Worksheet for Recursive relations and Master Theorem

1. (Rosen exercises 7-1(4)) Show that the sequence $\{a_n\}$ is a solution of the recurrence relation, $a_n = -3a_{n-1} + 4a_{n-2}$ if
   a. $a_n = 0$
   b. $a_n = 1$
   c. $a_n = (-4)^n$
   d. $a_n = 2(-4)^n + 3$

2. (Rosen exercises 7-1(9)) Find the solution to each of these recurrence relations and initial conditions. Use an iterative approach.
   a. $a_n = 3a_{n-1}$, $a_0 = 2$
   b. $a_n = a_{n-1} + 2$, $a_0 = 3$
   c. $a_n = na_{n-1}$, $a_0 = 5$
   d. $a_n = 2na_{n-1}$,$a_0 = 1$

3. (Rosen exercises 7-1(12)) Assume that the population of the world in 2002 was 6.2 billion and is growing at the rate of 1.3% a year

   a. Set up a recurrence relation for the population of the world n years after 2002.
   b. Find an explicit formula for the population of the world n years after 2002

4. (Rosen exercises 7-1(18)) Find a recurrence relation for the number of permutations of a set with n element

Use this recurrence relation to find the number of permutations of a set with n element using iterations.

5. (Rosen exercises 7-1(35)) Messages are transmitted over a communications channel using two signals. The transmittal of one signal requires 1 microsecond, and the transmittal of the other signal requires 2 microseconds.
   a. Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in n microseconds.
   b. What are the initial conditions?
   c. How many different messages can be sent in 10 microseconds using these two signals?

6. (Rosen exercises 7-1(40)) Find a recurrence relation for the number of bit sequences of length n with an even number of 0s.

7. (Rosen example 4 in 7-2) Find an explicit formula for the Fibonacci numbers

8. (Rosen exercises 7-3(14)) Suppose that there are $n = 2^k$ teams in an elimination tournament, where there are $n/2$ games in the first round, with the $n/2 = 2^{k-1}$ winners playing in the second round, and so on. Develop a recurrence relation for the number of rounds in the tournament.

9. (Rosen exercises 7-3(21)) Suppose that the function f satisfies the recurrence relation $f(n) = 2f(\sqrt{n}) + 1$ whenever n is a perfect square greater than 1 and $f(2) = 1$.

a. Find $f(16)$
b. Find a big-O estimate for $f(n)$.

10. (Rosen exercises 7-3(19))

a. Set up a divide-and-conquer recurrence relation for the number of multiplications required to compute x^n, where x is a real number and n is a positive integer.
b. Use the recurrence relation you found in part (a) to construct a big-O estimate for the number of multiplications used to compute x^n using the recursive algorithm.

11. Find the Big O estimation when n= 2^k, where f satisfies the recurrence relation f(n) = f(n/2) +1 with f(1) = 1 using the Master Theorem

12. Find the complexity of merge sort using the Master Theorem.

(1) Draw the recurrence tree (start with an array with 8 items)
(2) Build a recurrence relation for merge sort
(3) Find the complexity using the Master Theorem.