

CS 301 - Lecture 12 Pushdown Automata and Context Free Grammars

Fall 2008

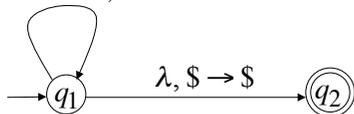
Review

- Languages and Grammars
 - Alphabets, strings, languages
- Regular Languages
 - Deterministic Finite and Nondeterministic Automata
 - Equivalence of NFA and DFA
 - Regular Expressions
 - Regular Grammars
 - Properties of Regular Languages
 - Languages that are not regular and the pumping lemma
- Context Free Languages
 - Context Free Grammars
 - Derivations: leftmost, rightmost and derivation trees
 - Parsing and ambiguity
 - Simplifications and Normal Forms
 - Nondeterministic Pushdown Automata
- Today:
 - Pushdown Automata and Context Free Grammars

Another NPDA example

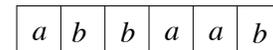
NPDA M
 $L(M) = \{w : n_a = n_b\}$

$a, \$ \rightarrow 0\$$ $b, \$ \rightarrow 1\$$
 $a, 0 \rightarrow 00$ $b, 1 \rightarrow 11$
 $a, 1 \rightarrow \lambda$ $b, 0 \rightarrow \lambda$

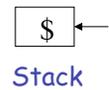


Execution Example: Time 0

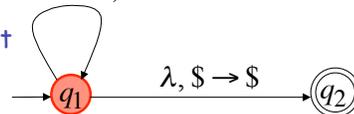
Input

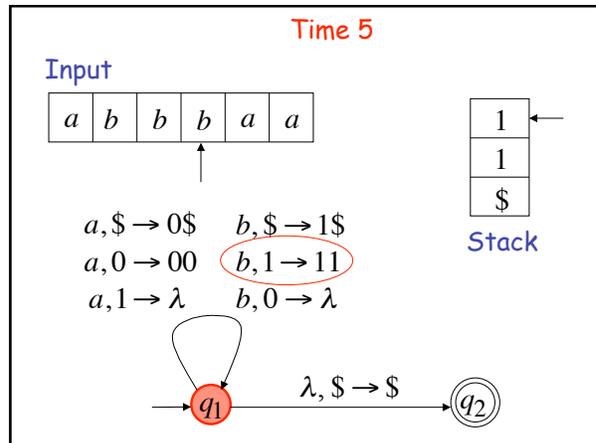
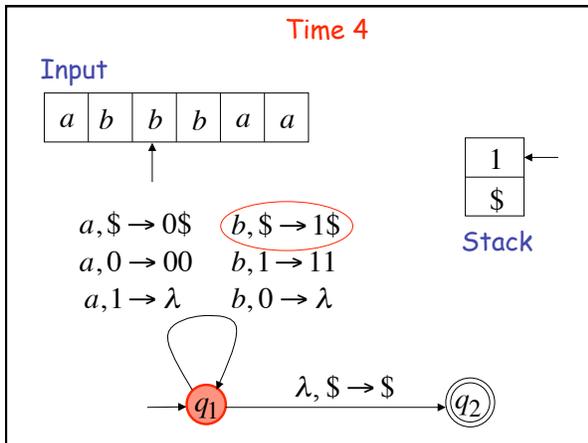
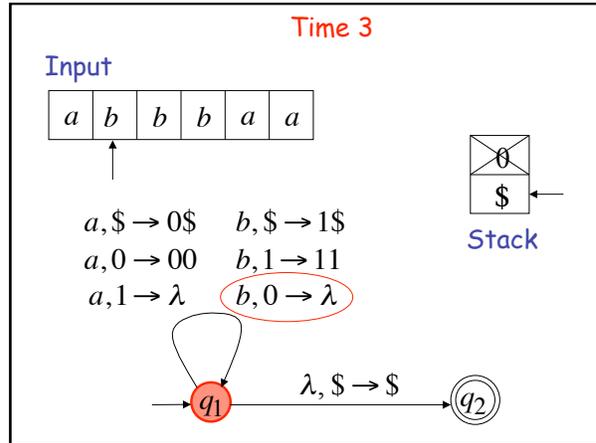
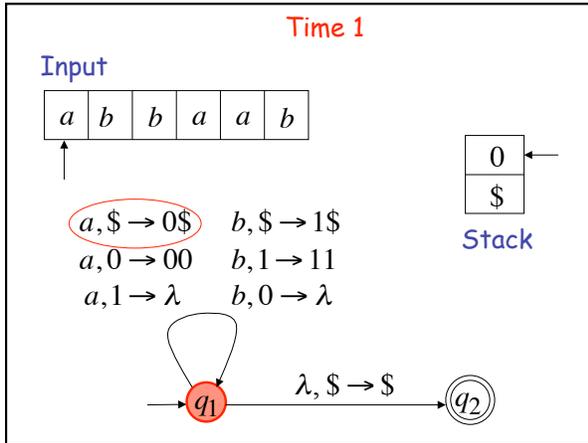


