Lemma 4.

If a language L is accepted by a PDA, then L is context-free.

Let M be the PDA (K, Σ, T, Δ, S, ∅) that accepts by empty store.

We use Z_0 to designate the symbol initially residing at the top of the stack.

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We will construct a context-free grammar G = (V, Σ, R, S)

Let $V = \{< S >\} \cup \{< q, A, p > \mid q, p \in K \text{ and } A \in T\}$

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The nonterminals <S> and <q, A, p>
represent elements of a lefthand derivation in G
which simulate the operation of M.

<q, A, p> responds to some string x
IFF x causes M to pop A from its stack
and move from state q to p.

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Construction:

1. Foreach q $\in$ K construct
   $< S > \longrightarrow < q_0, Z_0, q >$

   $< S > \longrightarrow$ <nonterminal>
   where
   <nonterminal> is of the form
   (current_state, stack, destination_state)
   $< S > \longrightarrow$ <initial_state, Z_0, all_other_states>

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For each new <nonterminal> state created,
we must generate new productions.

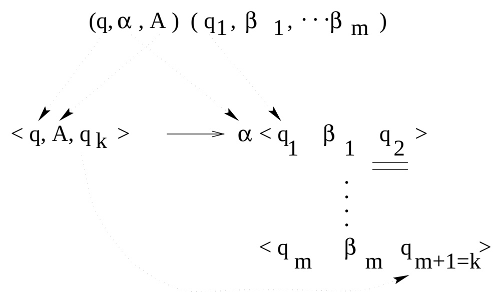
Intuition: the goal is to move from the initial state
to other states while emptying the stack ...
I.E., expand the "production tree" as much as possible,
prune deadends later.

2. For $(q, a, A)(q_1, \beta_1 \cdots \beta_m)$ generate

$$
\begin{aligned}
< q, A, q_{m+1} > \quad \longrightarrow a \quad & < q_1, \beta_1, q_2 >, \\
& < q_2, \beta_2, q_3 >, \ldots \\
& < q_m, \beta_m, q_{m+1} >
\end{aligned}
$$

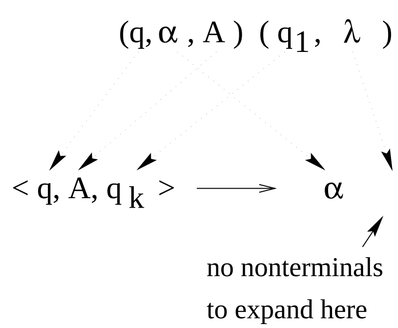
where a is the symbol scanned
$\beta_1 \cdots \beta_m$ are stack symbols
q, q_1, ..., $q_{m+1} \in$ K
A is a stack symbol

Each β_i will be expanded to a new nonterminal.



from q_2 to q_{k-1} we explore
all possible computation paths.

3. Foreach ((q, a, A), (q_1, λ)) $\in \Delta$
   Generate <q, A, q_1> $\longrightarrow$ a
   This is a special case of rule (2) in which symbol "a" is read and A is simply popped off the stack.



Example 5.3, page 119 Hopcroft and Ullman

$$
\begin{aligned}
M &= (K, \Sigma, T, \Delta, q_0, \emptyset) \\
K &= (q_0, q_1) \\
\Sigma &= (0, 1) \\
T &= (X, Z_0) \\
\Delta &= \{ \quad ((q_0, 0, Z_0), (q_0, XZ_0)), \\
& \qquad ((q_0, 0, X), (q_0, XX)), \\
& \qquad ((q_0, 1, X), (q_1, \lambda)), \\
& \qquad ((q_1, 1, X), (q_1, \lambda)), \\
& \qquad ((q_1, \lambda, X), (q_1, \lambda)), \\
& \qquad ((q_1, \lambda, Z_0), (q_1, \lambda))\}
\end{aligned}
$$

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All possible nonterminals

$<q_0, X, q_0>$ $<q_1, X, q_0>$

$<q_0, X, q_1>$ $<q_1, X, q_1>$

$<q_0, Z_0, q_0>$ $<q_1, Z_0, q_0>$

$<q_0, Z_0, q_1>$ $<q_1, Z_0, q_1>$

and, of course, $<S>$

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1. Rules starting from the distinguished symbol

R1 $\langle S \rangle \longrightarrow \langle q_0, Z_0, q_0 \rangle$

R2 $\langle S \rangle \longrightarrow \langle q_0, Z_0, q_1 \rangle$

Two new nonternimals now exist for which productions must be written.

$\langle q_0, Z_0, q_0 \rangle \ldots$ N1

$\langle q_0, Z_0, q_1 \rangle \ldots$ N2

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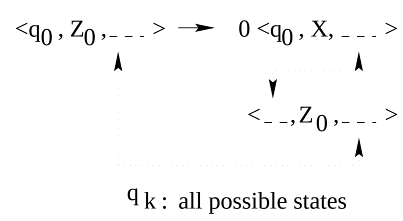
Transitions that match q_0 and Z_0 are relevant:

In this case:

$(\underline{q}_0, 0, \underline{Z}_0)\ (q_0, XZ_0)$

General Form of Productions



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New Nonterminal N1 $<q_0, Z_0, q_0>$

R3	$<q_0, Z_0, q_0>$	$\longrightarrow$	$0<q_0, X, q_0>$
			$<q_0, Z_0, q_0>$
R4	$<q_0, Z_0, q_0>$	$\longrightarrow$	$0<q_0, X, q_1>$
			$<q_1, Z_0, q_0>$

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New Nonterminal N2 $<q_0, Z_0, q_1>$

R5	$<q_0, Z_0, q_1>$	$\longrightarrow$	$0<q_0, X, q_0>$
			$<q_0, Z_0, q_1>$
R6	$<q_0, Z_0, q_1>$	$\longrightarrow$	$0<q_0, X, q_1>$
			$<q_1, Z_0, q_1>$

