheme as \( R = \text{EPBT and FDs as } EP \to T, P \to B, E = EMP\_ID, P = \text{PROJECT, } B = \text{PROJECT\_BUDGET, } T = \text{TIME\_SPENT\_BY\_PERSON\_ON\_PROJECT.} \)

(a) Since \( EP \) is the only candidate key of \( R \), both \( E \) and \( P \) are prime attributes while \( T \) and \( B \) are nonprime attributes.
(b) R is not in 3NF since $P \rightarrow B$ holds in $R$, $P$ is not a superkey and $B$ is not a prime attribute.

(c) R is not BCNF since it’s not 3NF.