Chapter 23 Sorting

CS1: Java Programming
Colorado State University

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Objectives

- To study and analyze time complexity of various sorting algorithms (§§23.2–23.7).
- To design, implement, and analyze insertion sort (§23.2).
- To design, implement, and analyze bubble sort (§23.3).
- To design, implement, and analyze merge sort (§23.4).

Why study sorting?

Sorting is a classic subject in computer science. There are three reasons for studying sorting algorithms.

- First, sorting algorithms illustrate many creative approaches to problem solving and these approaches can be applied to solve other problems.
- Second, sorting algorithms are good for practicing fundamental programming techniques using selection statements, loops, methods, and arrays.
- Third, sorting algorithms are excellent examples to demonstrate algorithm performance.

What data to sort?

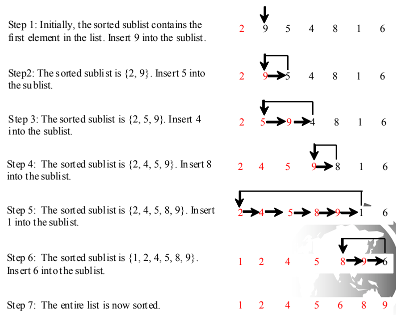
The data to be sorted might be integers, doubles, characters, or objects. §7.8, “Sorting Arrays,” presented selection sort and insertion sort for numeric values. The selection sort algorithm was extended to sort an array of objects in §11.5.7, “Example: Sorting an Array of Objects.” The Java API contains several overloaded sort methods for sorting primitive type values and objects in the java.util.Arrays and java.util.Collections class. For simplicity, this section assumes:

- data to be sorted are integers,
- data are sorted in ascending order, and
- data are stored in an array. The programs can be easily modified to sort other types of data, to sort in descending order, or to sort data in an ArrayList or a LinkedList.

Insertion Sort

int[] myList = {2, 9, 5, 4, 8, 1, 6}; // Unsorted

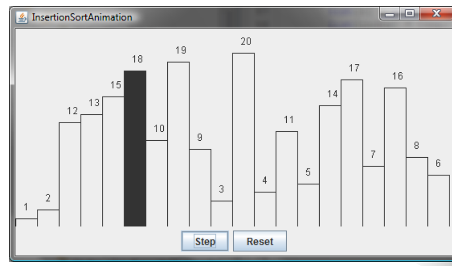
The insertion sort algorithm sorts a list of values by repeatedly inserting an unsorted element into a sorted sublist until the whole list is sorted.



animation

Insertion Sort Animation

http://www.cs.armstrong.edu/liang/animation/web/InsertionSort.html



animation

Insertion Sort

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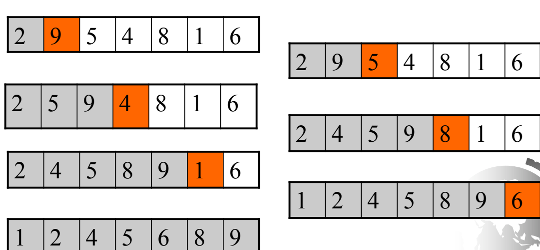
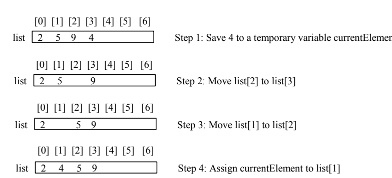


How to Insert?

The insertion sort algorithm sorts a list of values by repeatedly inserting an unsorted element into a sorted sublist until the whole list is sorted.



From Idea to Solution

```
for (int i = 1; i < list.length; i++) {
  insert list[i] into a sorted sublist list[0..i-1] so that
  list[0..i] is sorted
}
```

```
list[0]

list[0] list[1]

list[0] list[1] list[2]

list[0] list[1] list[2] list[3]

list[0] list[1] list[2] list[3] ...
```


From Idea to Solution

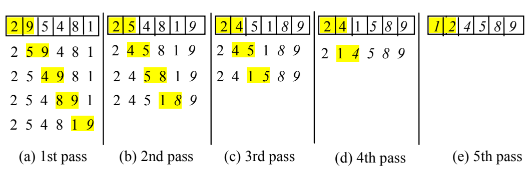
```
for (int i = 1; i < list.length; i++) {
  insert list[i] into a sorted sublist list[0..i-1] so that
  list[0..i] is sorted
}
```

