Reuse Optimization

Idea

- Eliminate redundant operations in the dynamic execution of instructions

How do redundancies arise?

- Loop invariant code (*e.g.,* index calculation for arrays)
- Sequence of similar operations (*e.g.,* method lookup)
- Lightning frequently strikes twice

Types of reuse optimization

- Value numbering
- Common subexpression elimination
- Partial redundancy elimination

Local Value Numbering

Idea

- Each variable, expression, and constant is assigned a unique number
- When we encounter a variable, expression or constant, see if it's already been assigned a number
  - If so, use the value for that number
  - If not, assign a new number
- Same number ⇒ same value

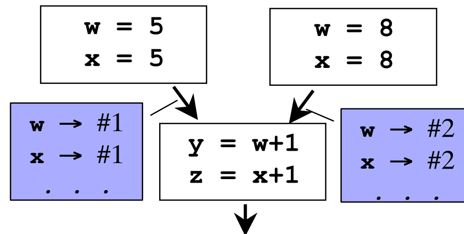
Example

```
a := b + c
d := b
b := a
e := d + c    a
```

b → ~~#1~~ #3
c → #2
b + c is #1 + # 2 → #3
a → #3
d → #1
d + c is #1 + # 2 → #3
e → #3

Global Value Numbering

How do we handle control flow?



Global Value Numbering (cont)

Idea [Alpern, Wegman, and Zadeck 1988]

- Partition program variables into **congruence classes**
- All variables in a particular congruence class have the same value
- SSA form is helpful

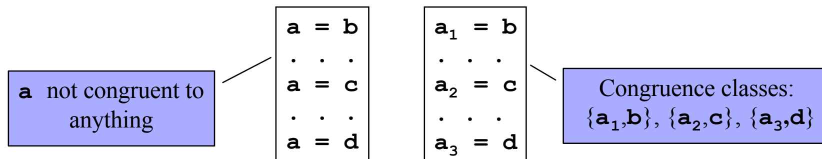
Approaches to computing congruence classes

- Pessimistic
  - Assume no variables are congruent (start with *n* classes)
  - Iteratively coalesce classes that are determined to be congruent
- Optimistic
  - Assume all variables are congruent (start with one class)
  - Iteratively partition variables that contradict assumption
  - Slower but better results

Role of SSA Form

SSA form is helpful

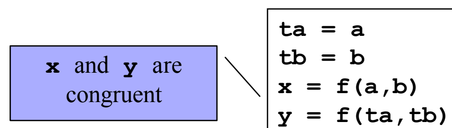
- Allows us to avoid data-flow analysis
- Variables correspond to values



Basis

Idea

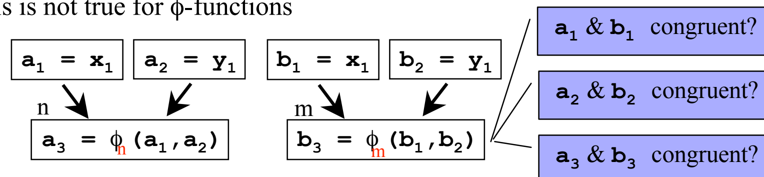
- If x and y are congruent then f(x) and f(y) are congruent



- Use this fact to combine (pessimistic) or split (optimistic) classes

Problem

- This is not true for ϕ-functions



Solution: Label ϕ-functions with join point

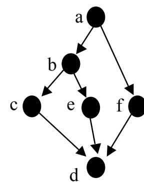
Pessimistic Global Value Numbering

Idea

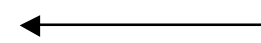
- Initially each variable is in its own congruence class
- Consider each assignment statement s (reverse postorder in CFG)
  - Update LHS value number with hash of RHS
- Identical value number ⇒ congruence

Why reverse postorder?

- Ensures that when we consider an assignment statement, we have already considered definitions that reach the RHS operands

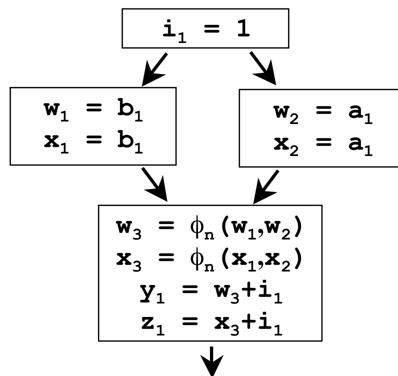


Postorder: d, c, e, b, f, a



Algorithm

for each assignment of the form: "**x = f(a,b)**"
  ValNum[x] ← UniqueValue()
for each assignment of the form: "**x = f(a,b)**" (in reverse postorder)
  ValNum[x] ← Hash(f ⊕ ValNum[a] ⊕ ValNum[b])



a_1 #1
b_1 #2
i_1 #3
w_1 ~~#4~~ #2
x_1 ~~#5~~ #2
w_2 ~~#6~~ #1
x_2 ~~#7~~ #1
w_3 ~~#8~~ ϕ_n(#2,#1) → #12
x_3 ~~#9~~ ϕ_n(#2,#1) → #12
y_1 ~~#10~~ +(#12,#3) → #13
z_1 ~~#11~~ +(#12,#3) → #13

Snag!