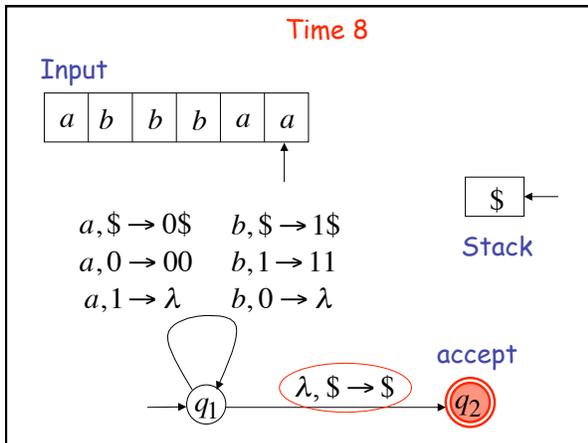
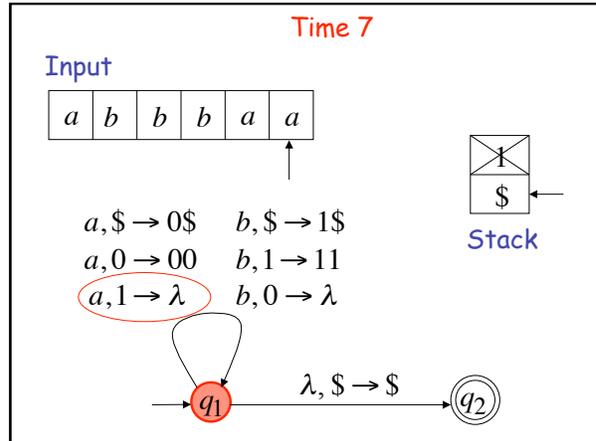
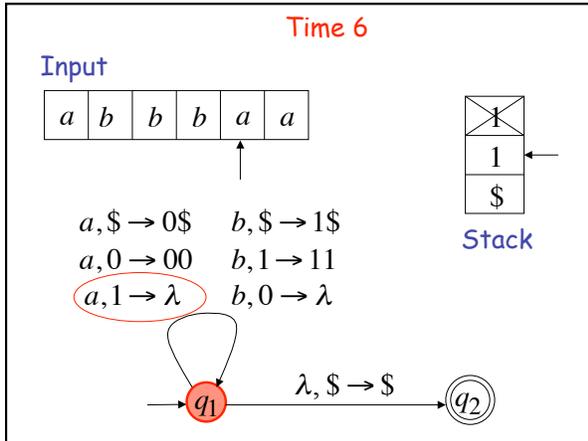
$a, \$ \rightarrow 0\$$ $b, \$ \rightarrow 1\$$
 $a, 0 \rightarrow 00$ $b, 1 \rightarrow 11$
 $a, 1 \rightarrow \lambda$ $b, 0 \rightarrow \lambda$