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New Nonterminals Introduced:

$<q_0, X, q_0>$	...	N3
$<q_0, X, q_1>$	...	N4
$<q_1, Z_0, q_0>$	...	N5
$<q_1, Z_0, q_1>$	...	N6

No transitions matching nonterminal N5

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Nonterminal N3 $<q_0, X, q_0>$
Matches transition $(q_0, 0, X), (q_0, XX)$

R7	$<q_0, X, q_0>$	$\longrightarrow$	$0 <q_0, X, q_0>$ $<q_0, X, q_0>$
R8	$<q_0, X, q_0>$	$\longrightarrow$	$0 <q_0, X, q_1>$ $<q_1, X, q_0>$

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Nonterminal N4 $<q_0, X, q_1>$

Matches transition $(q_0, 0, X), (q_0, XX)$

R9	$<q_0, X, q_1>$	$\longrightarrow$	$0 <q_0, X, q_0>$
			$<q_0, X, q_1>$
R10	$<q_0, X, q_1>$	$\longrightarrow$	$0 <q_0, X, q_1>$
			$<q_1, X, q_1>$

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Nonterminal N4 $<q_0, X, q_1>$

Also matches transition $((q_0, 1, X), (q_1, \lambda))$

R11	$<q_0, X, q_1>$	$\longrightarrow$	1

Nonterminal N6 $<q_1, Z_0, q_1>$

Also matches transition $((q_1, \lambda, Z_0), (q_1, \lambda))$

R12	$<q_1, Z_0, q_1>$	$\longrightarrow$	λ

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New Nonterminals Introduced:

$<q_1, X, q_1>$ ... N7

$<q_0, X, q_0>$ ... N8

No transitions matching nonterminal N8

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Nonterminal N7 $<q_1, X, q_1>$
Matches transition $((q_1, 1, X), (q_1, \lambda))$

R13 $<q_1, X, q_1> \longrightarrow 1$

Nonterminal N7 $<q_1, X, q_1>$
Matches transition $((q_1, \lambda, X), (q_1, \lambda))$

R14 $<q_1, X, q_1> \longrightarrow \lambda$

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There are no more new nonterminals.
But not all of the productions are meaningful.

E.G., nonterminals $<q_1, Z_0, q_0>$ (N5)
and $<q_1, X, q_0>$ (N8) are undefined.

1. Remove productions containing undefined nonterminals.
   this can cause other productions to be undefined.
2. We must also remove infinite recusions, since these will never yield all terminals.

This is analogous to traps: C = 0C + 1C

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R1	$<S>$	$\longrightarrow$	$<q_0, Z_0, q_0>$
R2	$<S>$	$\longrightarrow$	$<q_0, Z_0, q_1>$
R3	$<q_0, Z_0, q_0>$	$\longrightarrow$	$0<q_0, X, q_0> <q_0, Z_0, q_0>$
R4	$<q_0, Z_0, q_0>$	$\longrightarrow$	$0<q_0, X, q_1> <q_1,Z_0, q_0>$
R5	$<q_0, Z_0, q_1>$	$\longrightarrow$	$0<q_0, X, q_0> <q_0,Z_0, q_1>$
R6	$<q_0, Z_0, q_1>$	$\longrightarrow$	$0<q_0, X, q_1> <q_1,Z_0, q_1>$
R7	$<q_0, X, q_0>$	$\longrightarrow$	$0 <q_0, X, q_0> <q_0, X, q_0>$
R8	$<q_0, X, q_0>$	$\longrightarrow$	$0 <q_0, X, q_1> <q_1, X, q_0>$

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R9	$<q_0, X, q_1>$	$\longrightarrow$	$0 <q_0, X, q_0> <q_0, X, q_1>$
R10	$<q_0, X, q_1>$	$\longrightarrow$	$0 <q_0, X, q_1> <q_1, X, q_1>$
R11	$<q_0, X, q_1>$	$\longrightarrow$	1
R12	$<q_1, Z_0, q_1>$	$\longrightarrow$	λ
R13	$<q_1, X, q_1>$	$\longrightarrow$	1
R14	$<q_1, X, q_1>$	$\longrightarrow$	λ

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Since we don't need this complex form of nonterminals, we can rewrite them as follows:

R1	<S>	$\longrightarrow$	N1
R2	<S>	$\longrightarrow$	N2
R3	N1	$\longrightarrow$	0 N3 N1
R4	N1	$\longrightarrow$	0 N4 N5
R5	N2	$\longrightarrow$	0 N3 N2
R6	N2	$\longrightarrow$	0 N4 N6
R7	N3	$\longrightarrow$	0 N3 N3

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R8	N3	$\longrightarrow$	0 N4 N8
R9	N4	$\longrightarrow$	0 N3 N4
R10	N4	$\longrightarrow$	0 N4 N7
R11	N4	$\longrightarrow$	1
R12	N6	$\longrightarrow$	λ
R13	N7	$\longrightarrow$	1
R14	N7	$\longrightarrow$	λ

R4 references N5, which is undefined.

R8 references N8, which is undefined.

Drop these Rules

We now note that R7 is in an infinite loop.

Removing R7 makes N3 meaningless – remove R3, R5 and R9.

Since R3 and R4 are gone, N1 is meaningless, so remove R1.

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R2	<S>	$\longrightarrow$	N2
R6	N2	$\longrightarrow$	0 N4 N6
R10	N4	$\longrightarrow$	0 N4 N7
R11	N4	$\longrightarrow$	1
R12	N6	$\longrightarrow$	λ
R13	N7	$\longrightarrow$	1
R14	N7	$\longrightarrow$	λ

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