Expand

```
double currentElement = list[i];
int k;
for (k = i - 1; k >= 0 && list[k] > currentElement; k--) {
  list[k + 1] = list[k];
}
// Insert the current element into list[k + 1]
list[k + 1] = currentElement;
```

InsertSort Run

Bubble Sort



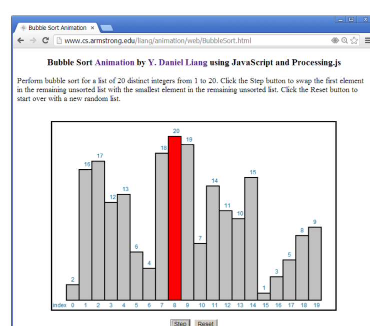
Bubble sort time: $O(n^2)$

$$(n-1)+(n-2)+...+2+1=\frac{n^2}{2}-\frac{n}{2}$$

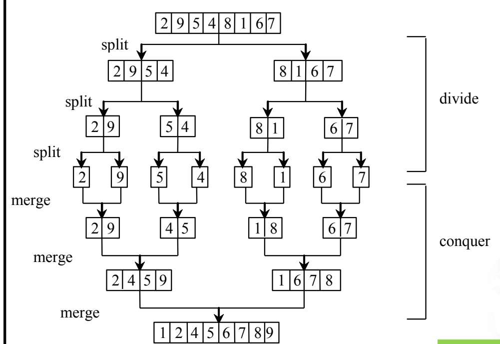
BubbleSort Run

Bubble Sort Animation

http://www.cs.armstrong.edu/liang/animation/web/BubbleSort.html



Merge Sort

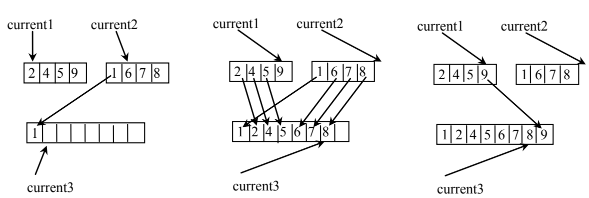


MergeSort Run

Merge Sort

mergeSort(list):
  firstHalf = mergeSort(firstHalf);
  secondHalf = mergeSort(secondHalf);
  list = merge(firstHalf, secondHalf);

Merge Two Sorted Lists



(a) After moving 1 to temp

(b) After moving all the elements in list2 to temp

(c) After moving 9 to temp

Animation for Merging Two Sorted Lists

Merge Sort Time

Let *T(n)* denote the time required for sorting an array of *n* elements using merge sort. Without loss of generality, assume *n* is a power of 2. The merge sort algorithm splits the array into two subarrays, sorts the subarrays using the same algorithm recursively, and then merges the subarrays. So,

$$T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + mergetime$$

$$T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + O(n)$$

Merge Sort Time

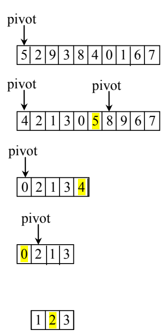
The first *T(n/2)* is the time for sorting the first half of the array and the second *T(n/2)* is the time for sorting the second half. To merge two subarrays, it takes at most *n-1* comparisons to compare the elements from the two subarrays and *n* moves to move elements to the temporary array. So, the total time is *2n-1*. Therefore,

$$T(n) = 2T(\frac{n}{2}) + 2n - 1 = 2(2T(\frac{n}{4}) + 2\frac{n}{2} - 1) + 2n - 1 = 2^2T(\frac{n}{2^2}) + 2n - 2 + 2n - 1$$
$$= 2^kT(\frac{n}{2^k}) + 2n - 2^{k-1} + ... + 2n - 2 + 2n - 1$$
$$= 2^{\log n}T(\frac{n}{2^{\log n}}) + 2n - 2^{\log n - 1} + ... + 2n - 2 + 2n - 1$$
$$= n + 2n\log n - 2^{\log n} + 1 = 2n\log n + 1 = O(n\log n)$$

Quick Sort

Quick sort, developed by C. A. R. Hoare (1962), works as follows: The algorithm selects an element, called the *pivot*, in the array. Divide the array into two parts such that all the elements in the first part are less than or equal to the pivot and all the elements in the second part are greater than the pivot. Recursively apply the quick sort algorithm to the first part and then the second part.