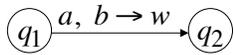
current
state





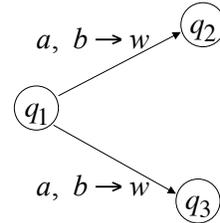


Formalities for NPDAs



Transition function:

$$\delta(q_1, a, b) = \{(q_2, w)\}$$



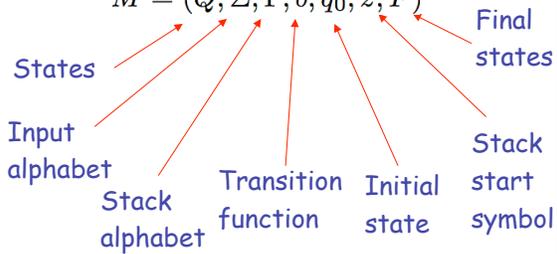
Transition function:

$$\delta(q_1, a, b) = \{(q_2, w), (q_3, w)\}$$

Formal Definition

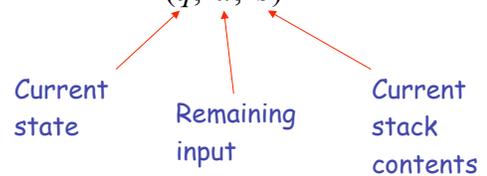
Non-Deterministic Pushdown Automaton
NPDA

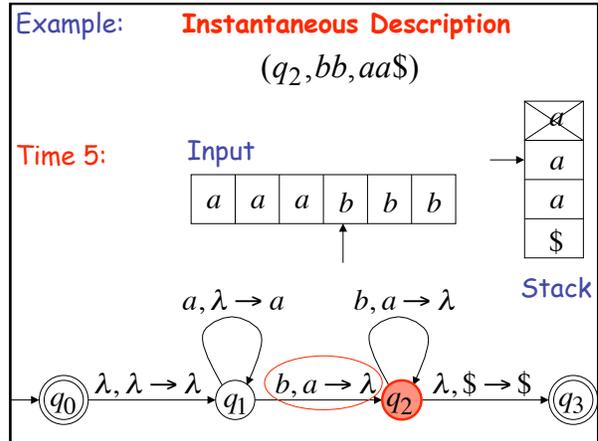
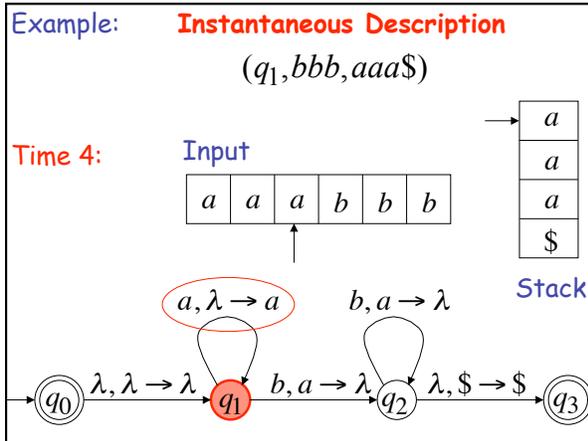
$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$



Instantaneous Description

$$(q, u, s)$$

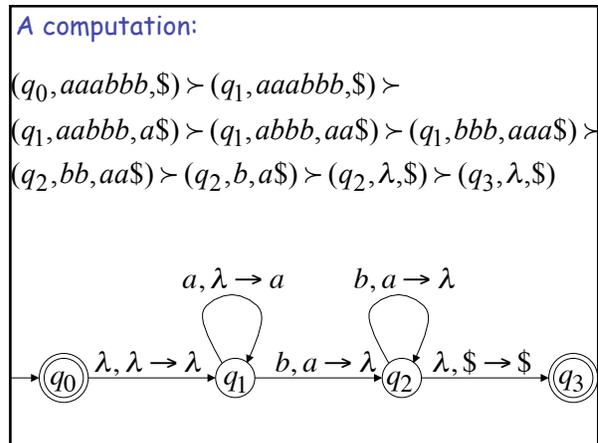




We write:

