Searching and Sorting (Savitch, Chapter 7.4)

TOPICS

- Algorithms
- Complexity
- Binary Search
- Bubble Sort
- Insertion Sort
- Selection Sort

What is an algorithm?

- A finite set of precise instructions for performing a computation or for solving a problem.
- Read about the history behind this word (http://en.wikipedia.org/wiki/Algorithm).

Examples

- Find the maximum in a finite sequence of integer values
- Locating an element in an ordered list – search problem
  - Linear search
  - Binary search
- Sorting

Characteristics of any algorithm

- Input and Output
- Definiteness: steps defined precisely
- Correctness: does it always work?
- Finiteness: Finite number of steps. Some people call this property: "An algorithm always terminates"
- Effectiveness: Each step must be performed exactly and in a finite amount of time
- Generality: Applicable to all instances of a problem, not just for a specific set of inputs

Linear search

- Find an element (e) in an array (array) of elements

```
public static boolean search(int[] array, int element) {
    for (int i = 0; i < array.length; i++ ) {
        if (array[i] == element)
            return true;
        }
        return false;
}
```

- Similar problems:
  - –Find the number of duplicate elements in a list.
  - –Find the maximum, minimum, median, mode.
  - –Palindrome detection.

Complexity

- The complexity (cost) of an algorithm is measured in instructions
  - – Time taken to execute the algorithm is too machine dependent
- But the number of instructions depends on the size of the input
- So complexity is measured in instructions per input

Example: Search

- Imagine a method that is given an array of integers and returns true iff at least one of the numbers is even.
  - – The method loops through the array
  - – For each integer, it uses the % operator to test if the integer is even
- How complex is this method?

Same Example in Java

```
public boolean evenp(int array[])
{
  int limit = array.length;
  for(int i=0; i < limit; i++) {
    if (0 == array[i] % 2)
        return true;
  }
  return false;
}
```

Complexity Example

- How complex is evenp?
  - Depends on the length of the array
  - Assume array.length == N
- How many instructions per integer?
  - Test if i < limit
  - Apply modulus operator
  - Test if result is zero
  - Increment i

Complexity Example (cont)

- What if the first number is even?
- What if none of the numbers are even?
- We typically analyze the *worst-case* scenario for its complexity
  - In this case, the worst case is that none of the integers are even
  - Average-case and best-case analyses are possible, but less useful

Complexity of evenp

- So the worst-case complexity of evenp is approximately 4n+2
  - Initializing limit: 1
  - Initializing i: 1
- Does the +2 matter? *No*
- Does the 4 matter? *Usually no*

Complexity of evenp

- Its cost is linear with the size of the array
- Double the array size, double the cost
- A fixed number of operations per input is what matters

Other algorithms of similar complexity

- Find the maximum in an (unsorted) array
- Sum the elements of an array
- Sum the squared elements of an array
- Match two strings

Binary search

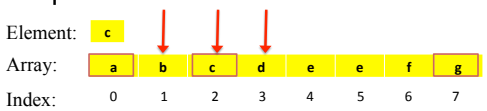
- Guess the number game: (from wikipedia)
  - "I'm thinking of an integer between forty and sixty inclusive, and to your guesses I'll respond 'High', 'Low', or 'Yes!' as might be the case."
  - If the numbers are between 1 and 16, how many guesses do you need?
- Use binary search when the array is already sorted

Binary Search Code

```
public static boolean binarySearch(int[] array, int element){
   int first = 0;
   int last = array.length-1;
   boolean found = false;
   while (! found && first < last) {
      int middle = first + (last – first) / 2;
      if (array[middle] == element) found = true;
      else if (array[middle] < element) first = middle + 1;
      else if (array[middle] > element) last = middle – 1;
   }
   return found;
}
```

Should this be first <= last?

Binary search example



1. Check the midpoint 'd' of array[0] and array[7]: Go low
2. Check the midpoint 'b' of array[0] and array[2]: Go high
3. Check the midpoint 'c' of array[2] and array[2]: Found it!

Binary Search Complexity

- Each iteration of the algorithm has a constant number of steps
- On each iteration, the data size is cut in half
- How many iterations?
  - Data sizes: n, n/2, n/4, …, 1
  - Number of iterations: log(n)

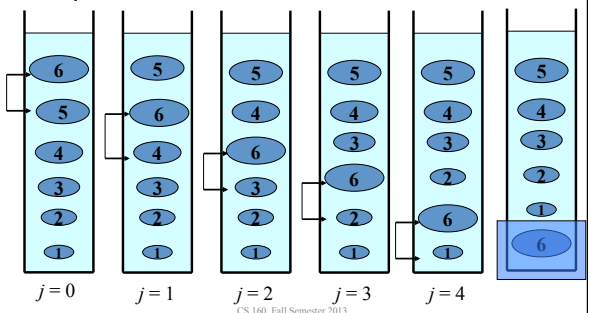
Sorting

- Sorting is the process of arranging a list of items into a particular order
- There must be some value on which the order is based
- There are many algorithms for sorting a list of items
- These algorithms vary in efficiency, and may depend on original ordering!

