Reuse Optimization

Last time

– Common subexpression elimination (CSE)

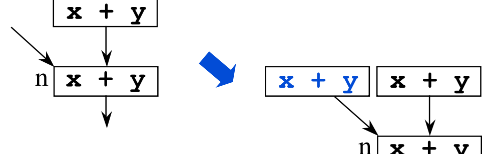
Today

– Partial redundancy elimination (PRE)

Partial Redundancy Elimination (PRE)

Partial Redundancy

– An expression (*e.g.,* **`x+y`**) is **partially redundant** at node n if **some** path from the entry node to n evaluates **`x+y`**, and there are no definitions of **`x`** or **`y`** between the last evaluation of **`x+y`** and n



Elimination

– Discover partially redundant expressions

– Convert them to fully redundant expressions

– Remove redundancy

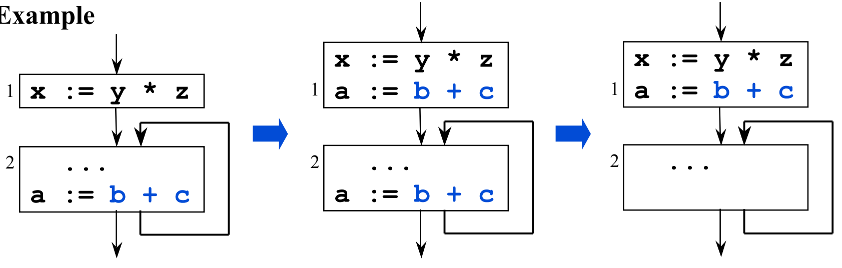
PRE subsumes CSE and loop invariant code motion

Loop Invariance Example

PRE removes loop invariants

- An invariant expression is partially redundant
- PRE converts this partial redundancy to full redundancy
- PRE removes the redundancy

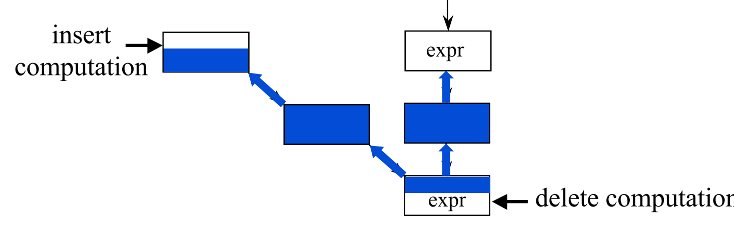
Example



Implementing PRE [Morel & Renvoise 1979]

Big picture

- Use local properties (**availability** and **anticipability**) to determine where redundancy can be created within a basic block
- Use global analysis (data-flow analysis) to discover where partial redundancy can be converted to full redundancy
- Insert code and remove redundant expressions



Local Properties

An expression is locally **transparent** in block b if its operands are not modified in b

An expression is locally **available** in block b if it is computed at least once and its operands are not modified after its last computation in b

An expression is locally **anticipated** if it is computed at least once and its operands are not modified before its first evaluation

Example

```
a := b + c
d := a + e
```

Transparent: {`b + c`}
Available: {`b + c, a + e`}
Anticipated: {`b + c`}

Local Properties (cont)

How are these properties useful?

– They tell us where we can introduce redundancy

Transparent — The expression can be redundantly evaluated anywhere in the block

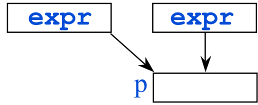
Available — `a = b + c` — The expression can be redundantly evaluated anywhere after its last evaluation in the block

Anticipated — `a = b + c` — The expression can be redundantly evaluated anywhere before its first evaluation in the block

Global Availability

Intuition

- Global availability is the same as Available Expressions
- If e is globally available at p, then an evaluation at p will create redundancy along all paths leading to p



Flow Functions

available_in[n] = $\bigcap_{p \in pred[n]}$ available_out[p]

available_out[n] = locally_available[n] $\cup$ (available_in[n] $\cap$ transparent[n])

