Homework 4, CS420, Fall 2011

Due 10/3 by class time

1. Write a Java, C, or C++ program that has the following menu that allows the user to select an operation, prompts for inputs for the operation, performs it, and displays the results, then redisplays itself.

```
        MENU

0.  Exit
1.  DFT
2.  Inverse DFT
3.  DFT followed by inverse
4.  Multiply two polynomials

  Which?
```

   In contrast to the FFT of the book, the precondition is that the degree bound is a power of three, instead of a power of two. The recursive strategy is to break the problem into three recursive calls, one for coefficients for powers that are divisible by 3, one for coefficients for powers that are 1 mod 3, and one for coefficients for powers that are 2 mod 3.

   (a) If the user prompts for 1, prompt for the order n of the polynomial (one greater than the highest power). Then prompt for the n coefficients. Pad with leading zeros, if necessary, find the DFT, and print it out.

   (b) The protocol for option 2 should be similar.

   (c) The protocol for option 3 should be similar.

   (d) The protocol for option 4 should be to prompt for the order n of the first polynomial, then the n coefficients, then the order n' of the second, then the coefficients. It should pad with zeros as necessary, and get the product in $O(n \log n)$ time, using the versions of the FFT and its inverse you have written above.

2. Prove that when you add the DFT's of two polynomials of length n together, then add the two DFT's as vectors, you get the same result as you do when you add the polynomials together and then take the DFT of the result.

   Think about how we expressed the DFT in terms of a product of a matrix and a column vector. A column vector is just the special case of a matrix that has only one column. So use matrix algebra to prove it. Recall that matrix algebra must use only the algebraic laws of a ring.

3. Prove that if r is a real number and $\overline{a}$ is a vector, then the DFT of $r\overline{a}$ is r times the DFT of $\overline{a}$.

4. What is the DFT of $(0, 1, 0, 0, 0, 0, 0, 0)$?

5. What is the DFT of $(1, 1, 1, 1, 1, 1, 1, 1)$? There is a long-winded way to get this, and an easy way to get it if you remember some facts we used in showing that the DFT matrix has an inverse.

6. What is the DFT of $(1, 0, 1, 1, 1, 1, 1, 1)$? There is a long-winded way to get this also, but there is an easy way that makes use of the last four things you proved, above.