Question 1-7 and 9: 10 points/each. Question 8 a) and b): 5 points/each. Question 8 c) and d): 10 points/each.

1. (a) YES. \( X^+ = XYWZ \)  
   \[ 5 \text{ pts} \]
   (b) No. \( (XW)^+ = XW, Z \) is not in there.  
   \[ 5 \text{ pts} \]

2. \( r \) satisfies \( AD \rightarrow B, C \rightarrow DE, CD \rightarrow A, AE \rightarrow B \). But \( r \) does not satisfy \( A \rightarrow B \) & \( AE \rightarrow B \) since the first two tuples of \( r \) have the same values for \( A \), but different values for \( B \).  
   \[ 2 \times 5 = 10 \text{ pts} \]

3. (a) \( A^+ = ABEC \)  
   (b) \( (AE)^+ = ABEC \)  
   (c) \( (ADE)^+ = ABCDEI \)  
   \[ 4 \text{ pts} \times 3 = 12 \text{ pts} \]

4. To see that \( F \) and \( G \) are equivalent, we need to verify that every FD in \( F \) is in \( G^+ \), and vice versa. We first check if \( F \subseteq G^+ \): (i) \( A^+ = ACD \), so \( A \rightarrow C \) is in \( G^+ \). (ii) \( AC_G^+ = ACD \), so \( AC \rightarrow D \) is in \( G^+ \). (iii) \( (E)_G^+ = ACDEH \), so \( E \rightarrow AD \) is in \( G^+ \).
   Next we verify if \( G \subseteq F^+ \): (i) \( (A)_G^+ = ACD \), so \( A \rightarrow CD \) is in \( F^+ \). (ii) \( (E)_F^+ = ACDE \), so \( E \rightarrow AHE \) is not in \( F^+ \).
   Thus \( F \) and \( G \) and NOT equivalent.  
   \[ 10 \text{ pts} \]

5. (a) Let \( F = \{ A \rightarrow BC, B \rightarrow C, AB \rightarrow D \} \). We need to obtain an equivalent set of FDs that satisfies the three properties of a minimal cover.
   - Right side of each FD in \( F \) must be a single attribute: so we replace \( F \) by \( F_1 = \{ A \rightarrow B, A \rightarrow C, B \rightarrow C, AB \rightarrow D \} \).
   - No extraneous attributes on the left side. We first check if \( A \) can be deleted from \( AB \rightarrow D \). We can do so if \( B \rightarrow D \) follows from \( F_1 \). Since \( (B)_F^+ = BC \), the answer is NO.
     We next check if \( B \) can be deleted from \( AB \rightarrow D \). We can do if \( A \rightarrow D \) follows from \( F_1 \). Since \( (A)_F^+ = ABCD \), the answer is YES. Let \( F_2 = \{ A \rightarrow BC, B \rightarrow C, A \rightarrow D \} \).
   - No redundant FDs: \( A \rightarrow C \) can be deleted from \( F_2 \). Minimal cover =  
     \[ \{ A \rightarrow B, B \rightarrow C, A \rightarrow D \} \]
(b) Let $F = \{A \to C, AB \to C, C \to D, I, EC \to AB, EI \to C\}$. We need to obtain an equivalent set of FDs that satisfies the three properties of a minimal cover.

- We replace $F$ by $F_1 = \{A \to C, AB \to C, C \to D, C \to I, EC \to A, EC \to B, EI \to C\}$.
- No extraneous attributes on left side: it can be checked that $B$ can be deleted from $AB \to C$. Let $F_2 = \{A \to C, C \to D, C \to I, EC \to A, EC \to B, EI \to C\}$.
  Similarly, $D$ can be removed from $CD \to I$.
- No redundant FDs: None of the FDs are redundant.

Minimal Cover is $\{A \to C, C \to D, C \to I, EC \to A, EC \to B, EI \to C\}$.

6. (a) $\rho$ is loss since $AB \cap BCD = B$, and neither $B \to AB$ nor $B \to BCD$ is true. (you can use chase algorithm too).

(b) The initial table:

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>a1</td>
<td>b4</td>
<td>b5</td>
<td>a4</td>
<td>a5</td>
<td>a6</td>
</tr>
<tr>
<td>a2</td>
<td>b6</td>
<td>a3</td>
<td>b2</td>
<td>b3</td>
<td>a6</td>
</tr>
<tr>
<td>a3</td>
<td>b7</td>
<td>a4</td>
<td>b5</td>
<td>a5</td>
<td>a6</td>
</tr>
<tr>
<td>a4</td>
<td>b1</td>
<td>a3</td>
<td>b3</td>
<td>b2</td>
<td>a6</td>
</tr>
<tr>
<td>a5</td>
<td>b2</td>
<td>a3</td>
<td>b4</td>
<td>b5</td>
<td>a6</td>
</tr>
<tr>
<td>a6</td>
<td>b3</td>
<td>a4</td>
<td>b1</td>
<td>b2</td>
<td>a5</td>
</tr>
</tbody>
</table>

By applying the three FDs, we obtain a tableau that has one row consisting entirely of $a$'s. Hence $\rho$ is lossless.

7. (a) No change if applying step 1.

- We can check that $F$ does not have any extraneous FDs.
- For step 3, we need only consider $AB \to C$. We can see that $B$ is redundant by considering $(A)^+_B$. Since $(A)^+_B = ABC$, $B$ is redundant. Thus, we replace $AB \to C$ by $B \to C$ to get the minimal cover $\{A \to C, C \to A, A \to B\}$. Unfortunately, this is not a minimal cover since the FD $A \to B$ is now extraneous.

8. (a) $IS$ is a candidate key since (i) $(IS)^+ = IBO$ and $S^+ = SD$.

(b) $IS$ is the only candidate key since neither $I$ nor $S$ appear in the right-hand side if any FD. Therefore any candidate key will have to contain both $I$ and $S$. But since $IS$ forms a candidate key, it is the only candidate key.

(c) One possible decomposition is obtained as follows:

(d) We first find the minimal cover $F = \{S \to D, I \to B, IS \to Q, B \to O\}$. It turns that $F$ is minimal. Thus $\{SD, IB, ISQ, BO\}$ is required decomposition. Notice $ISQ \to BOISQD$, hence the decomposition has lossless property.

9. Let’s rewrite the relation scheme as $R = EPBT$ and FDs as $EP \to T, P \to B, E = EMP_ID, P = PROJECT, B = PROJECT_BUDGET, T = TIME_SPENT_BY_PERSON_ON_PROJECT$.

(a) Since $EP$ is the only candidate key of $R$, both $E$ and $P$ are prime attributes while $T$ and $B$ are nonprime attributes.
Figure 1: 8.c decomposition

(b) R is not in 3NF since P → B holds in R, P is not a superkey and B is not a prime attribute.  
(c) R is not BCNF since it’s not 3NF.