Problem

- Our algorithm assumes that we consider operands before variables that depend upon it
- Can't deal with code containing loops!

Solution

- Ignore back edges
- Make conservative (worst case) assumption for previously unseen variable (*i.e.,* assume its in it's own congruence class)

Optimistic Global Value Numbering

Idea

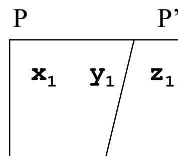
- Initially all variables in one congruence class
- Split congruence classes when evidence of non-congruence arises
  - Variables that are computed using different functions
  - Variables that are computed using functions with non-congruent operands

Splitting

Initially

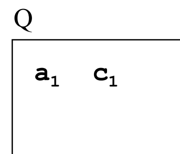
- Variables computed using the same function are placed in the same class

```
x1 = f(a1,b1)
. . .
y1 = f(c1,d1)
. . .
z1 = f(e1,f1)
```



Iteratively

- *Split* classes when corresponding operands are in different classes
- Example: a_1 and c_1 are congruent, but e_1 is congruent to neither

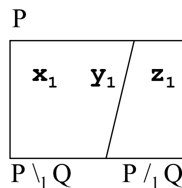


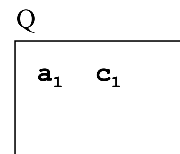
Splitting (cont)

Definitions

- Suppose P and Q are sets representing congruence classes
- Q **splits** P for each i into two sets
  - $P \backslash_i Q$ contains variables in P whose i^{th} operand is in Q
  - $P /_i Q$ contains variables in P whose i^{th} operand is not in Q
- Q **properly splits** P if neither resulting set is empty

```
x1 = f(a1,b1)
. . .
y1 = f(c1,d1)
. . .
z1 = f(e1,f1)
```





Algorithm

worklist ← ∅
for each function f
    C_f ← ∅
    for each assignment of the form "x = f(a,b)"
        $C_f \leftarrow C_f \cup \{ x \}$
    worklist ← worklist ∪ $\{C_f\}$
    CC ← CC ∪ $\{C_f\}$
while worklist ≠ ∅
    Delete some D from worklist
    for each class C properly split by D (at operand i)
        CC ← CC – C
        worklist ← worklist – C
        Create new congruence classes $C_j \leftarrow \{C \setminus_i D\}$ and $C_k \leftarrow \{C /_i D\}$
        CC ← CC ∪ $C_j \cup C_k$
        worklist ← worklist ∪ $C_j \cup C_k$

Note: see paper for optimization

Example

SSA code

```
x0 = 1
y0 = 2
x1 = x0+1
y1 = y0+1
z1 = x0+1
```

Congruence classes

S_0	$\{x_0\}$
S_1	$\{y_0\}$
~~S_2~~	~~$\{x_1, y_1, z_1\}$~~
S_3	$\{x_1, z_1\}$
S_4	$\{y_1\}$

Worklist: ~~$S_0=\{x_0\}$~~, ~~$S_1=\{y_0\}$~~, ~~$S_2=\{x_1, y_1, z_1\}$~~ ~~$S_3=\{x_1, z_1\}$~~, ~~$S_4=\{y_1\}$~~

S_0 psplit S_0? no S_0 psplit S_1? no S_0 psplit S_2? yes!

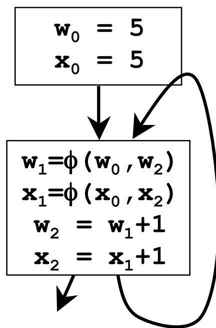
$S_2 \setminus_1 S_0 = \{x_1, z_1\} = S_3$

$S_2 /_1 S_0 = \{y_1\} = S_4$

Comparing Optimistic and Pessimistic

Differences

- Handling of loops
- Pessimistic makes worst-case assumptions on back edges
- Optimistic requires actual contradiction to split classes



Role of SSA

Single global result

- Single def reaches each use
- No data (flow value) at each point

No data flow analysis

- Optimistic: Iterate over congruence classes, not CFG nodes
- Pessimistic: Visit each assignment once

ϕ-functions

- Make data-flow merging explicit
- Treat like normal functions

Next Time

Lecture

- Common-Subexpression Elimination

Study Suggestion

- Read Alpern and Zadeck 1992 Chapter about Value Numbering
- Do the examples in Muchnick Section 12.4 and examples in paper using the others algorithm