CS 301 - Lecture 3
NFA DFA Equivalence
Regular Expressions

Fall 2008

Review

- Languages and Grammars
  - Alphabets, strings, languages
- Regular Languages
  - Deterministic Finite Automata
  - Nondeterministic Finite Automata
- Today:
  - Equivalence of NFA and DFA
  - Regular Expressions
  - Equivalence to Regular Languages

Equivalence of Machines

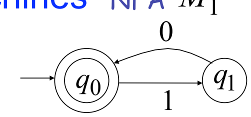
Machine M_1 is equivalent to machine M_2

if $L(M_1) = L(M_2)$

Example of equivalent machines

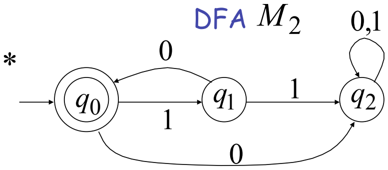
NFA M_1

$L(M_1) = \{10\}^*$

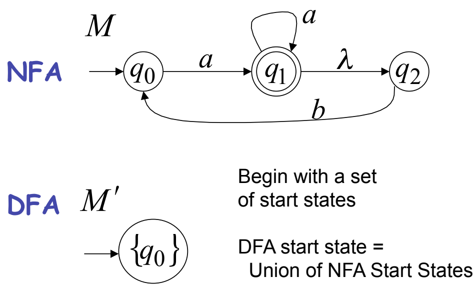


DFA M_2

$L(M_2) = \{10\}^*$



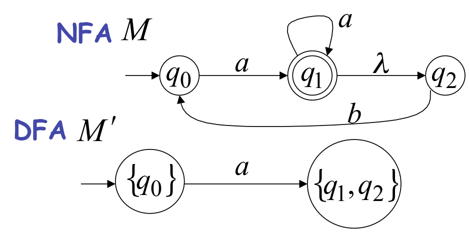
Convert NFA to DFA



Begin with a set of start states

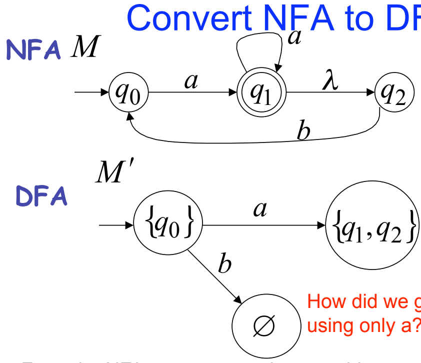
DFA start state =
Union of NFA Start States

Convert NFA to DFA



From the NFA start states, where could we get on a?
List the set of all possible states you can reach using just an a.

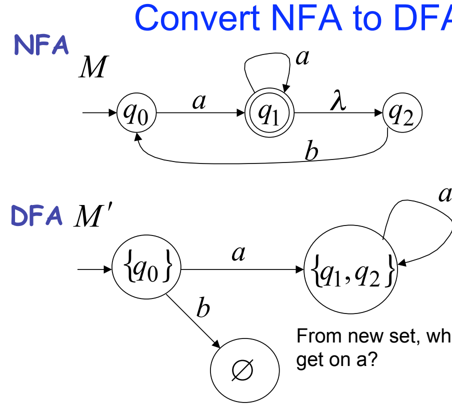
Convert NFA to DFA



How did we get from q0 to q2 using only a? (think λ)

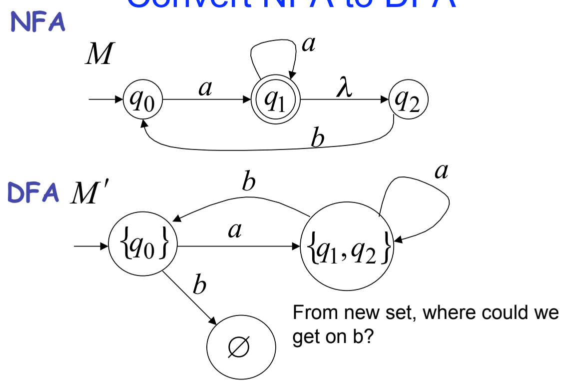
From the NFA start states, where could we get on b?
The ***set*** of all possible states you can reach may be empty!

