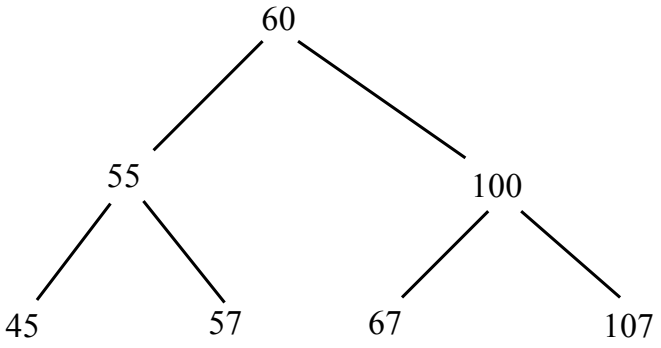
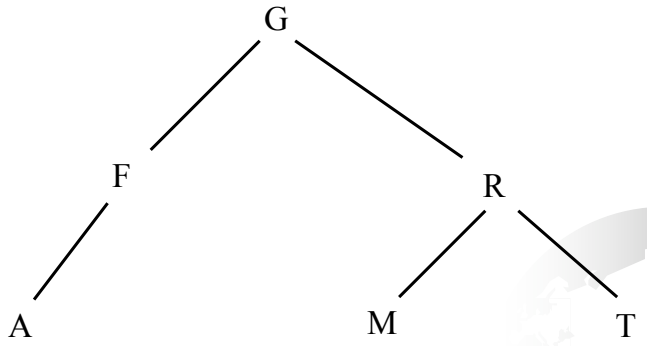
Chapter 25 Binary Search Trees
Original slides: Liang
updated by Wim Bohm and Sudipto Ghosh

Binary Trees

A list, stack, or queue is a linear structure that consists of a sequence of elements. A binary tree is a hierarchical structure. It is either empty or consists of an element, called the *root*, and two distinct binary trees, called the *left subtree* and *right subtree*.



(A)



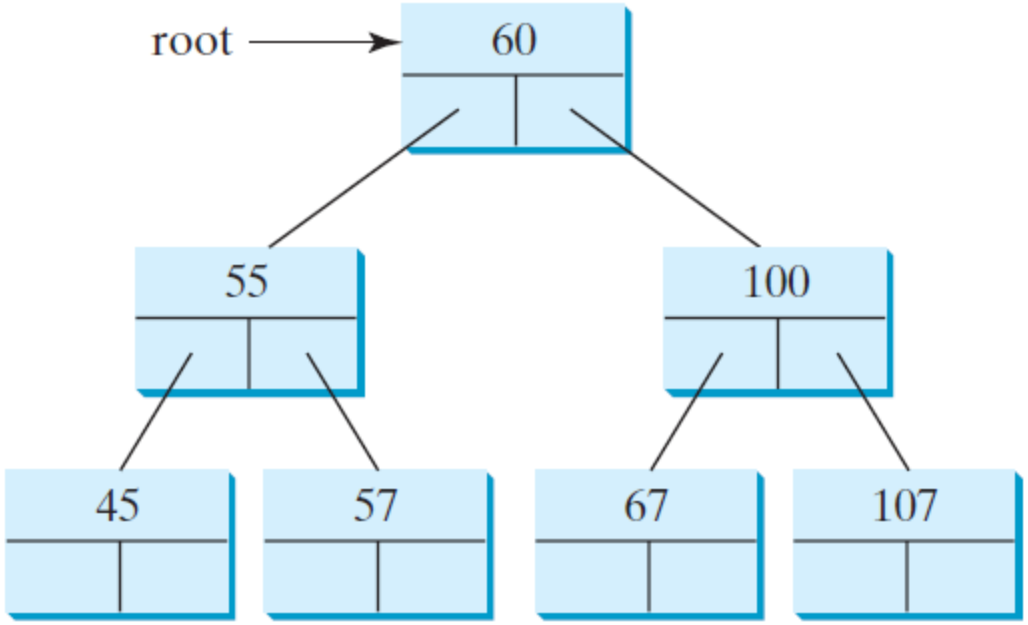
(B)

Binary Tree Terms

- A Binary consists of
  - A root
  - A left binary tree (left child)
  - A right binary tree (right child)
- A node without children is a *leaf*. A node has one parent, except for the root, which has no parents.

Representing Binary Trees

A binary tree can be represented using a set of linked nodes. Each node contains a value and two links named *left* and *right* that reference the left child and right child, respectively.




```
class TreeNode<E> {
  E element;
  TreeNode<E> left;
  TreeNode<E> right;

  public TreeNode(E o) {
    element = o;
  }
}
```

Binary Search Tree

- A binary search tree of (key, value) pairs, with no duplicate keys, has the following properties
- Every node in a left subtree has keys less than the key of the root
- Every node in a right subtree has keys greater than the key of the node.
- (often we only show the keys)
- What is the difference w.r.t heaps?

Searching an Element in a Binary Search Tree

```
public search(E element) {

  TreeNode<E> current = root; // Start from the root

  while (current != null)
    if (element key less than the key in current.element) {
      current = current.left; // Go left
    }

    else if (element value greater than the value in
             current.element) {
      current = current.right; // Go right
    }

    else // Element matches current.element
      return found ; // Element is found

  return not found; // Element is not in the tree
}
```

Inserting an Element to a Binary Tree

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
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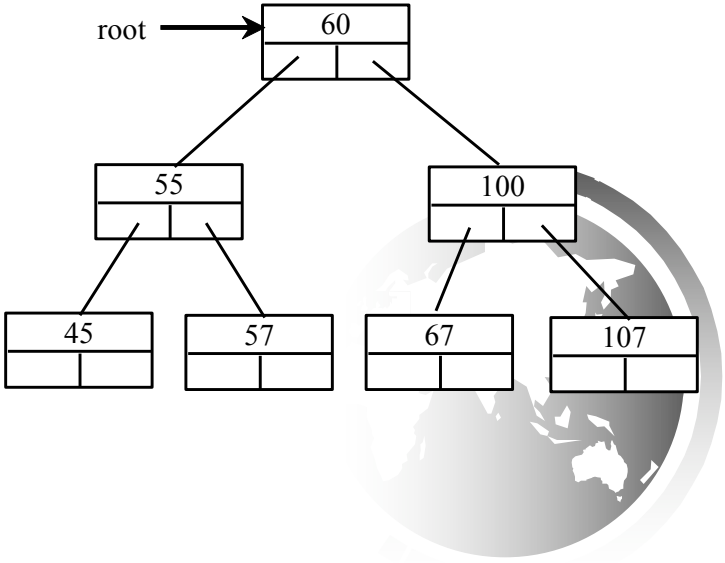
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(elemenet);

