

Rules of Inference

Modus Ponens

$$\frac{p \quad p \rightarrow q}{q}$$

Modus Tollens

$$\frac{\neg q \quad p \rightarrow q}{\neg p}$$

Hypothetical Syllogism

$$\frac{p \rightarrow q \quad q \rightarrow r}{p \rightarrow r}$$

Addition

$$\frac{p}{p \vee q}$$

Resolution

$$\frac{p \vee q \quad \neg p \vee r}{q \vee r}$$

Disjunctive Syllogism

$$\frac{p \vee q \quad \neg p}{q}$$

Simplification

$$\frac{p \wedge q}{p}$$

Conjunction

$$\frac{p \quad q}{p \wedge q}$$

Logical Equivalences

Idempotent Laws

$$p \vee p \equiv p$$
$$p \wedge p \equiv p$$

DeMorgan's Laws

$$\neg(p \wedge q) \equiv \neg p \vee \neg q$$
$$\neg(p \vee q) \equiv \neg p \wedge \neg q$$

Distributive Laws

$$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$$
$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Double Negation

$$\neg(\neg p) \equiv p$$

Absorption Laws

$$p \vee (p \wedge q) \equiv p$$
$$p \wedge (p \vee q) \equiv p$$

Associative Laws

$$(p \vee q) \vee r \equiv p \vee (q \vee r)$$
$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

Commutative Laws

$$p \vee q \equiv q \vee p$$
$$p \wedge q \equiv q \wedge p$$

Implication Laws

$$p \rightarrow q \equiv \neg p \vee q$$
$$p \rightarrow q \equiv \neg q \rightarrow \neg p$$

Biconditional Laws

$$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$$
$$p \leftrightarrow q \equiv \neg q \leftrightarrow \neg p$$

Identity Laws

$$p \wedge T \equiv p$$
$$p \vee F \equiv p$$

Domination Laws

$$p \vee T \equiv T$$
$$p \wedge F \equiv F$$

Negation Laws

$$p \vee \neg p \equiv T$$
$$p \wedge \neg p \equiv F$$