Tiling: A Data Locality Optimizing Algorithm

Previously
- Kelly & Pugh transformation framework
- Affine space partitions for parallelism

Today
- "Unroll and Jam" and Tiling
- Specifying tiling in the Kelly and Pugh transformation framework
- Status of code generation for tiling

Loop Unrolling

Motivation
- Reduces loop overhead
- Improves effectiveness of other transformations
  - Code scheduling
  - CSE

The Transformation
- Make n copies of the loop: n is the **unrolling factor**
- Adjust loop bounds accordingly

Loop Unrolling (cont)

Example

```
do i=1,n
   A(i) = B(i) + C(i)
enddo
```

```
do i=1,n-1 by 2
   A(i)   = B(i) + C(i)
   A(i+1) = B(i+1) + C(i+1)
enddo
if (i=n)
   A(i) = B(i) + C(i)
```

Details
- When is loop unrolling legal?
- Handle end cases with a cloned copy of the loop
  - Enter this special case if the remaining number of iteration is less than the unrolling factor

Loop Balance

Problem
- We'd like to produce loops with the right balance of memory operations and floating point operations
- The ideal balance is machine-dependent
  - *e.g.* How many load-store units are connected to the L1 cache?
  - *e.g.* How many functional units are provided?

Example

```
do j = 1,2*n
   do i = 1,m
      A(j) = A(j) + B(i)
   enddo
enddo
```

- The inner loop has 1 memory operation per iteration and 1 floating point operation per iteration
- If our target machine can only support 1 memory operation for every two floating point operations, this loop will be memory bound

What can we do?

Unroll and Jam

Idea

- Restructure loops so that loaded values are used many times per iteration

Unroll and Jam

- Unroll the outer loop some number of times
- Fuse (Jam) the resulting inner loops

Example

```
do j = 1,2*n
   do i = 1,m
      A(j) = A(j) + B(i)
   enddo
enddo
```

Unroll the Outer Loop

```
do j = 1,2*n by 2
   do i = 1,m
      A(j)   = A(j)   + B(i)
   enddo
   do i = 1,m
      A(j+1) = A(j+1) + B(i)
   enddo
enddo
```


Unroll and Jam Example (cont)

Unroll the Outer Loop

```
do j = 1,2*n by 2
   do i = 1,m
      A(j)   = A(j)   + B(i)
   enddo
   do i = 1,m
      A(j+1) = A(j+1) + B(i)
   enddo
enddo
```

- The inner loop has 1 load per iteration and 2 floating point operations per iteration
- We reuse the loaded value of **B(i)**
- The Loop Balance matches the machine balance

Jam the inner loops

```
do j = 1,2*n by 2
   do i = 1,m
      A(j)   = A(j)   + B(i)
      A(j+1) = A(j+1) + B(i)
   enddo
enddo
```


Unroll and Jam (cont)

Legality

- When is Unroll and Jam legal?

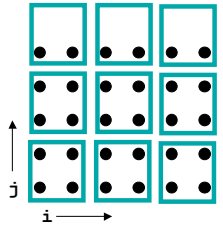
Disadvantages

- What limits the degree of unrolling?

Tiling

A non-unimodular transformation that ...

- groups iteration points into tiles that are executed atomically
- can improve spatial and temporal data locality
- can expose larger granularities of parallelism



Implementing tiling

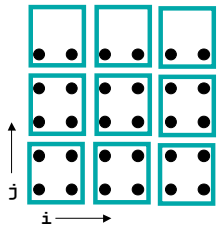
- how can we specify tiling?
- when is tiling legal?
- how do we generate tiled code?

```
do ii = 1,6, by 2
 do jj = 1, 5, by 2
  do i = ii, ii+2-1
   do j = jj, min(jj+2-1,5)
     A(i,j) = ...
```

Specifying Tiling

Rectangular tiling

- tile size vector $(s_1, s_2, ..., s_d)$
- tile offset, $(o_1, o_2, ..., o_d)$



Possible Transformation Mappings

- creating a tile space

$$\{[i,j] \rightarrow [ti,tj,i,j] \quad | \quad ti = floor((i-o_1)/s_1) \wedge tj = floor((j-o_2)/s_2)\}$$

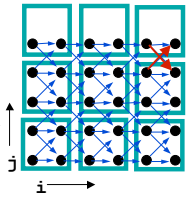
- keeping tile iterators in original iteration space

$$\{[i,j] \rightarrow [ii,jj,i,j] \quad | \quad ii = s_1 floor((i-o_1)/s_1) \wedge ii = s_1 floor((j-o_2)/s_2)\}$$

Legality of Tiling

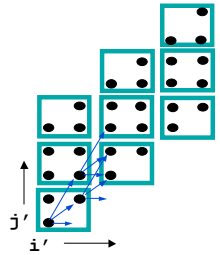
A legal rectangular tiling

- each tile executed atomically
- no dependence cycles between tiles
- Check legality by verifying that transformed data dependences are lexicographically positive



Fully permutable loops

- rectangular tiling is legal on fully permutable loops



Code Generation for Tiling

Fixed-size Tiles

- Omega library
- Cloog
- for rectangular space and tiles, straight-forward

```
do ii = 1,6, by 2
 do jj = 1, 5, by 2
  do i = ii, ii+2-1
   do j = jj, min(jj+2-1,5)
    A(i,j) = ...
```

Parameterized tile sizes

- Parameterized tiled loops for free, PLDI 2007
- TLOG - A Tiled Loop Generator, http://www.cs.colostate.edu/~ln/TLOG/

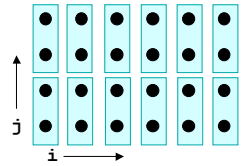
Overview of decoupled approach

- find polyhedron that may contain any loop origins
- generate code that traverses that polyhedron
- post process the code to start a tile origins and step by tile size
- generate loops over points in tile to stay within original iteration space and within tile

Unroll and Jam IS Tiling (followed by inner loop unrolling)

Original Loop

```
do j = 1,2*n
  do i = 1,m
    A(j)= A(j) + B(i)
  enddo
enddo
```



After Tiling

```
do jj = 1,2*n by 2
  do i = 1,m
    do j = jj, jj+2-1
      A(j)= A(j)+B(i)
    enddo
  enddo
enddo
```

After Unroll and Jam

```
do jj = 1,2*n by 2
  do i = 1,m
    A(j)= A(j)+B(i)
    A(j+1)= A(j+1)+B(i)
  enddo
enddo
```

Concepts

Unroll and Jam is the same as Tiling with the inner loop unrolled

Tiling can improve ...

- loop balance
- spatial locality
- data locality
- computation to communication ratio

Implementing tiling

- specification
- checking legality
- code generation

Next Time

Lecture

- Run-time reordering transformations

Suggested Exercises

- after array expansion of the scalar T, is it legal to tile the three loops in Figure 11.23? write the tiled code for a block size of your choice.