

Solution to the Ice Cream Challenge 14, Spring '05

The prisoners who escape are the ones in cell numbers that are perfect squares. That is, they are the ones in $\{1, 4, 9, 16, 25, 36, \dots\}$.

Why is this? Let m be a cell number. The number of times the lock was turned on cell m is equal to the number of divisors of m ; the jailer turns the lock on the k^{th} round if and only if k divides m . If $p_1^{n_1} p_2^{n_2} \dots p_j^{n_j}$ is the prime factorization of m , then the number of factors is $(n_1 + 1)(n_2 + 1) \dots (n_j + 1)$. Since the lock starts out locked, a prisoner will escape only if this number is odd, which happens if and only if each n_i is even. Each n_i is even if and only if m is a perfect square.