Reliability of Multi-component Systems

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Reliability of Multi-component Systems

- Software system: number of modules.
- Individual modules developed and tested differently: different defect densities and failure rates.
  - Sequential execution
  - Concurrent execution
  - N-version systems
Sequential execution

• Assume one module executed at a time.
• $f_i$: fraction of time module $i$ under execution; $\lambda_i$ its failure rate
• Mean system failure rate:

$$\lambda_{sys} = \sum_{i=1}^{n} f_i \lambda_i$$
Sequential Execution (cont.)

• **T**: mean duration of a single transaction

• module i is called $e_i$ times during $T$, each time executed for duration $d_i$

\[ f_i = \frac{e_i \cdot d_i}{T} \]
Sequential Execution (cont.)

- System reliability $R_{sys} = \exp(-\lambda_{sys} T)$

$$R_{sys} = \exp\left(- \sum_{i=1}^{n} e_i d_i \lambda_i \right)$$

- Since $\exp(-d_i \lambda_i)$ is $R_i$,

$$R_{sys} = \prod_{i=1}^{n} (R_i)^{e_i}$$

$$\lambda_{sys} = \sum_{i=1}^{n} f_i \lambda_i$$

$$f_i = \frac{e_i d_i}{T}$$
Concurrent execution

- Concurrently executing modules: all run without failures for system to run
- \( j \) concurrently executing modules

\[
\lambda_{sys} = \sum_{j=1}^{m} \lambda_j
\]
N-version systems

- Critical applications, like defense or avionics
- Each version is implemented and tested independently
- Common implementation uses triplication and voting on the result
**N-version Systems (Cont.)**

\[
R_{\text{sys}} = 1 - (1-R)^3 - 3R(1-R)^2
\]

- \( R = 0.9 \Rightarrow R_{\text{sys}} = 0.972 \)
- \( \overline{R} = 0.1 \Rightarrow \overline{R}_{\text{sys}} = 0.028 \)
N-version systems: Correlation

• Correlation significantly degrades fault tolerance
• Significant correlation common in N-version (Knight-Leveson)
• Is it cost effective?
N-version systems: Correlation

- 3-version system
- $q_3$: probability of all three versions failing for the same input.
- $q_2$: probability that any two versions will fail together.
- Probability $P_{sys}$ of the system failing for a transaction

$$P_{sys} = q_3 + 3q_2$$
N-version systems: Correlation

• Example: data collected by Knight-Leveson; computations by Hatton

• 3-version system, probability of a version failing for a transaction 0.0004

• in the absence of any correlated failures

\[ P_{sys} = (0.0004)^3 + 3(1 - 0.0004)(0.0004)^2 \]

\[ = 4.8 \times 10^{-7} \]

• Uncorrelated improvement factor of 0.0004/4.8 \times 10^{-7} = 833.3
N-version systems: Correlation

• Uncorrelated improvement factor of 0.0004/4.8 x 10^{-7} = 833.3

• Correlated: \( q_3 = 2.5 \times 10^{-7} \) and \( q_2 = 2.5 \times 10^{-6} \)

• \( P_{sys} = 2.5 \times 10^{-7} + 3 \times 2.5 \times 10^{-6} = 7.75 \times 10^{-6} \)

• Improvement factor: 0.0004/7.75 \times 10^{-6} = 51.6

• State-of-the-art techniques can reduce defect density only by a factor of 10!

• Thus 3-version system may be worth considering in some cases.