  return true; // Element inserted
}
```

Insert 101 into the following tree.



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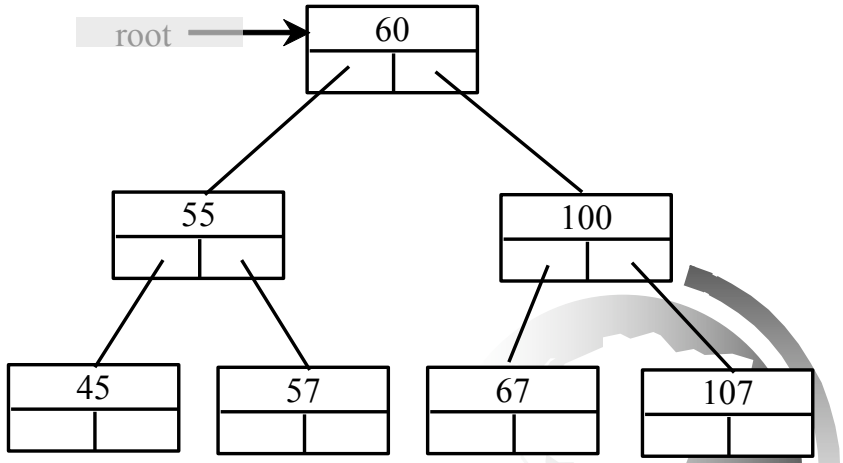

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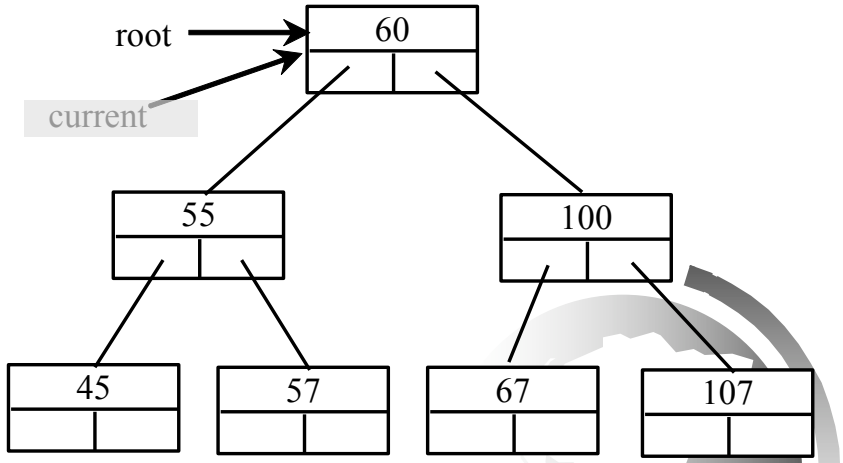

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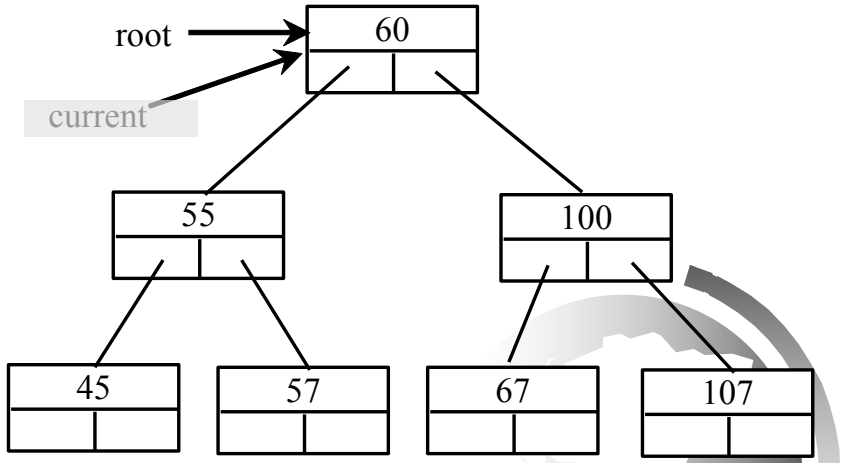

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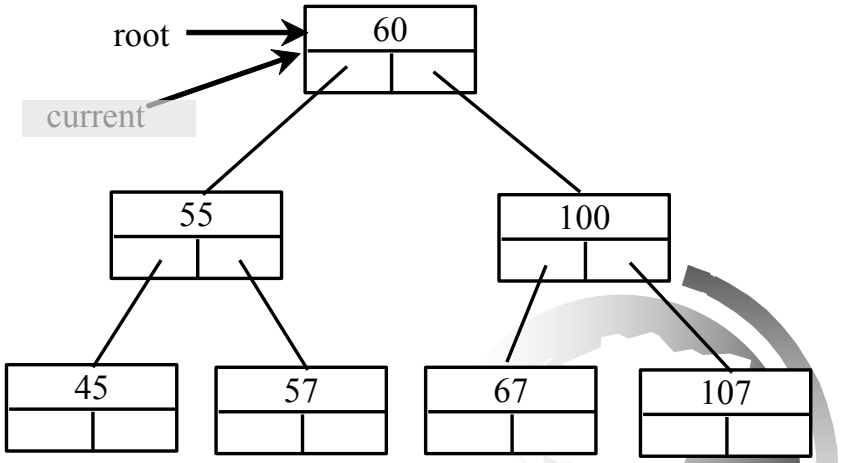

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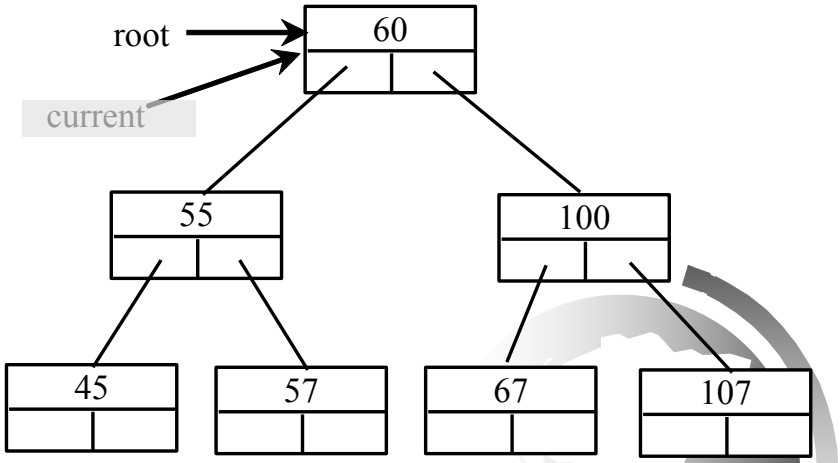

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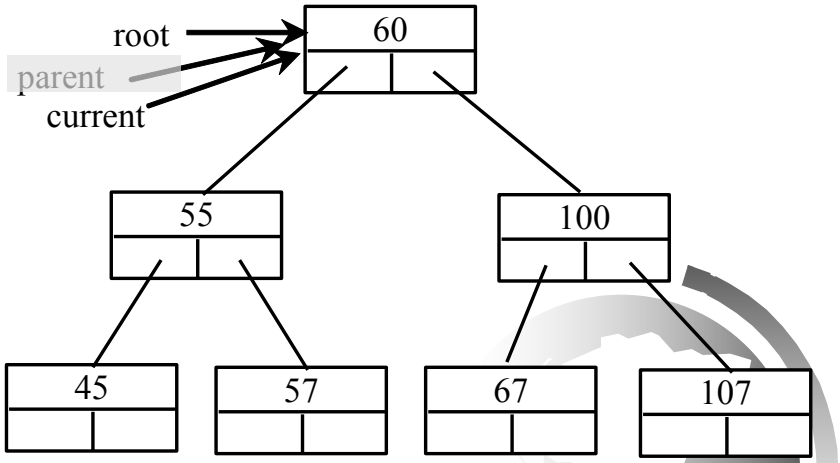

