

CS 160, Fall 2013  
**Homework 1 Answer Key**  
Propositional Proofs

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1. (30 points) Prove using Rules of Inference and Logical Equivalences. State specific rule for each step. Use as many steps as you need.

Given:  $\mathbf{p \vee q}$

$\mathbf{p \rightarrow r}$

$\mathbf{\neg r}$

Prove:  $\mathbf{q}$

**Step      Statement      Reason (Include Appropriate Step # if applicable)**

1.       $\mathbf{p \rightarrow r}$       **Given**

2.       $\mathbf{\neg r}$       **Given**

3.       $\mathbf{\neg p}$       **Modus Tollens (1, 2)**

4.       $\mathbf{p \vee q}$       **Given**

5.       $\mathbf{q}$       **Disjunctive Syllogism (3, 4)**

2. (40 points) Prove using Rules of Inference and Logical Equivalences. State specific rule for each step. Use as many steps as you need.

Given:  $(p \vee q) \rightarrow \neg r$

$r$

$\neg q$

Prove:  $\neg p$

Step      Statement      Reason (Include Appropriate Step # if applicable)

- |    |                                 |                      |
|----|---------------------------------|----------------------|
| 1. | $(p \vee q) \rightarrow \neg r$ | Given                |
| 2. | $r$                             | Given                |
| 3. | $\neg (p \vee q)$               | Modus Tollens (1, 2) |
| 4. | $\neg p \wedge \neg q$          | De Morgan's (3)      |
| 5. | $\neg p$                        | Simplification (4)   |

3. (30 points) Prove this equivalence using **Rules of Inference and** Logical Equivalences. State specific rule for each step. Use as many steps as you need.

$$((\neg(\mathbf{r} \rightarrow \mathbf{s})) \vee \mathbf{r}) \equiv \mathbf{r}$$

<u>Step</u>	<u>Statement</u>	<u>Reason (Include Appropriate Step # if applicable)</u>
1.	$(\neg(\mathbf{r} \rightarrow \mathbf{s}) \vee \mathbf{r})$	<b>Given</b>
2.	$(\neg(\neg\mathbf{r} \vee \mathbf{s}) \vee \mathbf{r})$	<b>Implication Law (1)</b>
3.	$(\neg\neg\mathbf{r} \wedge \neg\mathbf{s}) \vee \mathbf{r}$	<b>De Morgan's Law (2)</b>
4.	$(\mathbf{r} \wedge \neg\mathbf{s}) \vee \mathbf{r}$	<b>Double Negation (3)</b>
5.	$\mathbf{r} \vee (\mathbf{r} \wedge \neg\mathbf{s})$	<b>Commutative Law (4)</b>
6.	$\mathbf{r}$	<b>Absorption Law (5)</b>