Loop Iterator Live-out Value

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Objectives

- Solve a “real” compiler problem
- Write understandable algorithms (communication skill!)
- Prove the correctness of an algorithm
- Study the complexity of an algorithm

Problem: computation of the exit value of loop iterators
The Problem

Compute the exit value of all loop iterators of a given input C program

Example (Input program)

```c
for (i = 1; i < 3; ++i)
    for (j = 1; j < 3; ++j)
        S(i, j)
```

Example (Solution)

```c
i = 3;
j = 3;
```
Step-by-step methodology

1. **Problem definition:** Understand and define the problem
2. **Examples:** Find various example, and compute the desired output by hand
3. **Restriction:** Find an algorithm, maybe restricted to simpler cases
4. **Generalization:** Generalize the algorithm to work on all cases
5. **Proof:** Prove the algorithm is complete and correct
6. **Complexity:** Study the complexity of the algorithm
An Instance of the Problem

Problem: the exit value of loop iterators is not preserved when doing affine transformations

Example (Input code)

```c
for (i = 1; i < 3; ++i)
    for (j = 1; j < 3; ++j)
        S(i, j);
printf("%d", i); // prints '3'
```

Example (After shifting by -1)

```c
for (i = 0; i < 2; ++i)
    for (j = 0; j < 3; ++j)
        S1(i + 1, j);
printf("%d", i); // prints '2'
```
When Does This Problem Occur?

- When we perform transformations on the input code that changes the loop structure, or
- When we rename loop iterators

- **AND** when a loop iterator value is read after the transformed block execution

We want our transformation framework to generate correct code!
Problem definition: Loop Iterator Live-out Value

Context of Application: Affine Framework

Must work on any sub-program that can be represented/transformed in the affine framework:

- loop bounds are affine expressions of surrounding loop iterators and parameters
- same for conditionals
- we do not care about the actual statement body: no impact
  - property: if the program is a scop, then a loop iterator is not modified except by the loop increment statement
- loops iterator increase by step of 1 after 1 execution of the body, until the exit test is false

Example

```c
for (i = lbi; i < Ubi; ++i)
    for (j = lbj(i); j < Ubj(i); ++j)
        for (k = lbk(i,j); k < Ubk(i,j); ++k)
            if (expr(i, j, k))
                S1(i,j, k)
```

Here, `lb`, `Ub` and `expr` are affine expressions of global parameters and their arguments
Input and Output of the Algorithm

Input:
- The AST of the sub-program
- We also have at hand the affine representation of the sub-program

Output:
- An AST to be inserted after the sub-program
- This AST assigns to each loop iterator their exit value in the original program
Methodology

Before designing the algorithm:

- Are we clear on the input? On the output?

- Do we need to learn something about the chosen input/output?
  - AST representation
  - Input language (eg, specifics of for loops in C)

- Can we manually write a few input/output examples?

- Do we need additional restrictions on the input?
Examples of Input and Output [1/9]

Example (Input program)
for (i = 1; i < 3; ++i)
    for (j = 1; j < 3; ++j)
        S(i, j);

Example (desired output)
i = 3;
j = 3;
Examples of Input and Output [2/9]

Example (Input program)

for (i = 1; i < N; ++i)
    S(i, j);

Exercise: write the output
Examples of Input and Output [2/9]

Example (Input program)
for (i = 1; i < N; ++i)
    S(i,j);

Exercise: write the output

Example (Desired output)
if (1 >= N)
    i = 1;
else
    i = N;
Examples of Input and Output [3/9]

Example (Input program)

for (i = 1; i < N; ++i)
  for (j = 0; j < M; ++j)
    S(i, j);

Exercise: write the output
Examples of Input and Output [3/9]

Exercise: write the output

Example (Input program)

```c
for (i = 1; i < N; ++i)
    for (j = 0; j < M; ++j)
        S(i, j);
```

Example (Desired output)

```c
if (1 >= N)
    i = 1;
else {
    i = N;
    if (0 >= M)
        j = 0;
    else
        j = M;
}
Examples of Input and Output [4/9]

Example (Input program)

```c
for (i = 1; i < N; ++i) {
    for (j = 0; j < M; ++j)
        S(i, j);
    for (j = 0; j < P; ++j)
        S(i, j);
}
```

Exercise: write the output
Examples of Input and Output [4/9]

Example (Input program)

for (i = 1; i < N; ++i) {
    for (j = 0; j < M; ++j)
        S(i,j);
    for (j = 0; j < P; ++j)
        S(i,j);
}

Exercise: write the output

Example (Desired output)

if (1 >= N)
    i = 1;
else {
    i = N;
    if (0 >= M)
        j = 0;
    else
        j = M;
    if (0 >= P)
        j = 0;
    else
        j = P;
}
Examples of Input and Output [5/9]

Example (Input program)