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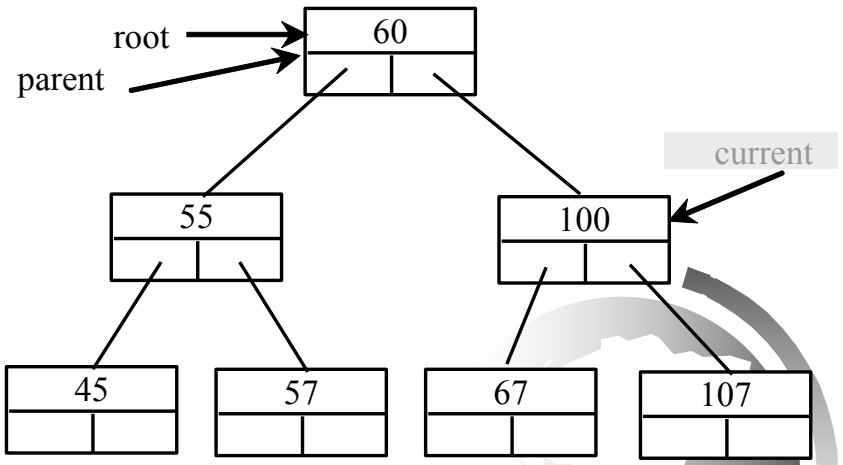

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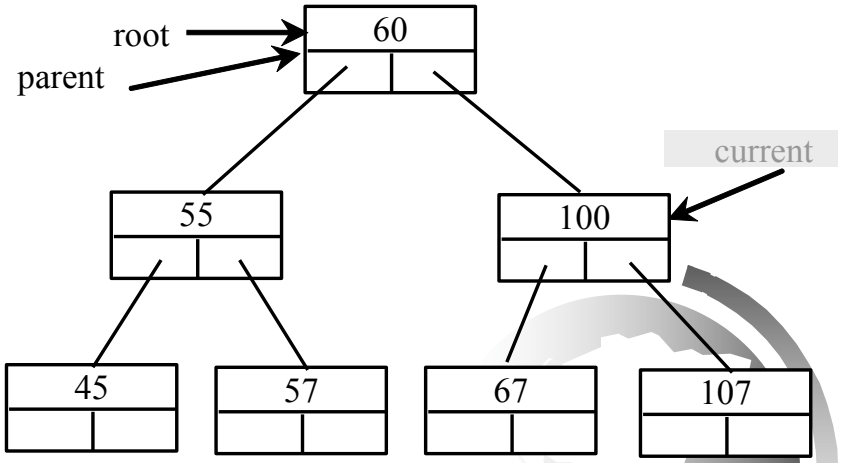

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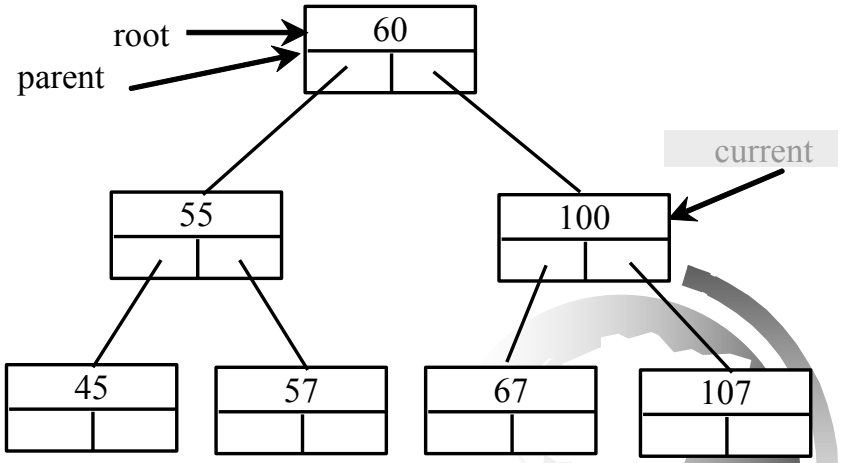

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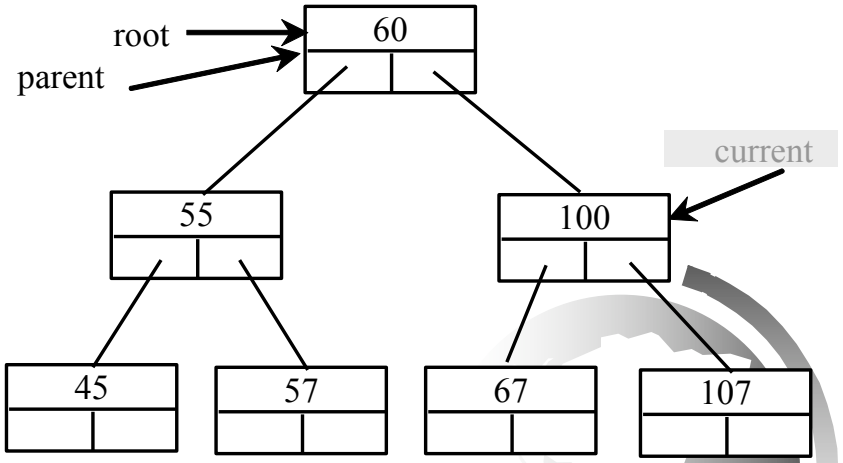

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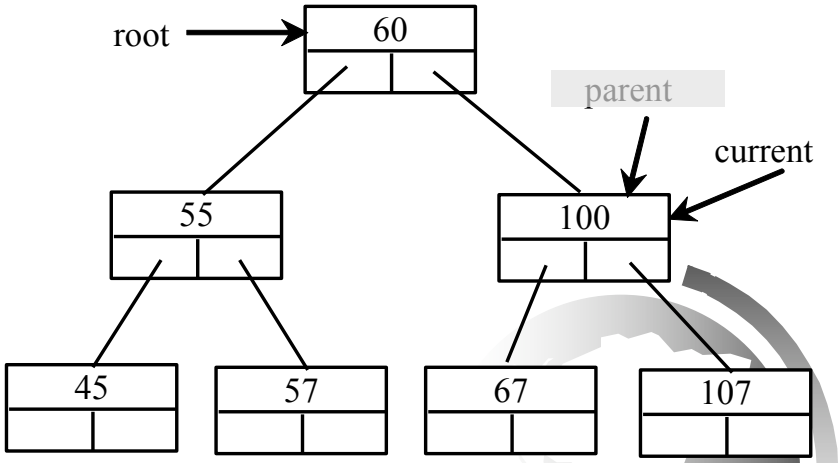

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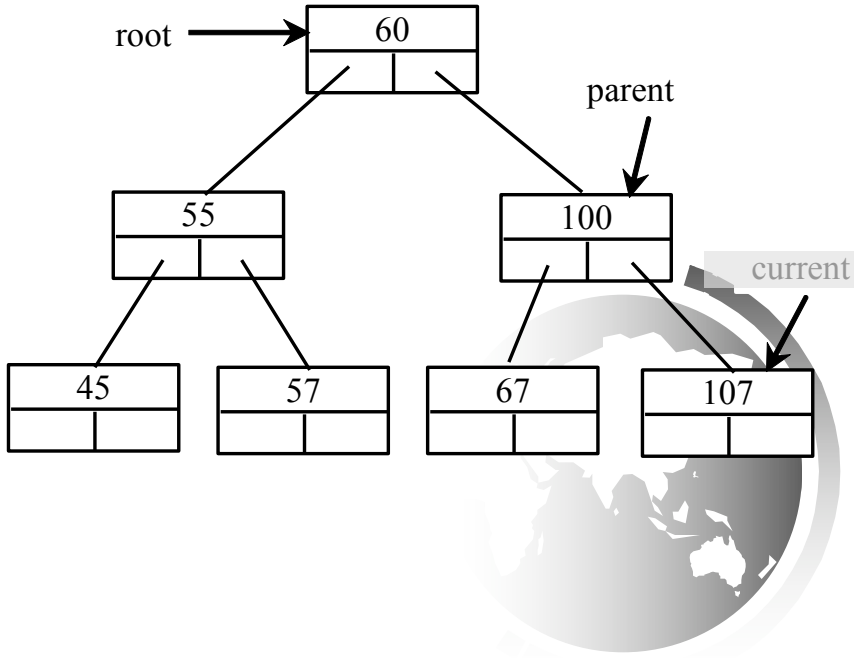

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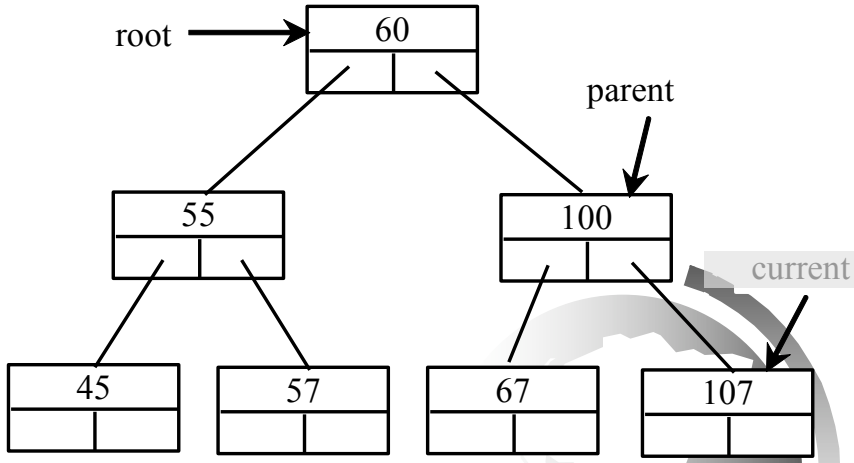