$$(q_1, bbb, aaa\$) \succ (q_2, bb, aa\$)$$

Time 4 Time 5



$(q_0, aaabbb, \$) \succ (q_1, aaabbb, \$) \succ$
 $(q_1, aabbb, a\$) \succ (q_1, abbb, aa\$) \succ (q_1, bbb, aaa\$) \succ$
 $(q_2, bb, aa\$) \succ (q_2, b, a\$) \succ (q_2, \lambda, \$) \succ (q_3, \lambda, \$)$

For convenience we write:

$$(q_0, aaabbb, \$) \overset{*}{\succ} (q_3, \lambda, \$)$$

Formal Definition

Language $L(M)$ of NPDA M :

$$L(M) = \{w : (q_0, w, s) \overset{*}{\succ} (q_f, \lambda, s')\}$$

Initial state

Final state

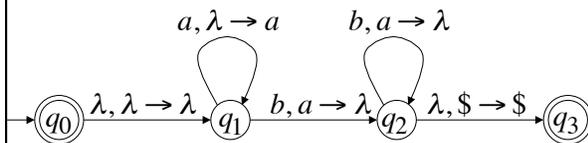
Example:

$$(q_0, aaabbb, \$) \overset{*}{\succ} (q_3, \lambda, \$)$$



$$aaabbb \in L(M)$$

NPDA M :

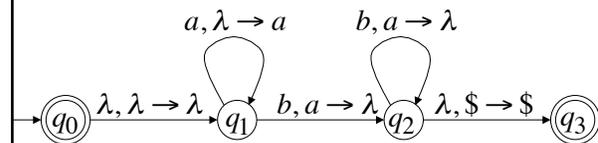


$$(q_0, a^n b^n, \$) \overset{*}{\succ} (q_3, \lambda, \$)$$



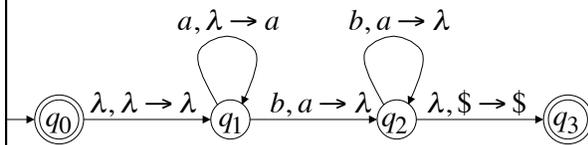
$$a^n b^n \in L(M)$$

NPDA M :



Therefore: $L(M) = \{a^n b^n : n \geq 0\}$

NPDA M :



NPDA's Accept
Context-Free Languages

Theorem:

$$\left\{ \begin{array}{l} \text{Context-Free} \\ \text{Languages} \\ \text{(Grammars)} \end{array} \right\} = \left\{ \begin{array}{l} \text{Languages} \\ \text{Accepted by} \\ \text{NPDA's} \end{array} \right\}$$

Proof - Step 1:

$$\left\{ \begin{array}{l} \text{Context-Free} \\ \text{Languages} \\ \text{(Grammars)} \end{array} \right\} \subseteq \left\{ \begin{array}{l} \text{Languages} \\ \text{Accepted by} \\ \text{NPDA's} \end{array} \right\}$$

Convert any context-free grammar G
to a NPDA M with: $L(G) = L(M)$

Proof - Step 2:

$$\left\{ \begin{array}{l} \text{Context-Free} \\ \text{Languages} \\ \text{(Grammars)} \end{array} \right\} \equiv \left\{ \begin{array}{l} \text{Languages} \\ \text{Accepted by} \\ \text{NPDAs} \end{array} \right\}$$

Convert any NPDA M to a context-free grammar G with: $L(G) = L(M)$

Proof - step 1

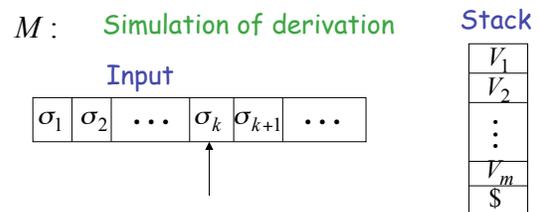
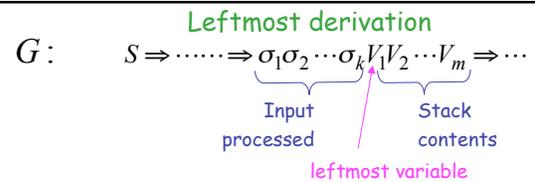
**Converting
Context-Free Grammars
to
NPDAs**

