

Solution to Challenge 9, Spring '05

You can get out of the maze without leaving any coins behind!

Consider the suggested algorithm that leaves coins behind. Let us say that a room is *white* if it has never been entered, and it turns *gray* when you enter it for the first time. Eventually, you will retreat to it and find all of the corridors marked, and you will retreat from it through the unique corridor marked with heads. Let us say that it turns *black* at this point.

Treat the exterior of the maze as a “room”. It is easy to see that the invariant is maintained that no white room is adjacent to a black room and that a gray room will eventually be blackened if you don’t reach the exit first. Therefore, the algorithm can only halt if you reach the exit, or all nodes reachable from your starting room have been blackened. In the latter case, the exit is unreachable, which we have assumed is not the case. You will find the exit with the suggested algorithm.

How can we make it cheaper? Any corridor from a gray room to a black room is marked with a coin at the gray room, which serves to block you from entering a black room from a gray room. The coins in the black room serve no purpose, so you can take them with you when you retreat from a room as it turns black.

Notice that if you follow this regimen, only the gray rooms have coins, and the gray rooms form a path of retreat from your current location to your starting location. This is even true when you have found the exit. Re-enter the maze, “retreat” down this path, marking a new retreat back to the exit with new heads. Then re-retreat back to the exit collecting all the coins on the path as you go!