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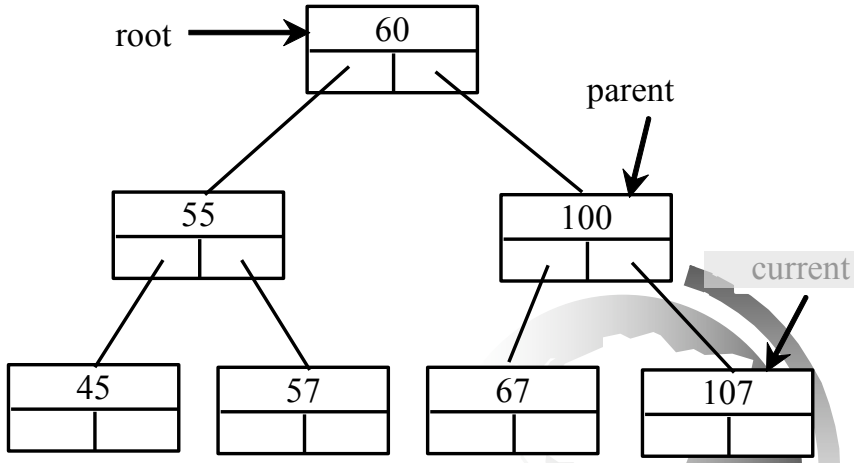

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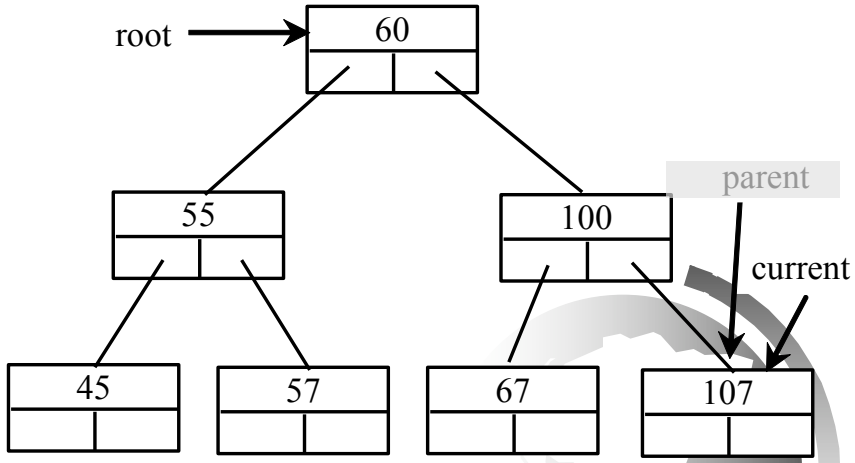

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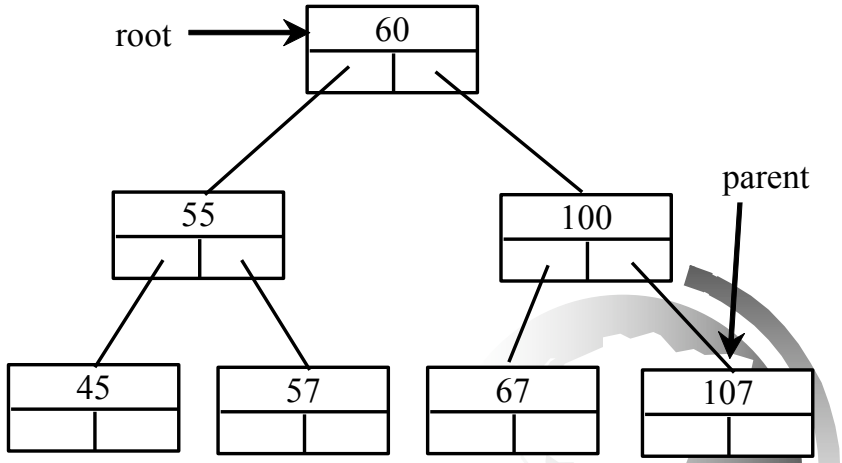

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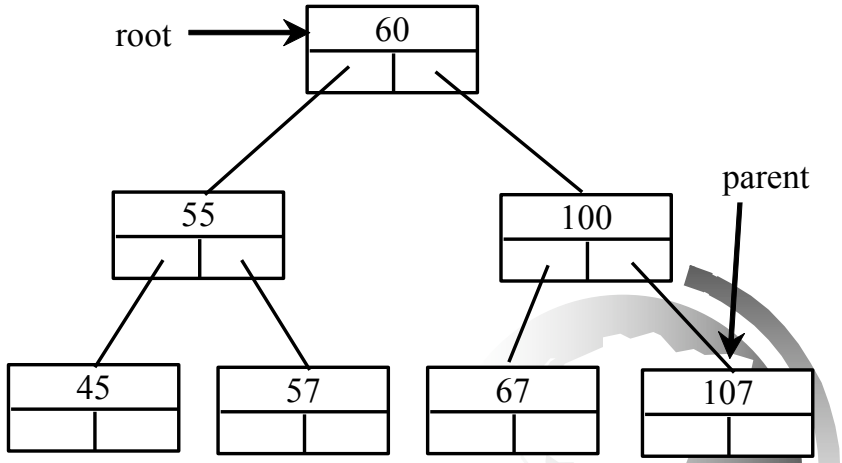

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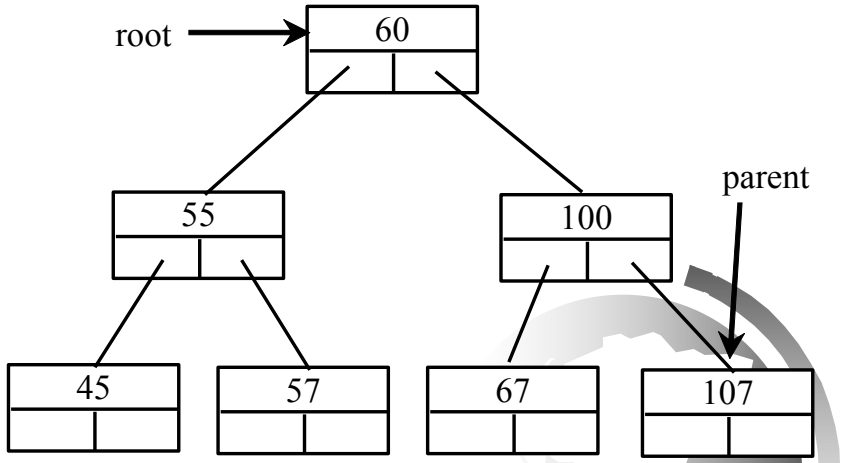

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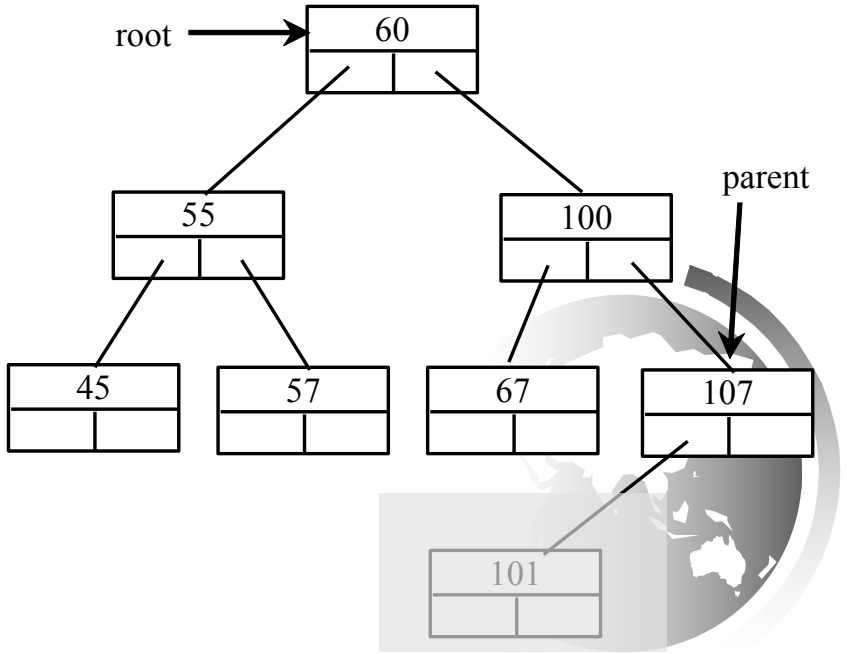
  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(elemenet);
  else
    parent.right = new TreeNode(element);

  return true; // Element inserted
}
```