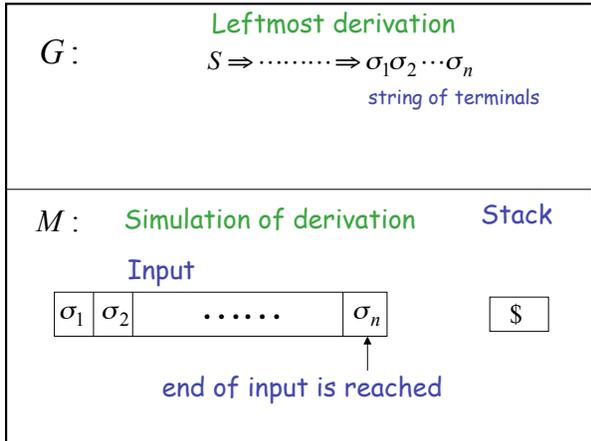
We will convert any context-free grammar G

to an NPDA automaton M

Such that:

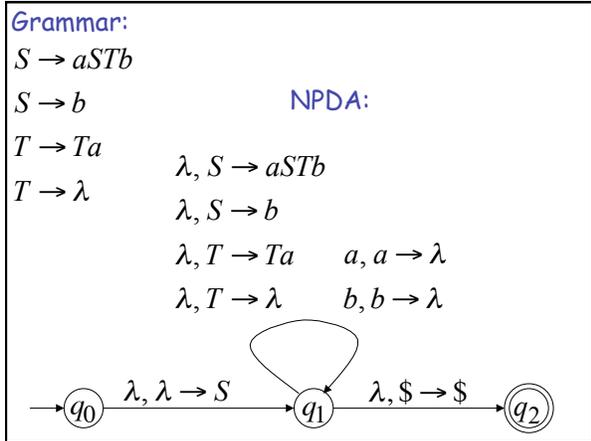
M Simulates leftmost derivations of G





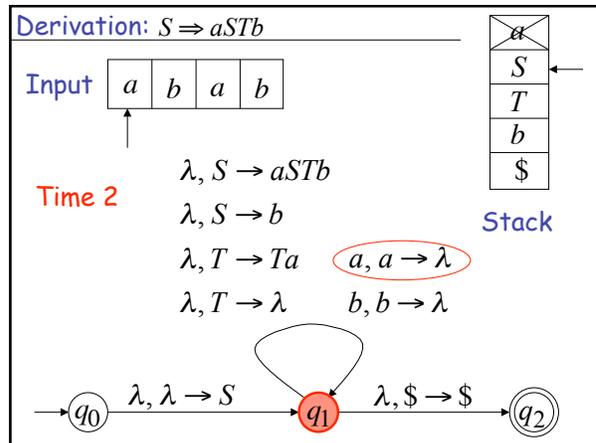
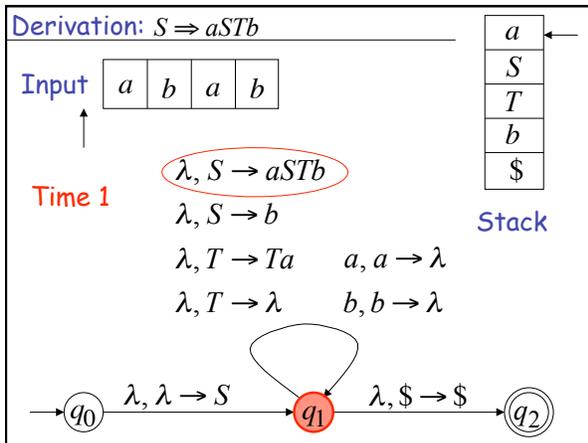
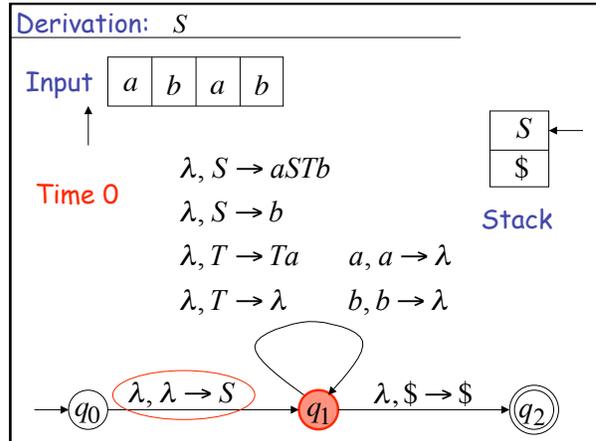