Convert NFA to DFA



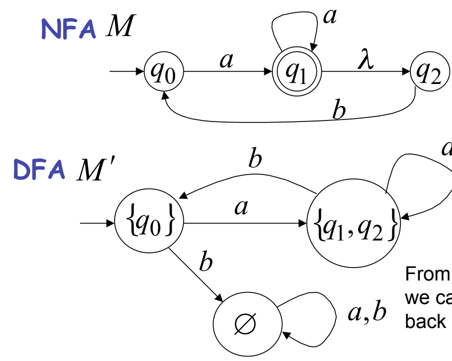
From new set, where could we get on a?

Convert NFA to DFA



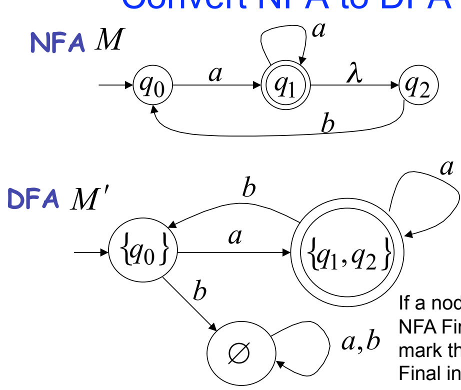
From new set, where could we get on b?

Convert NFA to DFA



From empty set, we can only get back to empyt set.

Convert NFA to DFA



If a node has any NFA Final state, mark the node as Final in the DFA.

Could This Produce Infinite States?

If the NFA has states

$$q_0, q_1, q_2, \ldots$$

There are a finite number of NFA states by definition

the DFA has states in the powerset

$$\varnothing, \{q_0\}, \{q_1\}, \{q_1, q_2\}, \{q_3, q_4, q_7\}, \ldots.$$

Powerset of finite set can be big, but it is not infinite!

Procedure NFA to DFA

1. Initial state of NFA: q_0

$\Downarrow$

Initial state of DFA: $\{q_0\}$

Procedure NFA to DFA

2. For every DFA's state $\{q_i, q_j, \ldots, q_m\}$

Compute in the NFA

$$\left.\begin{array}{l} \delta^*(q_i, a), \\ \delta^*(q_j, a) \\ \ldots \end{array}\right\} = \{q'_i, q'_j, \ldots, q'_m\}$$

Add transition to DFA

$$\delta(\{q_i, q_j, \ldots, q_m\},\ a) = \{q'_i, q'_j, \ldots, q'_m\}$$

Procedure NFA to DFA

Repeat Step 2 for all letters in alphabet, until no more transitions can be added.

Procedure NFA to DFA

3. For any DFA state $\{q_i, q_j, \ldots, q_m\}$

If some q_j is a final state in the NFA

Then, $\{q_i, q_j, \ldots, q_m\}$ is a final state in the DFA

Theorem

Take NFA M

Apply procedure to obtain DFA M'

Then M and M' are equivalent :