Insert 101 into the following tree.

101 < 107 true



Trace Inserting 101 into the following tree, cont.

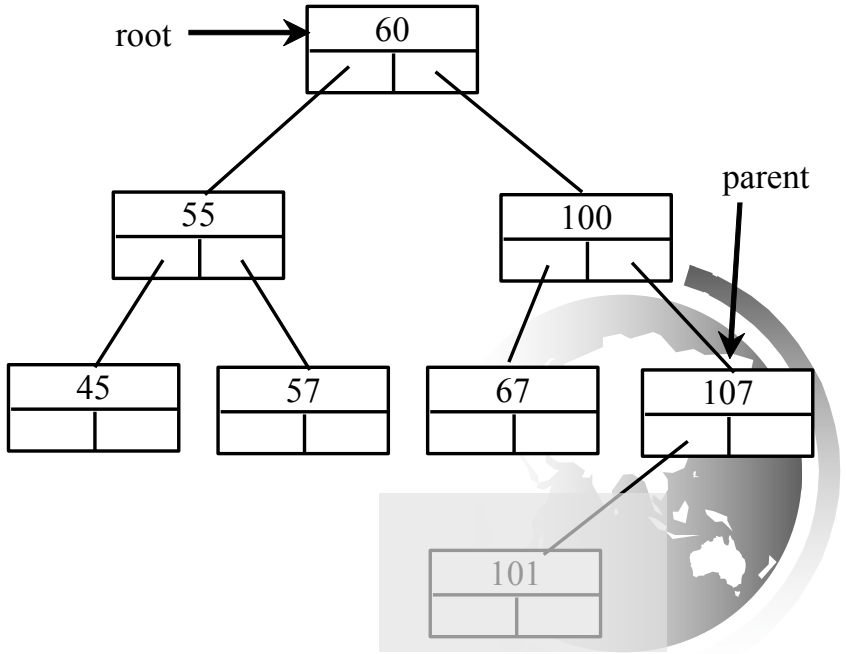
```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(element);
  else
    parent.right = new TreeNode(element);

  return true; // Element inserted
}
```

Insert 101 into the following tree.

