Solution to the Barnes and Noble Challenge 12, Spring '05

If there were no gas/restroom combinations, the problem would be easy: stop at a restroom only when you can't make it to the next one, and similarly for gas stations. When the gas/restroom combinations are introduced, this puts you in a dilemma: should you stop a little early at a combination stop in order to take advantage of the added efficiency that it offers?

Greedy considerations dictate that when you stop at a gas-only stop, it doesn't hurt to stop at the farthest one within gas range. A similar observation applies to restroom-only stops. Whenever you stop at a gas/restroom combination, it can't hurt to stop at the farthest combination stop within both ranges. Let us assume that you adopt this strategy.

Let us create a finite state machine to model the problem. A *state* consists of a stop and remaining gas and restroom ranges as you leave the stop. There is only one state you can leave a combination stop in. There are $O(n)$ states that you could leave a restroom-only stop in; one for each possible last prior gas stop, which determines how much gas remains in your tank. Similarly, there are $O(n)$ states that you could leave a gas-only stop in. Given this strategy, there are only two transitions out of any state: one to the farthest combination stop in range, and one to the farthest one-service stop you can reach before you are forced to stop.

The answer to the problem is just the length of a shortest path from the start state to the destination state. There are $O(n)$ states for each stop, for a total of $O(n^2)$, and twice this many transitions (also $O(n^2)$). It is not hard to construct the graph of the transition diagram in $O(n^2)$ time. The length of a shortest path in a graph can be found in linear time by breadth-first search, for example. This gives an $O(n^2)$ time bound to find an optimum itinerary.

(Faster time bounds are possible, using the observation that the distances from the states at a stop to the destination state can differ by at most one.)