(Global) Partial Availability

Intuition

- An expression is partially available if it is available along some path
- If e is partially available at p, then ∃ a path from the entry node to p such that the evaluation of e at p would give the same result as the previous evaluation of e along the path

expr

p

Flow Functions

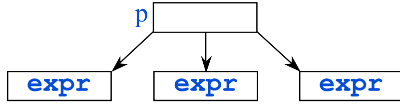
partially_available_in[n] = $\bigcup_{p \in pred[n]}$ partially_available_out[p]

partially_available_out[n] = locally_available[n] $\cup$ (partially_available_in[n] $\cap$ transparent[n])

Global Anticipability

Intuition

– If e is globally anticipated at p, then an evaluation of e at p will make the next evaluation of e redundant along all paths from p



Flow Functions

anticipated_out[n] = $\bigcap_{s \in succ[n]}$ anticipated_in[s]

anticipated_in[n] = locally_anticipated[n] $\cup$ (anticipated_out[n] $\cap$ transparent[n])

Global Possible Placement

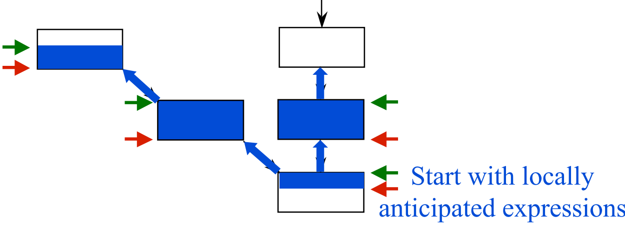
Goal

– Convert partial redundancies to full redundancies

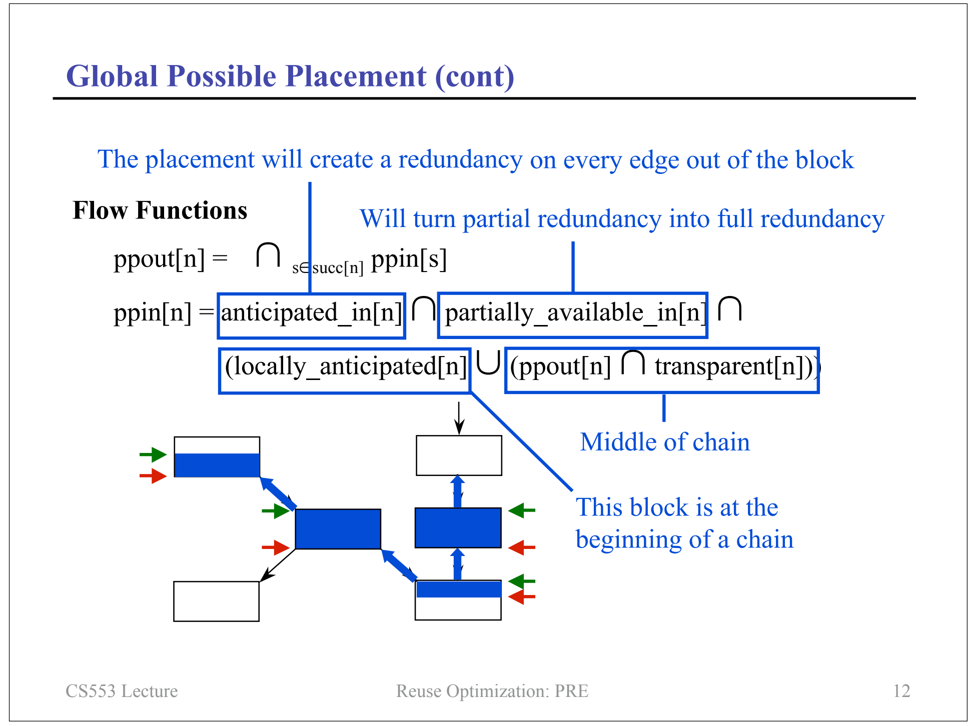
– Possible Placement uses a backwards analysis to identify locations where such conversions can take place