101 < 107 true

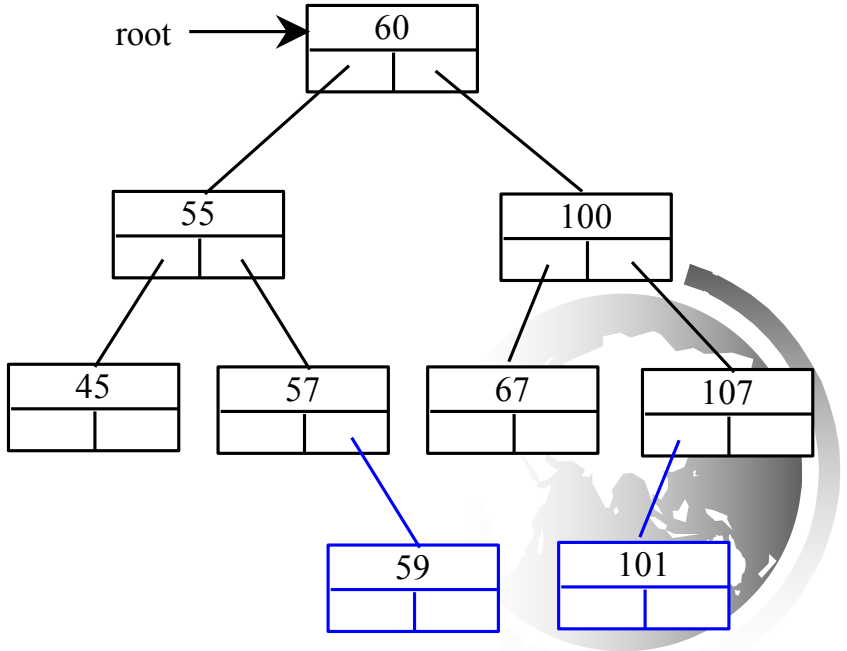


Inserting 59 into the Tree

```
if (root == null)
  root = new TreeNode(element);
else {
  // Locate the parent node
  current = root;
  while (current != null)
    if (element value < the value in current.element) {
      parent = current;
      current = current.left;
    }
    else if (element value > the value in current.element) {
      parent = current;
      current = current.right;
    }
    else
      return false; // Duplicate node not inserted

  // Create the new node and attach it to the parent node
  if (element < parent.element)
    parent.left = new TreeNode(element);
  else
    parent.right = new TreeNode(element);

  return true; // Element inserted
}
```



Tree Traversal

Tree traversal is the process of visiting each node in the tree exactly once. There are several ways to traverse a tree. This section presents *depth-first: in-*, *pre-*, *post order and breadth-first: level order* traversals.

✦InOrder

- The inorder traversal is to visit the left subtree of the current node first recursively, then the current node itself, and finally the right subtree of the current node recursively.

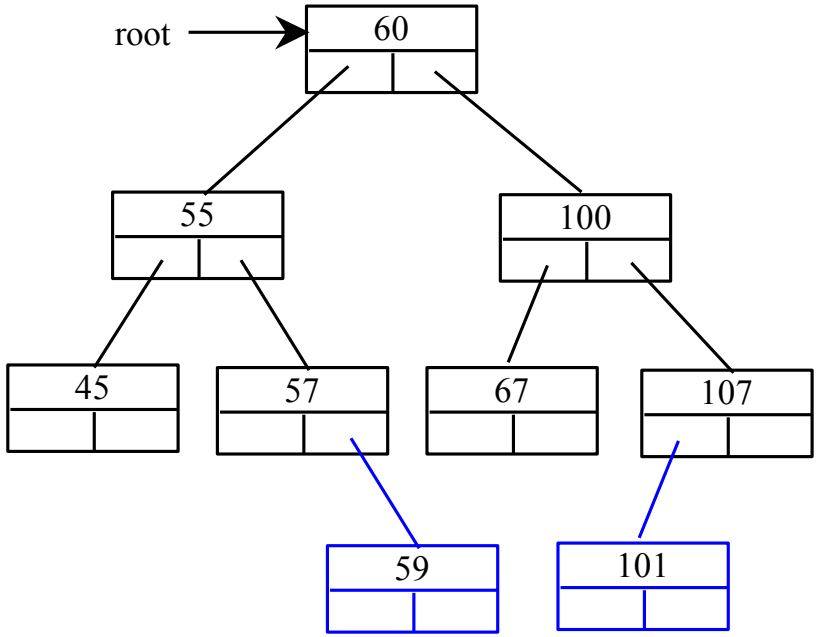
✦Postorder

- The postorder traversal is to visit the left subtree of the current node first, then the right subtree of the current node, and finally the current node itself.

Tree Traversal, cont.

- Preorder
  - The preorder traversal is to visit the current node first, then the left subtree of the current node recursively, and finally the right subtree of the current node recursively.
- Level order
  - The level order (breadth-first) traversal is to visit the nodes level by level. First visit the root, then all children of the root from left to right, then grandchildren of the root from left to right, and so on.

Tree Traversal, cont.



Inorder: 45 55 57 59 60 67 100 101 107

Postorder: 45 59 57 55 67 101 107 100 60

Preorder: 60 55 45 57 59 100 67 107 101

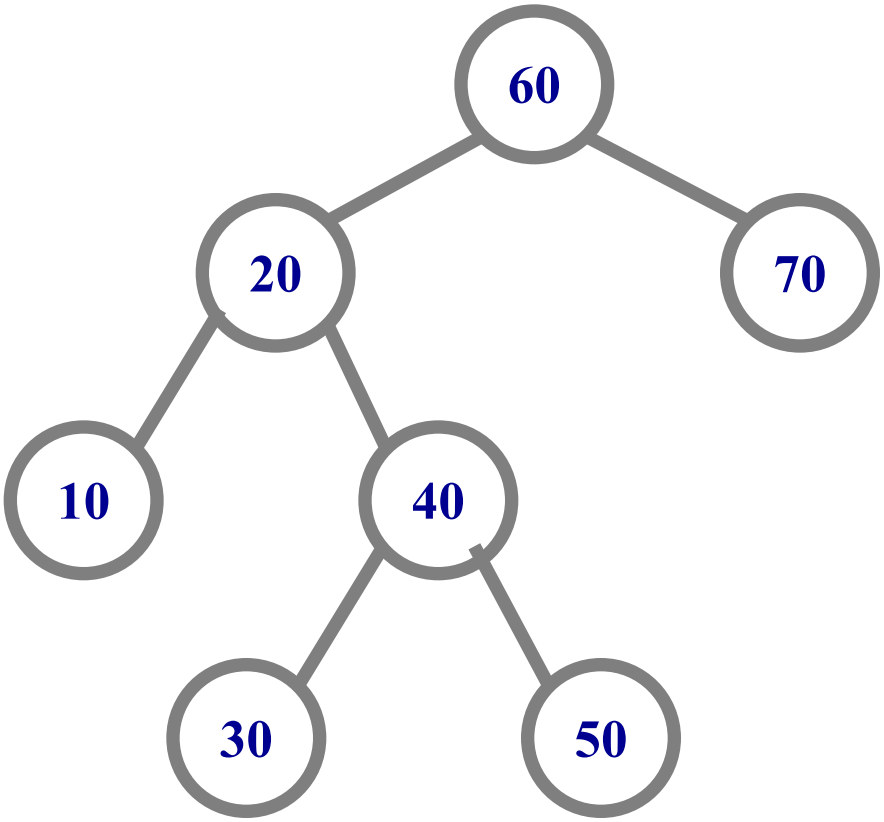
Level order: 60 55 100 45 57 67 107 59 101

Breadth-first traversal (BFS)

- Breadth-first processes the tree **level by level** starting at the root and handling all the nodes at a particular level from **left to right**.