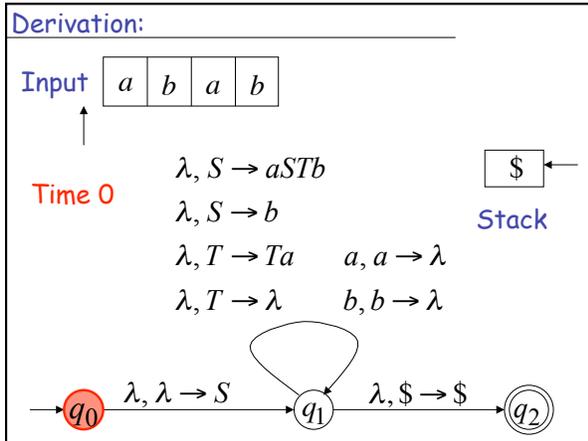
An example grammar: $S \rightarrow aSTb$
 $S \rightarrow b$
 $T \rightarrow Ta$
 $T \rightarrow \lambda$

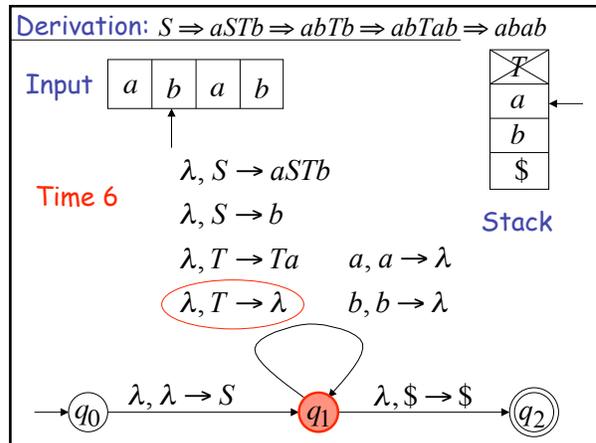
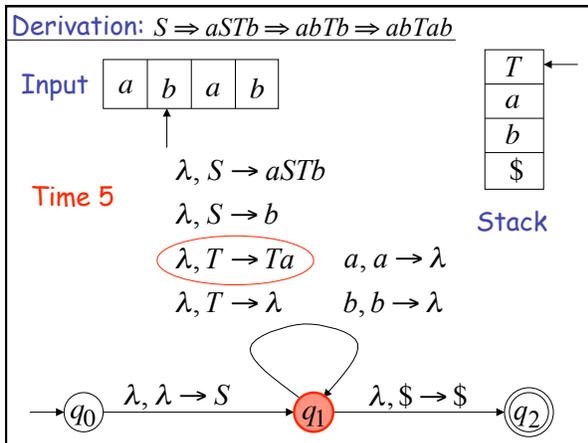
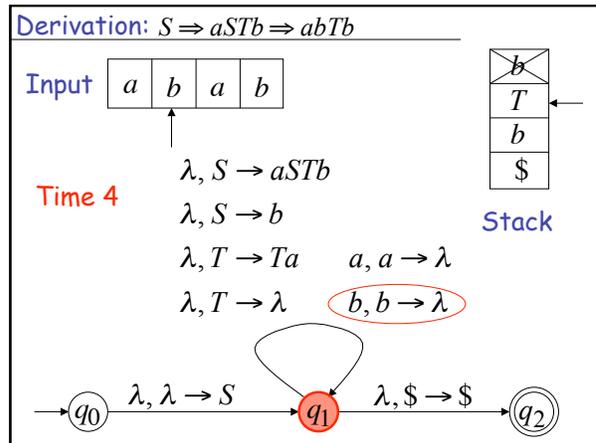
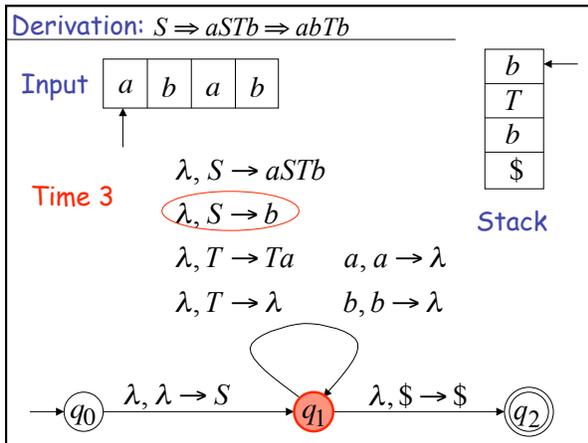
What is the equivalent NPDA?