– e ∈ ppin[n] can be placed at entry of n

– e ∈ ppout[n] can be placed at exit of n



Push Possible Placement backwards as far as possible


Global Possible Placement (cont)
The placement will create a redundancy on every edge out of the block
Flow Functions
Will turn partial redundancy into full redundancy
ppout[n] = ⋂ s∈succ[n] ppin[s]
ppin[n] = anticipated_in[n] ⋂ partially_available_in[n] ⋂
(locally_anticipated[n] ⋃ (ppout[n] ⋂ transparent[n]))
Middle of chain
This block is at the beginning of a chain
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Reuse Optimization: PRE
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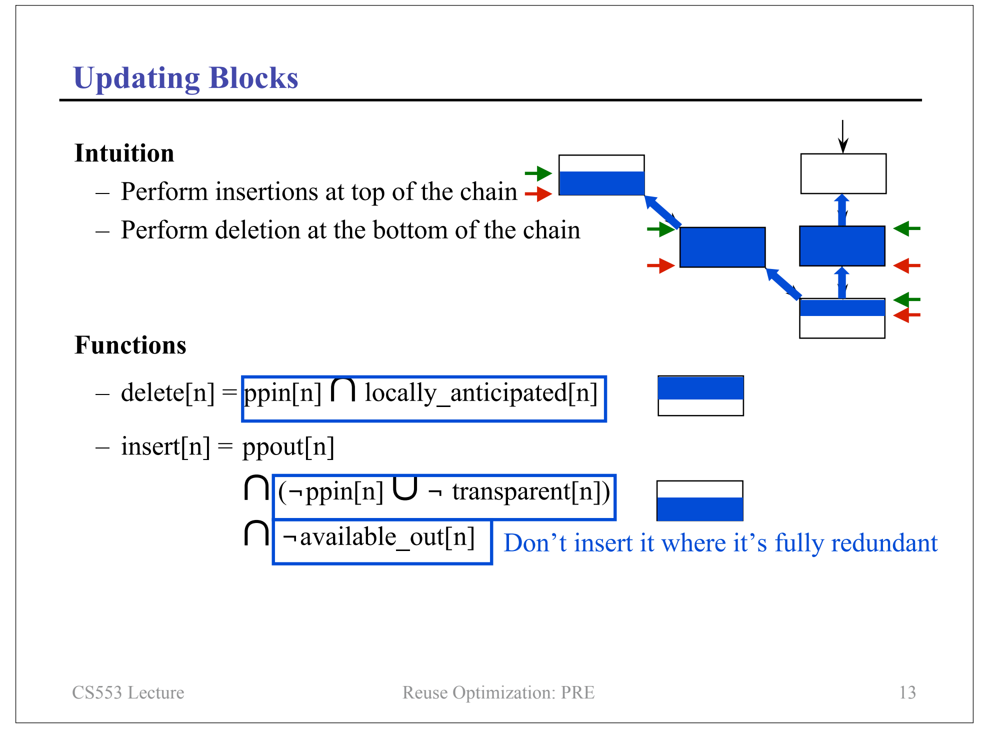
Updating Blocks
Intuition
– Perform insertions at top of the chain
– Perform deletion at the bottom of the chain
Functions
– delete[n] = ppin[n] ⋂ locally_anticipated[n]
– insert[n] = ppout[n]
⋂ (¬ppin[n] ⋃ ¬ transparent[n])
⋂ ¬available_out[n]
Don't insert it where it's fully redundant
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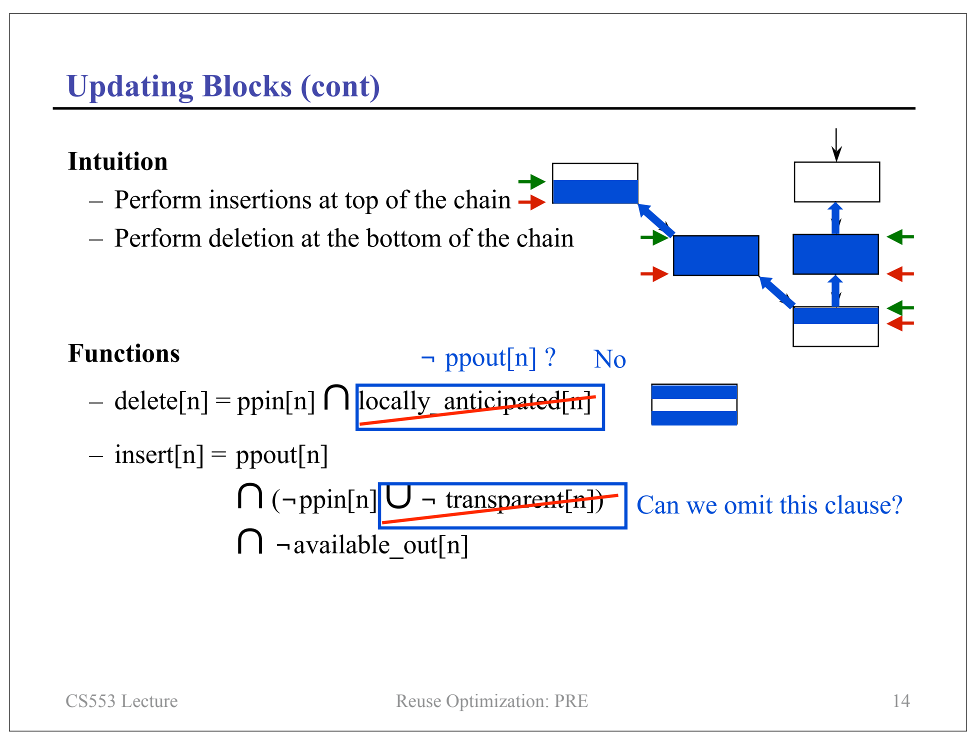
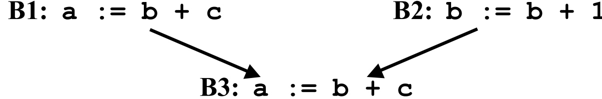