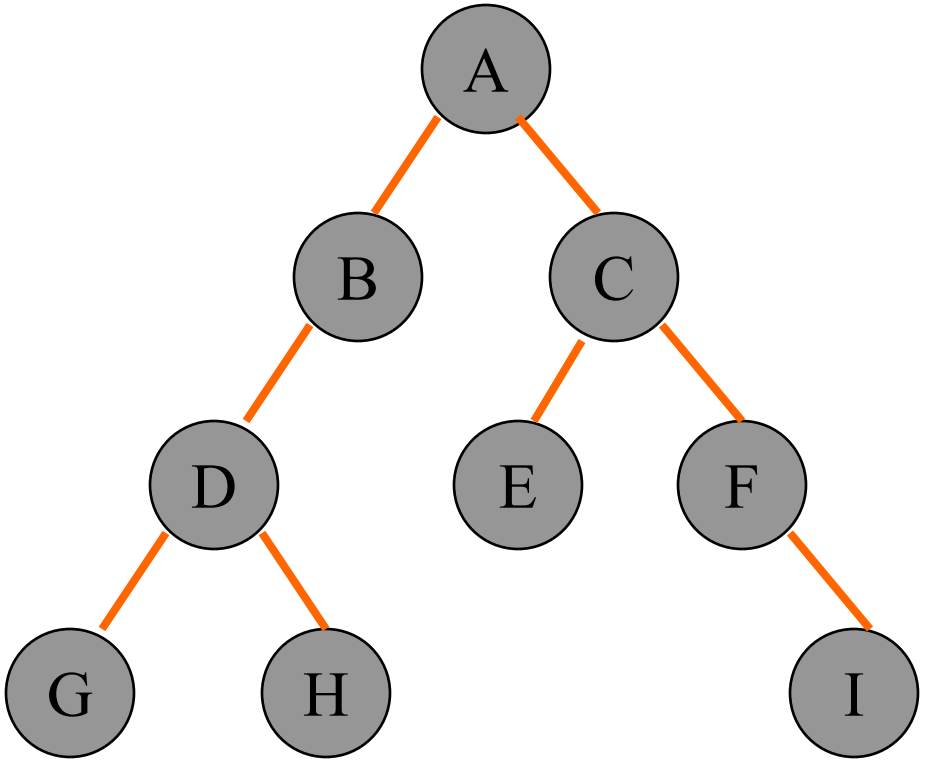
- To achieve this, we use a Queue, because the parent child references are not sufficient

Breadth-first traversal



60 – 20 – 70 – 10 – 40 – 30 – 50

LevelOrder



	Queue	Output
Init	[A]	-
Step 1	[B,C]	A
Step 2	[C,D]	A B
Step 3	[D,E,F]	A B C
Step 4	[E,F,G,H]	A B C D
Step 5	[F,G,H]	A B C D E
Step 6	[G,H,I]	A B C D E F
Step 7	[H,I]	A B C D E F G
Step 8	[I]	A B C D E F G H
Step 9	[ ]	A B C D E F G H I

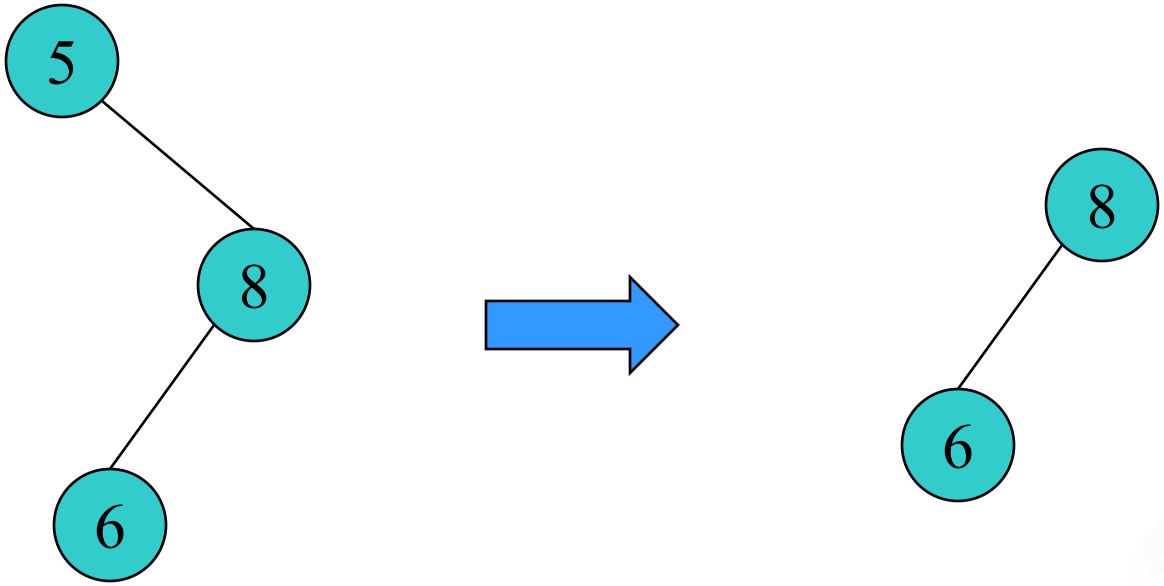
Deleting a BST node

- What is the problem?
- What to do?
- Cases to Consider
  - Delete something that is not there
    - Throw exception
  - Delete a leaf
    - Easy, just set link from parent to null
  - Delete a node with one child
  - Delete a node with two children

Delete

Case 1: one child

delete(5)



Child becomes root

Delete

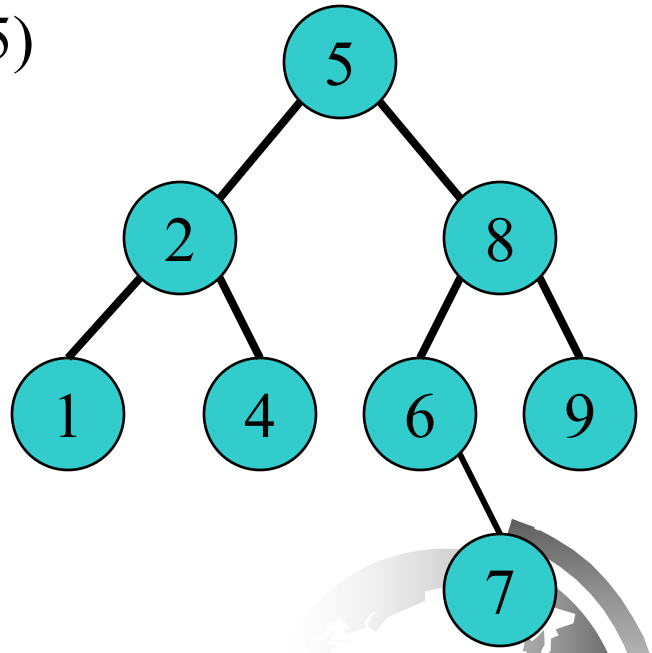
Case 2: two children

Which are valid replacement nodes?