$$L(M) = L(M')$$

Proof

$$L(M) = L(M')$$

$$\Updownarrow$$

$L(M) \subseteq L(M')$ AND $L(M) \supseteq L(M')$

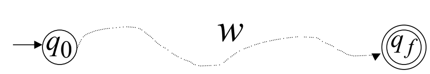
First we show: $L(M) \subseteq L(M')$

Take arbitrary: $w \in L(M)$

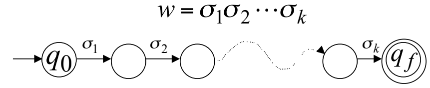
We will prove: $w \in L(M')$

$$w \in L(M)$$

$$\Downarrow$$

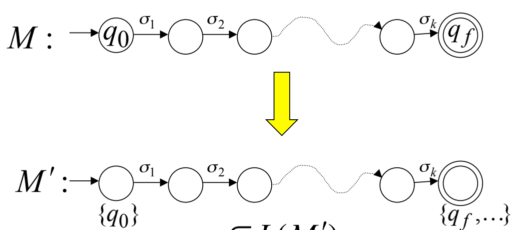
$M :$ 

$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$

$M :$ 

We will show that if $w \in L(M)$

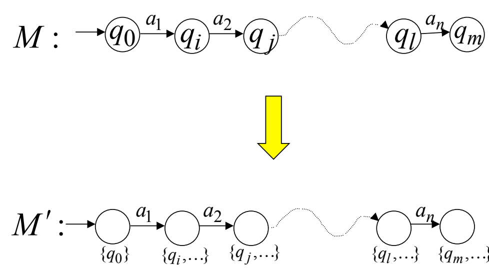
$w = \sigma_1 \sigma_2 \cdots \sigma_k$



$w \in L(M')$

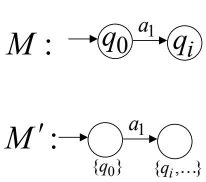
More generally, we will show that if in M:

(arbitrary string) $v = a_1 a_2 \cdots a_n$



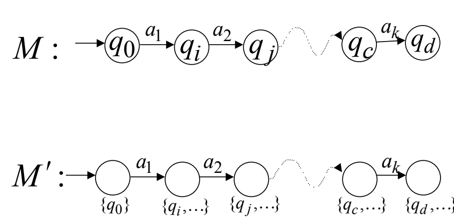
Proof by induction on $|v|$

Induction Basis: $v = a_1$



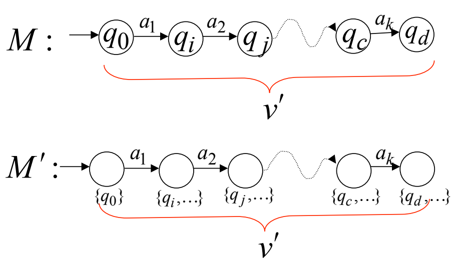
Induction hypothesis: $1 \leq |v| \leq k$

$v = a_1 a_2 \cdots a_k$



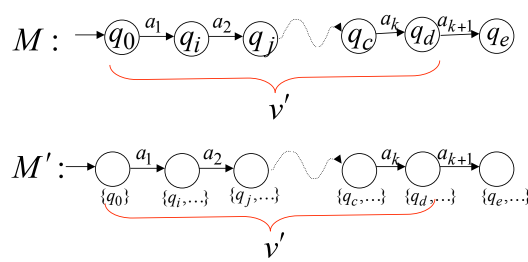
Induction Step: $|v| = k+1$

$$v = \underbrace{a_1 a_2 \cdots a_k}_{v'} a_{k+1} = v' a_{k+1}$$



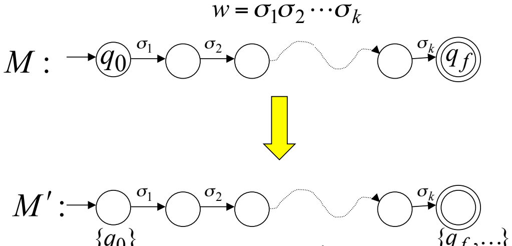
Induction Step: $|v| = k+1$

$$v = \underbrace{a_1 a_2 \cdots a_k}_{v'} a_{k+1} = v' a_{k+1}$$



Therefore if $w \in L(M)$

$$w = \sigma_1 \sigma_2 \cdots \sigma_k$$



$w \in L(M')$

We have shown: $L(M) \subseteq L(M')$

We also need to show: $L(M) \supseteq L(M')$

(proof is similar)