Quick Sort



(a) The original array

(b)The original array is partitioned

(c) The partial array (4 2 1 3 0) is partitioned

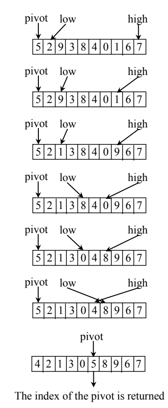
(d) The partial array (0 2 1 3) is partitioned

(e) The partial array (2 1 3) is partitioned

Partition

Animation for partition

QuickSort Run



(a) Initialize pivot, low, and high

(b) Search forward and backward

(c) 9 is swapped with 1

(d) Continue search

(e) 8 is swapped with 0

(f) when high < low, search is over

(g) pivot is in the right place

Quick Sort Time

To partition an array of *n* elements, it takes *n-1* comparisons and *n* moves in the worst case. So, the time required for partition is *O(n)*.

Worst-Case Time

In the worst case, each time the pivot divides the array into one big subarray with the other empty. The size of the big subarray is one less than the one before divided. The algorithm requires $O(n^2)$ time:

$$(n \quad 1) + (n \quad 2) + ... + 2 + 1 = O(n^2)$$

Best-Case Time

In the best case, each time the pivot divides the array into two parts of about the same size. Let *T(n)* denote the time required for sorting an array of elements using quick sort. So,

$$T(n) = T(\frac{n}{2}) + T(\frac{n}{2}) + n = O(n \log n)$$

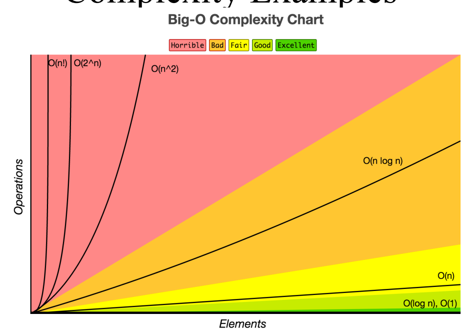
Average-Case Time

On the average, each time the pivot will not divide the array into two parts of the same size nor one empty part. Statistically, the sizes of the two parts are very close. So the average time is *O(nlogn)*. The exact average-case analysis is beyond the scope of this book.

Computational Complexity (Big O)

- ✦ $T(n)=O(1)$ // constant time
- ✦ $T(n)=O(\log n)$ // logarithmic
- ✦ $T(n)=O(n)$ // linear
- ✦ $T(n)=O(n\log n)$ // linearithmic
- ✦ $T(n)=O(n^2)$ // quadratic
- ✦ $T(n)=O(n^3)$ // cubic

Complexity Examples



http://bigocheatsheet.com/

Complexity Examples

Array Sorting Algorithms

Algorithm	Time Complexity			Space Complexity
	Best	Average	Worst	Worst
Quicksort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(log(n))
Mergesort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(n)
Timsort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)
Heapsort	Ω(n log(n))	Θ(n log(n))	O(n log(n))	O(1)
Bubble Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Insertion Sort	Ω(n)	Θ(n^2)	O(n^2)	O(1)
Selection Sort	Ω(n^2)	Θ(n^2)	O(n^2)	O(1)
Tree Sort	Ω(n log(n))	Θ(n log(n))	O(n^2)	O(n)
Shell Sort	Ω(n log(n))	Θ(n(log(n))^2)	O(n(log(n))^2)	O(1)
Bucket Sort	Ω(n+k)	Θ(n+k)	O(n^2)	O(n)
Radix Sort	Ω(nk)	Θ(nk)	O(nk)	O(n+k)
Counting Sort	Ω(n+k)	Θ(n+k)	O(n+k)	O(k)
Cubesort	Ω(n)	Θ(n log(n))	O(n log(n))	O(n)

http://bigocheatsheet.com/

Why does it matter?

Algorithm	10	20	50	100	1,000	10,000	100,000
$O(1)$	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s
$O(\log(n))$	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s
$O(n)$	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s
$O(n*\log(n))$	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s	<1 s
$O(n^2)$	<1 s	<1 s	<1 s	<1 s	<1 s	2 s	3 m
$O(n^3)$	<1 s	<1 s	<1 s	<1 s	20 s	6 h	232 d
$O(2^n)$	<1 s	<1 s	260 d	∞	∞	∞	∞
$O(n!)$	<1 s	∞	∞	∞	∞	∞	∞
$O(n^n)$	3 m	∞	∞	∞	∞	∞	∞