4 and 6, WHY?

max of left, min of right

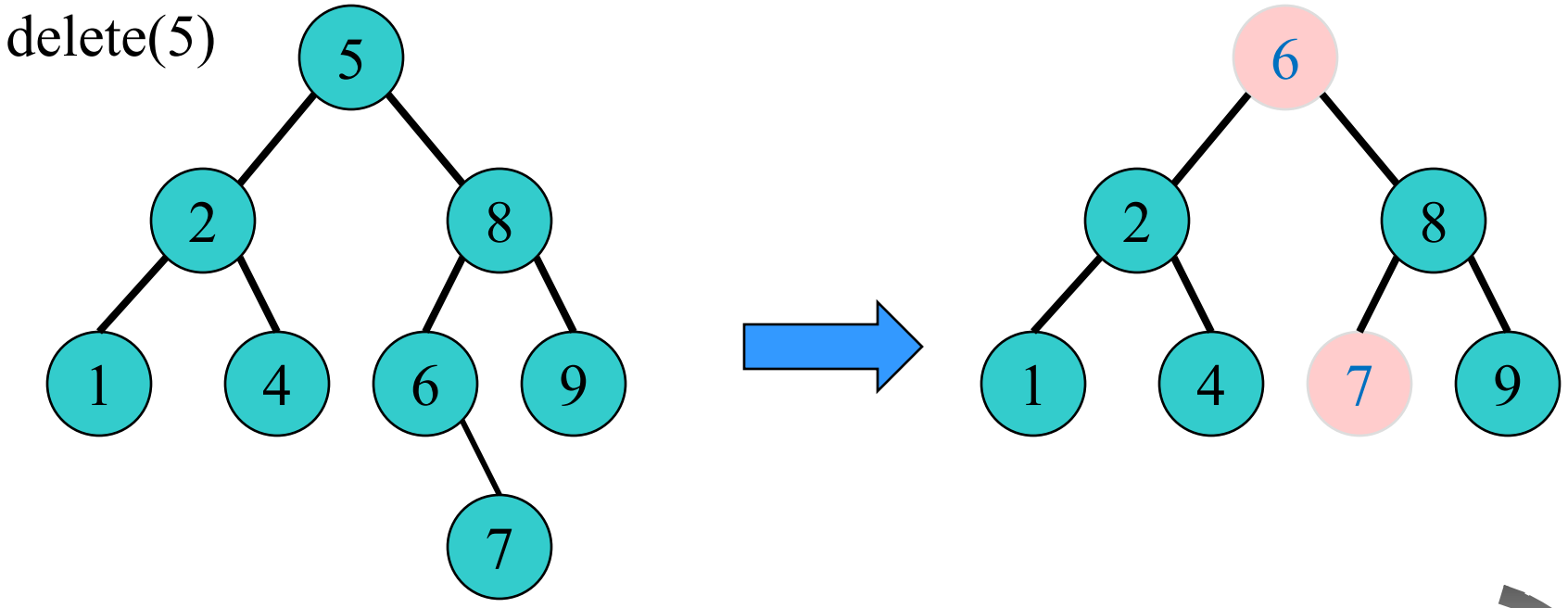
delete(5)



Digression: inorder traversal of BST

- In order:
  - go left
  - visit the node
  - go right
- The keys of an inorder traversal of a BST are in sorted order!

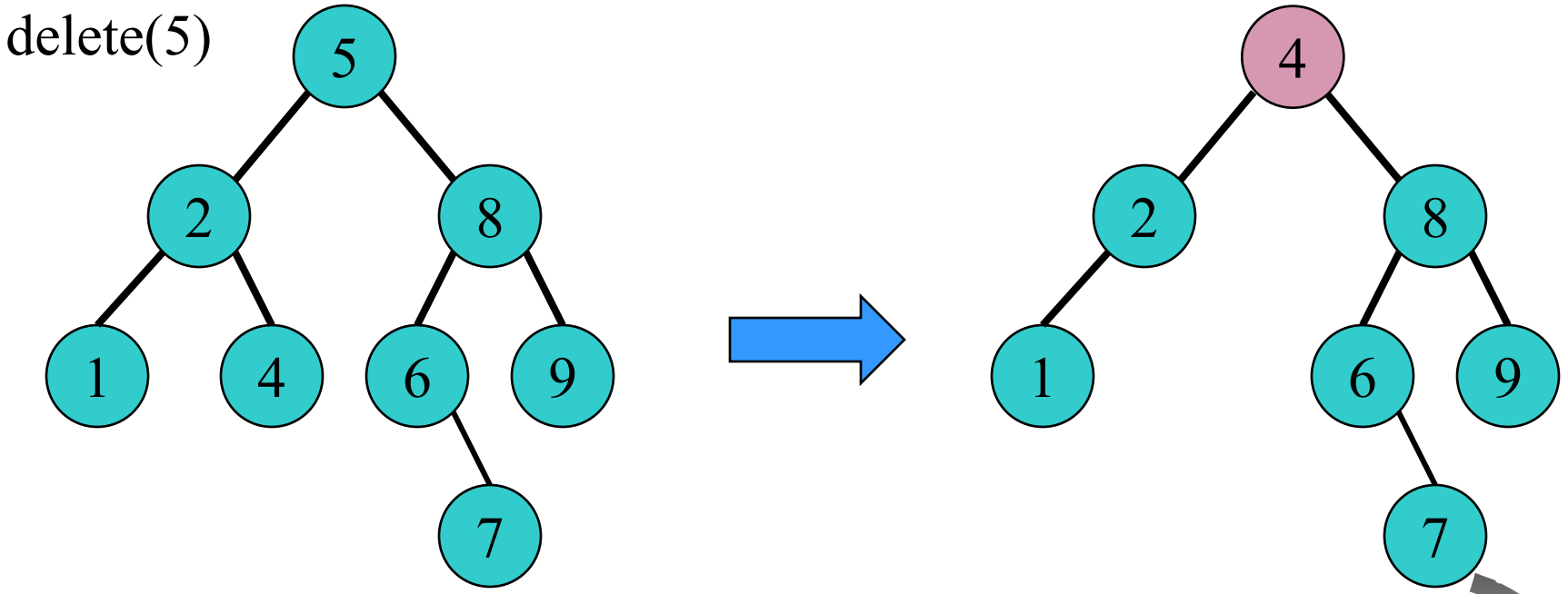
Replace with successor



Replace root with its **leftmost right descendant** and replace that node **with its right child,** if necessary (an easy delete case).
That node is the inorder successor of the root.

Can that node have two children? A left child?

Replace with predecessor



Replace root with its **rightmost leftt descendant** and replace that node **with its left child,** if necessary (an easy delete case).
That node is the inorder predecessor of the root.

Can that node have two children? A right child?

Delete
Case 2: two children

1. Find the ***inorder successor or predecessor M*** of N's search key.
   - The node whose search key comes immediately after or before N's search key
2. Copy the item of M, to the deleting node N.
3. Remove the node M from the tree.