CS 160, Spring 2013
Homework Assignment H3
Propositional Proofs and Predicate Logic

Please turn in a paper copy of this assignment in class on Mar. 15, no electronic submissions. Late assignments can be submitted under the door of my office in COMSC 256 until 10PM.

1. **Translation**: Find and list 3 propositions in the following English sentence, then construct a compound statement in propositional logic to match the English meaning (10 points).

   If the tire is flat then change the tire or call road service.

   p)  
   q)  
   r)  

   Compound Proposition: ________________________________

2. **Translation**: Translate the following propositional logic into an English statement (10 points).

   \((p \land q) \rightarrow r\)

   \(p\): the Java syntax of the program is legal
   \(q\): the logic of the Java code works correctly
   \(r\): the Java program will run without errors
3. **Logical Equivalences**: Given the equivalence proof shown below, find the name of the logical equivalence from p. 27 of Rosen used to perform each transformation (15 points).

\[
\neg (q \lor F) \land (p \lor \neg p)
\]

\[
\neg (q \lor F) \land (p \lor \neg p)
\equiv \neg q \land (p \lor \neg p)
\equiv \neg q \land T
\equiv \neg q
\]

4. **Logical Equivalences**: Given the equivalence proof shown below, find the name of the logical equivalence from p. 27 of Rosen used to perform each transformation (10 points).

\[
(p \land \neg q) \lor (p \land F)
\]

\[
(p \land \neg q) \lor (p \land F)
\equiv p \land (\neg q \lor F)
\equiv p \land \neg q
\]
5. **Logical Proof**: Translate the following logic problem from English to propositional logic, you should have exactly three axioms and one conclusion (15 points).

If the weather is bad or the freeway is closed, then we cannot go skiing tomorrow. Given that we go skiing tomorrow and the freeway is open, prove that the weather must not be bad.

$p$: the weather is bad

$q$: the freeway is closed

$r$: we go skiing tomorrow

**AXIOMS**:

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**CONCLUSION**: 

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If the weather is bad or the freeway is closed, then we cannot go skiing tomorrow. Given that we go skiing tomorrow and the freeway is open, prove that the weather must not be bad.

$p$: the weather is bad

$q$: the freeway is closed

$r$: we go skiing tomorrow

**AXIOMS**:

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**CONCLUSION**: 

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6. **Logical Proof**: Prove the conclusion based on the axioms, using the definitions in the previous problem. You may use the inference rules and logical equivalences posted on the course website, but you must identify what you use at each step of your proof (20 points). You may use a different number of steps than listed below, as long as your proof is correct!

1. ____________________________

2. ____________________________

3. ____________________________

4. ____________________________

5. ____________________________

6. ____________________________

7. ____________________________

8. ____________________________

9. ____________________________
7. **Predicate Logic**: Show the translation to predicate logic, including the domain, for the following English statements, and specify whether the statement is true. If the statement is false, show a counterexample (5 points each = 10 points).

There exists a positive integer greater than -2 and less than 2.

The square of all integers is greater than -1.

8. **Predicate Logic**: Show the translation to English statements of the following predicate logic, and specify whether the predicate is true. If the statement is true, show an example, if false, show a counterexample (5 points for each = 10 points).

\[ \exists x \in \mathbb{N}, x^2 > 60 \land x^2 < 80 \]

\[ \forall x \in \mathbb{N}, (x \times 5) > (x + 5) \]