CS 200 Algorithms and Data Structures
Written Assignment #3
Due on Nov. 17 at the beginning of the class

[Problem 1] 5pts
List the 16 different relations on the set \{0,1\}.

[Problem 2] 10 pts (2+2+2+2+2)
For each of these relations on the set \{1,2,3,4\}, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, and whether it is transitive.

1) \{(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}
2) \{(1,1),(1,2),(2,1),(2,2),(3,3),(4,4)\}
3) \{(2,4),(4,2)\}
4) \{(1,2),(2,3),(3,4)\}
5) \{(1,1),(2,2),(3,3),(4,4)\}

[Problem 3] 5 pts (1+1+1+2)
Let R be the relation on the set \{1,2,3,4\} containing the ordered pairs (1,1), (1,2), (1,3), (2,3), and (2,4). Find

1) \(R^2\)
2) \(R^3\)
3) \(R^4\)
4) \(R^5\)

[Problem 4] 5 pts
Let R be a reflexive relation. Show that \(R^n\) is reflexive for all positive integers n.
(Hint: use mathematical induction on n)

[Problem 5] 5 pts
What do you obtain when you apply the selection operator Sc, where C is the condition Room = A100, to the database in following Table?

<table>
<thead>
<tr>
<th>Professor</th>
<th>Department</th>
<th>Course Number</th>
<th>Room</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cruz</td>
<td>Zoology</td>
<td>335</td>
<td>A100</td>
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<td>Psychology</td>
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<tr>
<td>Farber</td>
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<td>618</td>
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<td>Physics</td>
<td>544</td>
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<td>518</td>
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<td>Computer Science</td>
<td>575</td>
<td>N502</td>
<td>3:00PM</td>
</tr>
</tbody>
</table>
[Problem 6] 5 pts (2+1+1+1)
Represent each of these relations on \{1,2,3,4\} with a matrix
1) \{(1,2),(1,3),(1,4),(2,3),(2,4),(3,4)\}
2) \{(1,1),(1,4),(2,2),(3,3),(4,1)\}
3) \{(1,2),(1,3),(1,4),(2,1),(2,3),(2,4),(3,1),(3,2),(3,4),(4,1),(4,2),(4,3)\}
4) \{(2,4),(3,1),(3,2),(3,4)\}

[Problem 7] 5 pts (2+1+1+1)
Draw the directed graphs representing each of the relations from the Problem 6.

[Problem 8] 5 pts (1+1+1+1+1)
How many nonzero entries does the matrix representing the relation R on \(A = \{1,2,3,4...,10\}\) consisting of the first 10 positive integers have in R is
1) \{(a,b)|a \leq b\}?
2) \{(a,b)|a = b\pm 1\}?
3) \{(a,b)|a+b = 10\}?
4) \{(a,b)|a+b \leq 10\}?
5) \{(a,b)|a \neq 0\}?

[Problem 9] 5 pts (2+3)
The hash table has size 2,048. The keys are strings that begin with a letter. The hash function is
\[ h(key) = (\text{position in alphabet of first letter of key}) \mod 2048 \]
Thus, string “BUT” maps to 2.

1) How appropriate is this hash function if the strings are random (i.e. each of the characters in the word is picked randomly)?

2) What if the strings are English words? (hint: think about the words in the English dictionary.)