Solution to Challenge 15, Fall '04

Solution: Squeal to the husbands of two of the other cheating wives.

Here is why this works. If there are three unfaithful wives, including you, the other two will get packed off the island that evening because you told on them. Your husband won't see them the next day, and will therefore be deprived of the evidence against you that he would have if you didn't take this action. (Indeed, he may even take their departure as evidence that you have been faithful!) This breaks the induction from last week's problem at $k = 3$. For higher values of k, it follows that only the two wives that you told on will have to leave the island.

Remarks: *Don't delay in taking action – if you wait a day, you will have to rat out three of of the other wives, and if you wait two days, you will have rat out four of them, etc. Indeed, some solvers suggested waiting until day $k-2$ and then telling on all of the other wives; this breaks the induction at the last moment, at the expense of all of the unfaithful wives except for yourself.*

Doug Beeman noted that if the wife dislikes n of the k cheating wives, she should wait until day $n-1$ to tell their husbands, thereby also satisfying a secondary goal of getting rid of them from the island.

The most devious solution came from Florian Hulpke, who proposed that the wife, Alice, should tell her husband that an innocent wife, Eve, had been unfaithful, and then wait until night $k-1$ to tell Eve's husband that Eve had been unfaithful. This would ensure that the k wives that Alice's husband now thought were unfaithful, including Eve, were ushered off the island on night $k-1$, giving him the impression that everything was normal in his marriage.

Update: *Florian's solution clearly takes a couple of minor liberties with the guidelines of the problem. For instance, it doesn't examine whether Alice's husband might wonder why he had only heard about Eve through Alice, and not through the usual grapevine. Also, it does not examine whether Eve's husband, possessing infinite smarts and knowledge of Alice's infidelity, might not see through her last-minute "announcement."*

My solution – having Alice truthfully tell two husbands about their wives – doesn't suffer this defect, since the husbands can see right through Alice's plan and still come to the conclusion that she's telling the truth. However, Rocke Verser of Loveland has pointed out that it violates a strict reading of the ground rules, which is that the natives don't mention a wife's infidelities to her spouse.

I had heard Challenge 14 elsewhere; Challenge 15 was my own invention. Here is perhaps a better way to formulate the puzzle so that it gets around Rocke's objection:

> *After having the husbands raise their hands, the missionary figures out that the incident will cause all of the unfaithful wives to get caught. For reasons of his own, he wants to protect one of the wives' secret. How can he accomplish this? Assume that he is not bound by the cultural norms of the island and that there are at least three unfaithful wives.*

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