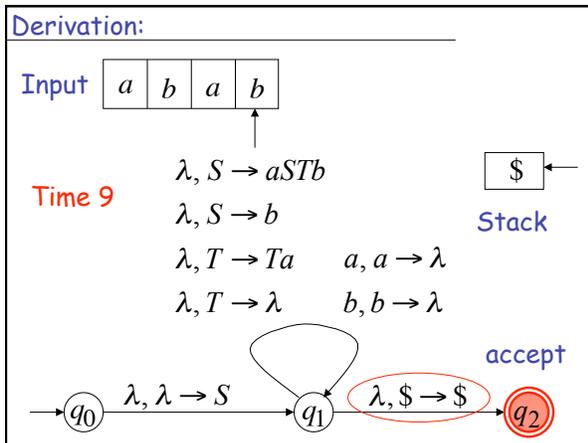
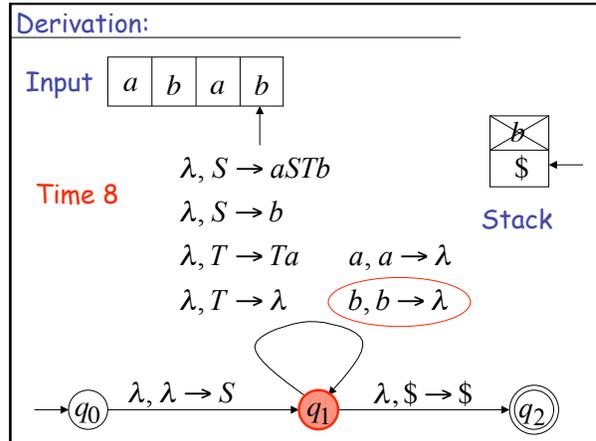
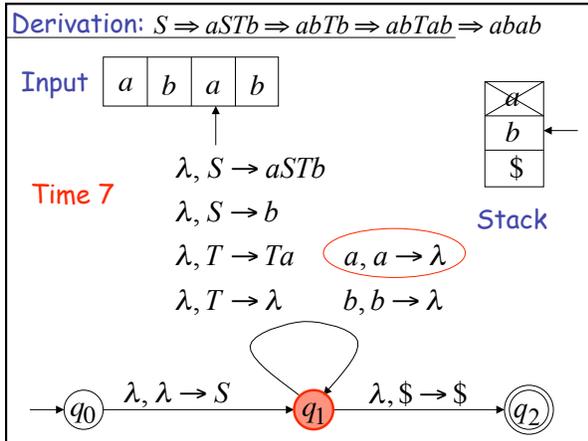


