

Solution to Challenge 3 Problem 1, Spring '04

Number the coins from 1 to 12 in Base 3 as follows:

- (001, 002, 010, 011, 012, 020, 021, 022, 100, 101, 102, 110)

On the first weighing, consider only the rightmost digits of the coins. Put a coin in the left pan if its rightmost digit is 0, leave it on the table if it is 1, and put it in the rightmost pan if it's two. That is, put {010, 020, 100, 110} on the left pan, leave {001, 011, 021, 101} on the table, and put {002, 012, 022, 102} on the right pan.

On the second weighing, repeat the operation with the second-to-rightmost digits, and on the last weighing, repeat it with the leftmost digits. Use trusted coins to counterbalance any mismatch in the number of coins in the two pans.

Suppose that the counterfeit coin is heavy. If the scale tips left, the last digit of the counterfeit's label is 0, if it balances, the last digit is 1, and if it tips right, the last digit is 2. The other two weighings "read" the other two digits of the counterfeit's label in the same way, uniquely identifying the counterfeit.

On the other hand, if interpreting the outcomes as digits yields a number that is larger than the largest label, 110, the counterfeit is light; complement the roles of digits 0 and 2 and you'll get the label of the counterfeit.