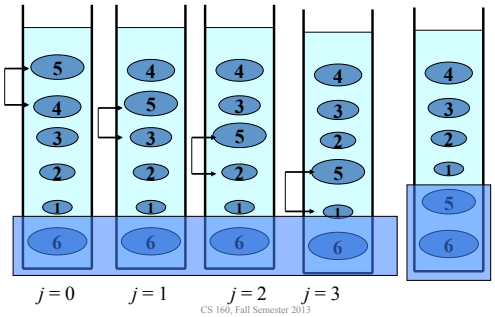
Java animations

- http://www.sorting-algorithms.com/
- Find out about other sites using Google
- We will examine three specific algorithms:
  - Bubble Sort
  - Selection Sort
  - Insertion Sort

Bubble sort (First pass)
i = 0



Bubble sort (Second pass)
i = 1



Bubble sort

- Complete the rest of the passes as an exercise

Bubble sort code

```
public static void sort(int a[])  {
      for (int i = 0; i < a.length – 1; i++) {
          for (int j = 0; j < a.length – 1 – i;  j++) {
              if (a[j] > a[j+1]) {
                  // swap the elements
                  int tmp = a[j];
                  a[j] = a[j+1];
                  a[j+1] = tmp;
              }
          }
      }
}
```


Bubble main method

```
public static void main(String[] args) {
      int[] numbers = {6, 5, 4, 3, 2, 1};
      sort(numbers);
      for (int i=0; i<numbers.length; i++)
            System.out.print(numbers[i] + " ");
      System.out.println();
  }
```

Bubble sort

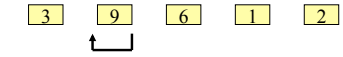
- The algorithm keeps executing the loops even if the numbers are already sorted.
- Can you improve the algorithm so that it stops in such cases?

Insertion Sort

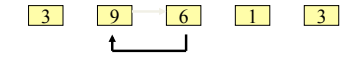
- The approach of Insertion Sort:
  - pick any item and insert it into a sorted sublist
  - repeat until all items have been inserted
- In more detail:
  - consider the first item to be a sorted sublist (of one item)
  - insert the second item into the sorted sublist
  - shift items as necessary to make room to insert new item
  - insert the third item into the sorted sublist (of two items)
  - shift items as necessary to make room to insert new item
  - repeat until all values are inserted into their proper position

Example

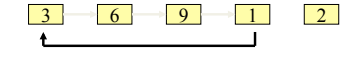
3 is sorted
Shift nothing.
Insert 9.



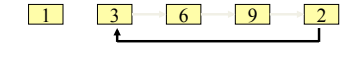
3 and 9 are sorted.
Shift 9 to the right.
Insert 6.

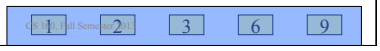


3, 6, 9 are sorted.
Shift 9, 6, 3, to the right.
Insert 1.



1, 3, 6, 9 are sorted.
Shift 9, 6, 3 to the right.
Insert 2.





Insertion Sort

- Example summary (without animation):

```
original:    3  9  6  1  2
insert 9:    3  9  6  1  2
insert 6:    3  6  9  1  2
insert 1:    1  3  6  9  2
insert 2:    1  2  3  6  9
```

Developing the code: Insertion sort

- At any point some part of the list is already sorted.
  - In the beginning, the first element by itself can be considered as a (trivial) sorted list.
- Insert one by one, the subsequent elements in this sorted list.
- In other words, at the j^{th} step, the j^{th} element is inserted into the correct position of the list of previously sorted $j - 1$ elements.

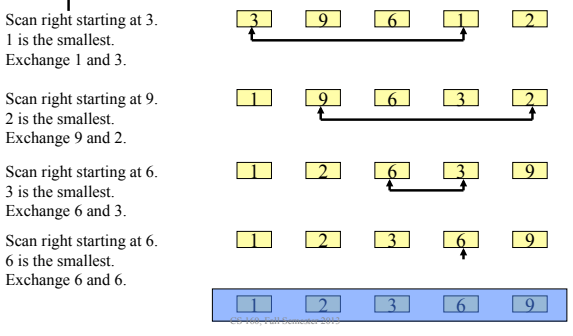
Insertion sort code (Lewis & Loftus)

```
// Sorts the specified array of integers using the insertion sort algorithm.
public static void insertionSort (int[] numbers) {

    for (int index = 1; index < numbers.length; index++) {

        int key = numbers[index];
        int position = index;

        // shift larger values to the right
        while (position > 0 && numbers[position-1] > key) {
            numbers[position] = numbers[position-1];
            position--;
        }

        numbers[position] = key;
    }
}
```


Selection Sort

- The approach of Selection Sort:
  - select one value and put it in its final place in sorted list
  - repeat for all other values
- In more detail:
  - find the smallest value in the list
  - switch it with the value in the first position
  - find the next smallest value in the list
  - switch it with the value in the second position
  - repeat until all values are placed

Example



Selection Sort

- Example summary (without animation):

```
original:          3   9   6   1   2
smallest is 1:     1   9   6   3   2
smallest is 2:     1   2   6   3   9
smallest is 3:     1   2   3   6   9
smallest is 6:     1   2   3   6   9
```


Selection sort code (Lewis & Loftus)

```
// Sorts the specified array of integers using the selection sort algorithm.
public static void selectionSort (int[] numbers) {

    int min, temp;
    for (int index = 0; index < numbers.length-1; index++) {

        minimum = index;
        for (int scan = index+1; scan < numbers.length; scan++)
            if (numbers[scan] < numbers[minimum])
                minimum = scan;

        // Swap the values
        temp = numbers[minimum ];
        numbers[minimum ] = numbers[index];
        numbers[index] = temp;
    }
}
```