Markov Modeling: Outline

- System states, transition probability rates, differential equations
- Temporary faults: presence, activity
- Single module and TMR
- Solving system of differential equations
- Use of computational tools
Temporary Faults, Markov Modeling

- Here we consider both temporary and permanent failures.
- We will look at general methods for Markov modeling of systems.
- Time-independent model:
  \[ p = P(\text{fault active}), \quad 1-p = P(\text{fault not active}) \]
- Time dependent Markov model: parameters \( \lambda, \mu \)

\[ \lambda \\
\text{Good} \\
0 \\
\mu \\
\text{Bad} \\
1 \]

\[ p(t) = p_0(0) e^{-\lambda t} + \frac{\mu}{\lambda + \mu} (1 - e^{-(\lambda + \mu) t}) \]

- If observed at intervals \( \gg 1/\lambda \), reduces to
  \[ p_0 = \frac{\mu}{\lambda + \mu} = 1 - p \]

Generalization: Fault not present, not active (but present), active:

\[ \text{justification} \]

Note: if \( \mu \to 0 \), permanent fault
Markov Modeling

- Identify transition probability rates
- Identify good and bad states
- Construct a state model
- Solve resulting system of differential equations to obtain probability of being in a good (or bad) state.

TMR: Markov Model

\[ \begin{align*}
\mu' &= \mu / (\lambda + \mu) \\
\lambda' &= \lambda / (\lambda + \mu)
\end{align*} \]

Availability: all transitions considered.

“Reliability”: Remove transitions back from bad states.
TMR: Markov Model (cont.)

- Availability (instantaneous):
  \[ A(t) = 3G^3 - 2G^2 \text{ where } G = e^{-\lambda t} + \mu (1 - e^{-\lambda t}) \]

- Reliability (duration 1):
  - remove transitions from bad to good states
  - get system of equations

\[
\frac{dP_{2,1}}{dt} = 2\nu \lambda' P_{1,0} + 2\lambda P_{2,0} + 2\nu \mu' P_{1,1} + 2\mu P_{2,2} - (\mu + \lambda + \nu \mu' + \nu \lambda') P_{2,1}
\]

Chapman-Kolmogorov differential equations

TMR: Markov Model (cont.)

- Algebraic solution:
  \[ \dot{P}(t) = T.P(t) \]
  where \( T \): state transition matrix,
  \( P(t) \): vector containing state probabilities

Let \( L[P(t)] = p(s), \) then
  \[ s.p(s) - P(0) = T.p(s) \]
  \[ [sI-T].p(s) = P(0) \] [here \( I \) is identity matrix]
  \[ p(s) = [sI-T]^1P(0) \] [matrix inversion]
  \[ P(t) = L^{-1}[[sI-T]^1P(0)] \]

- The system of differential equations may be "stiff" requiring Geary's method etc.
Markov Software Packages

- A number of tools for reliability computation are available (Sharpe, SURF, etc)
  - http://www.ee.duke.edu/~chirel/research1.html
  - http://www.laas.fr/surf/surf-uk.html
- Generally provide graphical interface.
- Limitations:
  - Number of states etc
  - Computational accuracy
  - Performance
- Some allow complex failure types etc.

Ex: Markov Modeling (Mathcad)

\[
\frac{dp_s(t)}{dt} = -\lambda p_s(t) + \mu p_1(t)
\]
\[
\frac{dp_1(t)}{dt} = +\lambda p_0(t) - \mu p_1(t)
\]
\[
\dot{P}(t) = T\cdot P(t) \quad T = \begin{bmatrix}
-\lambda & \mu \\
\lambda & -\mu
\end{bmatrix}
\]
\[
[sI - T]^{-1} = \begin{bmatrix}
s + \lambda & -\mu \\
-\lambda & s + \mu
\end{bmatrix}^{-1}
\]
\[
= \begin{bmatrix}
s + \mu & \mu \\
\lambda & s + \mu + \lambda \\
\end{bmatrix}
\begin{bmatrix}
s(s + \mu + \lambda) \\
\lambda s(s + \mu + \lambda)
\end{bmatrix}
\]

Good 0

\mu

Bad 1
Ex: Markov Modeling (Mathcad)

\[
[sI - T]^3 P(0) = \begin{bmatrix}
\frac{s + \mu}{\lambda} \\
\frac{s(s + \mu + \lambda)}{\lambda}
\end{bmatrix}
\]

\[
L^t([sI - T]^3 P(0)) = \begin{bmatrix}
\frac{\mu + \lambda}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu)t} \\
\frac{\mu + \lambda}{\mu + \lambda} - \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu)t}
\end{bmatrix} = \begin{bmatrix}
P_0(t) \\
P_1(t)
\end{bmatrix}
\]