CS 160, Fall 2013

Homework 2 Answer Key

Math Proofs

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1. (25 points) **Direct Proof:** Use a direct proof to show that ***5x + 3y*** is even when ***x*** is an odd integer and ***y*** is an odd integer. **Note**: Use as many steps as necessary.

	Step	***Reason***
1.	**$O(x) \wedge O(y) \rightarrow E(5x + 3y)$**	Hypothesis
2.	$O(x) = 2j + 1$, $O(y) = 2k + 1$	Definition of odd
3.	$E(5(2j + 1) + 3(2k + 1))$	Substitution
4.	$E(10j + 5 + 6k + 3)$	Algebra
5.	$E(10j + 6k + 8)$	Algebra
6.	$E(2(5j + 3k + 4))$ = true	Definition of even
7.	$E(5x + 3y)$ = true	Proves hypothesis

Direct proof is complete.

2. (25 points) **Contrapositive Proof:** Use a contrapositive proof to show that if ***x*** and ***y*** are integers, and *5xy* is even, then ***x*** is even or ***y*** is even. **Note**: Use as many steps as necessary.

	Step	***Reason***
1.	$E(5xy) \rightarrow E(x) \vee E(y)$	Hypothesis
2.	$\neg(E(x) \vee E(y)) \rightarrow \neg E(5xy)$	Contrapositive
3.	$O(x) \wedge O(y) \rightarrow O(5xy)$	De Morgan's
4.	$O(x) = 2j + 1, O(y) = 2k + 1$	Definition of odd
5.	$O(5(2j + 1)(2k + 1))$	Substitution
4.	$O(5(4jk + 2k + 2j + 1))$	Algebra
5.	$O(20jk + 10j + 10k + 5)$	Algebra
6.	$O(2(10jk + 5j + 5k + 2) + 1) = true$	Definition of odd
7.	$O(5xy) = true$	Proves contrapositive

Proving contrapositive is equivalent to proving hypothesis!

3. (25 points) **Contradictive Proof:** Use a contrapositive proof to show that if ***x*** and ***y*** are integers, and *5xy* is even, then ***x*** is even or ***y*** is even. **Note**: Use as many steps as necessary.

	Step	***Reason***
1.	E(5xy) → E(x) ∨ E(y)	Hypothesis
2.	**E(5xy) ∧ ¬(E(x) ∨ E(y))**	Contradiction
3.	E(5xy) ∧ O(x) ∧ O(y))	De Morgan's
4.	O(x) = 2j + 1, O(y) = 2k + 1	Definition of odd
3.	E(5(2j + 1)(2k + 1))	Substitution
4.	E(5(4jk + 2k + 2j + 1))	Algebra
5.	E(20jk + 10j + 10k + 5)	Algebra
6.	E(2(10jk + 5j + 5k + 2) + 1) = false	Definition of odd
7.	E(5xy) = false	Disproves contradiction

Disproving contradiction is equivalent to proving hypothesis!

4. (25 points) **Proof by Cases**: Use proof by cases to show that if x and y are real numbers, then $|xy| = |x||y|$. Hint: there are four cases.

   Definition of absolute value: if $x < 0$, $|x| = -x$, if $x >= 0$, $|x| = x$

   CASE 1: $x >= 0$, $y >= 0$

   Proof: The product of two positive numbers is positive, so $|xy| = xy$, and $x > 0$, so $|x| = x$, and $y > 0$, so $|y| = y$ thus $|x||y| = xy = |xy|$.

   CASE 2: $x < 0$, $y >= 0$

   Proof: The product of a negative and positive number is negative, so $|xy| = -xy$, and $x < 0$, so $|x| = -x$, and $y > 0$, so $|y| = y$ thus $|x||y| = -xy = |xy|$.

   CASE 3: $x >= 0$, $y < 0$

   Proof: The product of positive and negative number is negative, so $|xy| = -xy$, and $x >= 0$, so $|x| = x$, and $y < 0$, so $|y| = -y$ so $|x||y| = -xy = |xy|$.

   CASE 4: $x < 0$, $y < 0$

   Proof: The product of two negative numbers is positive, so $|xy| = xy$, and $x < 0$, so $|x| = -x$, and $y < 0$, so $|y| = -y$ so $|x||y| = (-x)(-y) = xy = |xy|$.