Sandwich Example

	B0	B1	B2	B3	B4
transparent		a+b		a+b	a+b
locally_available		a+b	a+b	a+b	a+b
locally_anticipated	a+b	a+b	a+b	a+b	a+b
available_in					a+b
available_out		a+b	a+b	a+b	a+b
partially_available_in	a+b		a+b		a+b
partially_available_out		a+b	a+b	a+b	a+b
anticipated_out	a+b	a+b	a+b	a+b	
anticipated_in	a+b	a+b	a+b	a+b	a+b
ppout	a+b	a+b	a+b	a+b	
ppin	a+b		a+b		a+b
insert	a+b				
delete			a+b		a+b

Example



	B1	B2	B3
transparent	{b+c}		{b+c}
locally_available	{b+c}		{b+c}
locally_anticipated	{b+c}	{b+1}	{b+c}
available_in			
available_out	{b+c}		{b+c}
partially_available_in			{b+c}
partially_available_out	{b+c}		{b+c}
anticipated_out	{b+c}	{b+c}	
anticipated_in	{b+c}	{b+1}	{b+c}
ppout	{b+c}	{b+c}	
ppin			{b+c}
insert		{b+c}	
delete			{b+c}

Comparing Redundancy Elimination

Value numbering
- Examines values not expressions
- Symbolic

CSE
- Examines expressions

PRE
- Examines expressions
- Subsumes CSE and loop invariant code motion
- Other implementations are now available

Constant propagation
- Requires that values be statically known

PRE Summary

What's so great about PRE?

- A modern optimization that subsumes earlier ideas
- Composes several simple data-flow analyses to produce a powerful result
  - Finds earliest and latest points in the CFG at which an expression is anticipated

Next Time

Assignments

- HW2 has been posted, start it now!

Lecture

- Alias analysis