```c
for (i = 1; i < N; ++i) {
    for (j = 0; j < M; ++j)
        S(i, j);
    for (j = 0; j < min(N, P); ++j)
        S(i, j);
}
```

Exercise: write the output
Examples of Input and Output [5/9]

Example (Input program)

```c
for (i = 1; i < N; ++i) {
    for (j = 0; j < M; ++j)
        S(i,j);
    for (j = 0; j < min(N, P); ++j)
        S(i,j);
}
```

Exercise: write the output

Example (Desired output)

```c
if (1 >= N)
    i = 1;
else {
    i = N;
    if (0 >= M)
        j = 0;
    else
        j = M;
    if (N > P) {
        if (0 >= P)
            j = 0;
        else
            j = P;
    } else {
        if (0 >= N)
            j = 0;
        else
            j = N;
    }
}
```
Examples of Input and Output [6/9]

Example (Input program)

```c
for (i = 1; i < N; ++i) {
    for (j = 0; j < M; ++j)
        S(i,j);
    if (N > M)
        for (j = 0; j < min(N, P); ++j)
            S(i,j);
}
```

Exercise: write the output
Examples of Input and Output [6/9]

**Example (Input program)**

```c
for (i = 1; i < N; ++i) {
    for (j = 0; j < M; ++j)
        S(i, j);
    if (N > M)
        for (j = 0; j < min(N, P); ++j)
            S(i, j);
}
```

**Exercise: write the output**

**Example (Desired output)**

```c
if (1 >= N)
    i = 1;
else {
    i = N;
    if (0 >= M)
        j = 0;
    else
        j = M;
    if (N > M) {
        if (N > P) {
            if (0 >= P)
                j = 0;
            else
                j = P;
        }
    }
    else {
        if (0 >= N)
            j = 0;
        else
            j = N;
    }
}
```
Examples of Input and Output [7/9]

Example (Input program)

```c
for (i = 1; i < N; ++i) {
    for (j = i; j < M; ++j)
        S(i,j);
    if (i > M)
        for (j = i; j < min(N, P); ++j)
            S(i,j);
}
```

Exercise: write the output
Examples of Input and Output [7/9]

Example (Input program)

```c
for (i = 1; i < N; ++i) {
    for (j = i; j < M; ++j)
        S(i, j);
    if (i > M)
        for (j = i; j < min(N, P); ++j)
            S(i, j);
}
```

Exercise: write the output

Example (Desired output)

```c
if (1 >= N)
    i = 1;
else {
    i = N;
    if (N - 1 >= M)
        j = N - 1;
    else
        j = M;
    if (N > M) {
        if (N > P) {
            if (N - 1 >= P)
                j = N - 1;
            else
                j = P;
        }
        else {
            if (N - 1 >= N)
                j = N - 1;
            else
                j = N;
        }
    }
}
```
Examples of Input and Output [8/9]

Example (Input program)

```c
for (i = 1; i < N; ++i)
    for (j = max(N, M); j < P; ++j)
        S(i, j);
```

Exercise: write the output
Examples of Input and Output [8/9]

Example (Input program)

```c
for (i = 1; i < N; ++i)
    for (j = max(N, M); j < P; ++j)
        S(i,j);
```

Exercise: write the output

Example (Desired output)

```c
if (1 >= N)
    i = 1;
else {
    i = N;
    if (N >= M) {
        if (N < P)
            j = P;
        else
            j = N;
    } else {
        if (M < P)
            j = P;
        else
            j = M;
    }
```
Examples of Input and Output [9/9]

Example (Input program)

```plaintext
for (i = 1; i < N; ++i)
    for (j = max(i, M); j < P; ++j)
        S(i, j);
```

Exercise: write the output
Examples of Input and Output [9/9]

Example (Input program)
for (i = 1; i < N; ++i)
    for (j = max(i, M); j < P; ++j)
        S(i, j);

Exercise: write the output

Example (Desired output)
if (1 >= N)
    i = 1;
else {
    i = N;
    if (N - 1 >= M) {
        if (N < P) {
            j = P;
        }
        else {
            j = N - 1;
        }
    }
    else {
        if (M < P) {
            j = P;
        }
        else {
            j = M;
        }
    }
}
Examples of Input and Output

Exercise: write sample input and the corresponding output