Solution to Challenge 14, Fall '04

We can show by induction on k that the k kuckolds flee the island on the $(k-1)^{th}$ night following the handraising incident.

Since all the husbands raised their hands, there must be at least two husbands of unfaithful wives. As a base case, if there are exactly two, A and B, then A will see B's hand raised, and know that B knows that A's own dear wife has been unfaithful. A will leave the island that night. By similar reasoning, so will B.

If there are exactly three, then if they find out that there are three, they will know that their own wives have been unfaithful. We have just figured out that if there were just two, they would leave the island that night. Being infinitely smart, the husbands can figure this out too. On day 2 when they see that this hasn't happened, each will know that there are three, not two, as they had hoped. They will all leave that night, which is night 2.

If there are exactly four, then if they find out that there are four, they will know that their own wives have been unfaithful. We have just figured out that if there were just three, they would leave the island on night 2. Being infinitely smart, the husbands can figure this out too. On day 3 when they see that this hasn't happened, each will know that there are four, not three, as they had hoped. They will all leave that night, which is night 3, etc, etc.

We can advance through higher and higher values of k, proving for each that if k of the ladies are unfaithful, their husbands will flee with them on night $k-1$.

Chris Hoover had a novel idea: If the husbands whose wives were unfaithful get together and agree to stay on an extra night, they will fool all of the other husbands on the island, and end up with the whole place for themselves!