NFAs Accept Regular Languages

We will prove:

$$\left\{\begin{array}{l}\text{Languages}\\\text{accepted}\\\text{by NFAs}\end{array}\right\} = \left\{\begin{array}{l}\text{Regular}\\\text{Languages}\end{array}\right\}$$

Languages accepted by DFAs

NFAs and DFAs have the same computation power

Step 1

$$\left\{\begin{array}{l}\text{Languages}\\\text{accepted}\\\text{by NFAs}\end{array}\right\} \supseteq \left\{\begin{array}{l}\text{Regular}\\\text{Languages}\end{array}\right\}$$

Proof: Every DFA is trivially an NFA

⇓

Any language L accepted by a DFA is also accepted by an NFA

Step 2

$$\left\{\begin{array}{l}\text{Languages}\\\text{accepted}\\\text{by NFAs}\end{array}\right\} \subseteq \left\{\begin{array}{l}\text{Regular}\\\text{Languages}\end{array}\right\}$$

Proof: Any NFA can be converted to an equivalent DFA

⇓

Any language L accepted by an NFA is also accepted by a DFA

Regular Expressions

- Regular expressions describe regular languages

- Example: $(a+b\cdot c)^*$

  describes the language

$$\{a,bc\}^* = \{\lambda, a, bc, aa, abc, bca, \ldots\}$$

Recursive Definition

Primitive regular expressions: $\varnothing, \quad \lambda, \quad \alpha$

Given regular expressions r_1 and r_2

$$\left.\begin{array}{l} r_1 + r_2 \\ r_1 \cdot r_2 \\ r_1 ^* \\ (r_1) \end{array}\right\} \text{Are regular expressions}$$

Examples

A regular expression:

$$(a+b\cdot c)^* \cdot (c+\varnothing)$$

Not a regular expression: $(a+b+)$

Languages of Regular Expressions

- $L(r)$: language of regular expression r

- Example

$$L((a+b\cdot c)^*) = \{\lambda, a, bc, aa, abc, bca, \ldots\}$$

Formal Definition

- For primitive regular expressions:

$$L(\varnothing) = \varnothing$$

$$L(\lambda) = \{\lambda\}$$

$$L(a) = \{a\}$$

Definition (continued)

- For regular expressions r_1 and r_2

$$L(r_1 + r_2) = L(r_1) \cup L(r_2)$$

$$L(r_1 \cdot r_2) = L(r_1)\,L(r_2)$$

$$L(r_1 *) = (L(r_1))*$$

$$L((r_1)) = L(r_1)$$

Example

$$\begin{aligned}
L((a+b)\cdot a*) &= L((a+b))\,L(a*) \\
&= L(a+b)\,L(a*) \\
&= (L(a) \cup L(b))(L(a))* \\
&= (\{a\} \cup \{b\})(\{a\})* \\
&= \{a,b\}\{\lambda, a, aa, aaa, \ldots\} \\
&= \{a, aa, aaa, \ldots, b, ba, baa, \ldots\}
\end{aligned}$$

Example

- Regular expression

$$r = (a+b)*(a+bb)$$

$$L(r) = \{a, bb, aa, abb, ba, bbb, \ldots\}$$

What’s Next

- Read
  - Linz Chapter 1, 2.1, 2.2, 2.3, (skip 2.4), Chapter 3
  - JFLAP Startup, Chapter 1, 2.1, (skip 2.2) 3, 4
- Next Lecture Topics from Chapter 3.2 and 3.3
  - Regular Expressions and Regular Languages
  - Regular Grammars and Regular Languages
- Quiz 1 in Recitation on Wednesday 9/17
  - Covers Linz 1.1, 1.2, 2.1, 2.2, 2.3, and JFLAP 1, 2.1
  - Closed book, but you may bring one sheet of 8.5 x 11 inch paper with any notes you like.
  - Quiz will take the full hour
- Homework
  - Homework 2 is Due Thursday