Grammar: $S \rightarrow aSTb$
 $S \rightarrow b$
 $T \rightarrow Ta$
 $T \rightarrow \lambda$

A leftmost derivation:
 $S \Rightarrow aSTb \Rightarrow abTb \Rightarrow abTab \Rightarrow abab$





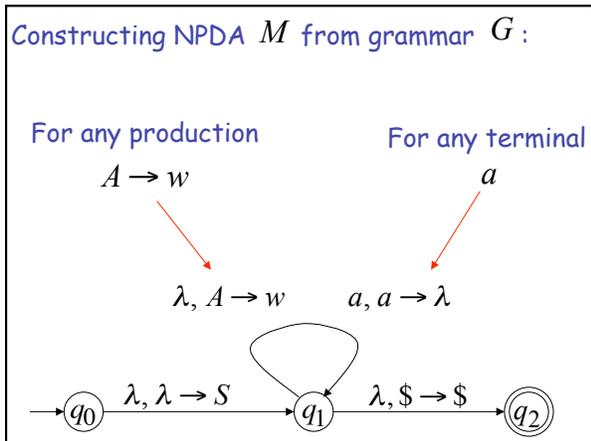


In general:

Given any grammar G

We can construct a NPDA M

With $L(G) = L(M)$



Grammar G generates string w

if and only if

NPDA M accepts w

↓

$L(G) = L(M)$

Therefore:

For any context-free language
there is a NPDA
that accepts the same language

$\left\{ \begin{array}{l} \text{Context-Free} \\ \text{Languages} \\ \text{(Grammars)} \end{array} \right\} \cong \left\{ \begin{array}{l} \text{Languages} \\ \text{Accepted by} \\ \text{NPDAs} \end{array} \right\}$

Proof - step 2

**Converting
NPDAs
to
Context-Free Grammars**

For any NPDA M

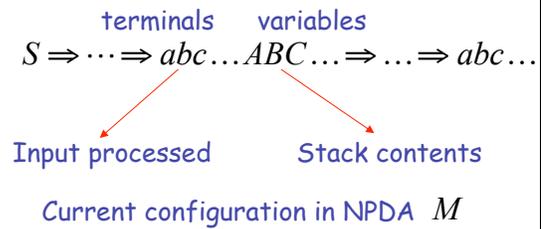
we will construct

a context-free grammar G with

$$L(M) = L(G)$$

Intuition: The grammar simulates the machine

A derivation in Grammar G :



Some Necessary Modifications

Modify (if necessary) the NPDA so that:

- 1) The stack is never empty
- 2) It has a single final state and empties the stack when it accepts a string
- 3) Has transitions in a special form

What's Next

- Read
 - Linz Chapter 1, 2.1, 2.2, 2.3, (skip 2.4), 3, 4, 5, 6.1, 6.2, (skip 6.3), 7.1, 7.2, and 7.3 (skip 7.4)
 - JFLAP Chapter 1, 2.1, (skip 2.2), 3, 4, 5, 6, 7
- Next Lecture Topics from Chapter 7.2 and 7.3
 - Converting NPDAs to Context Free Grammars
 - Deterministic Pushdown Automata
- Midterm in Lecture on Tuesday 10/14
 - Covers Linz 1.1, 1.2, 2.1, 2.2, 2.3, (skip 2.4), 3, 4, 5, 6.1, 6.2 and JFLAP 1, 2.1, (skip 2.2), 3, 4, 6.1
 - Closed book, but you may bring one sheet of 8.5 x 11 inch paper with any notes you like.
 - Midterm will take the full lecture
- Homework
 - Homework Due Today
 - New Homework Assigned Friday
 - New Homework Due Next Thursday