Put both your name and your lab section at the top of your submission.

1. [15 points] Use mathematical induction to show that whenever $n$ is a positive integer:
   \[
   \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \cdots + \frac{1}{(2n - 1)(2n + 1)} = \frac{n}{2n + 1}
   \]
   a) What is the basis step?
   b) What is the inductive step?

2. [20 points] Give a big-$O$ estimate for each of these functions. For the function $g$ in your estimate $f(x)$ is $O(g)$, use a simple function $g$ of smallest order.
   a. $6x^4 + 3^4 + 12$
   b. $2 + 4x + 8x^2$
   c. $(x^3 + 2x)/(2x + 1)$
   d. $\log x + 27$
   e. $(2^x + x^2)(x^4 + 3^4)$

3. [12 points] In the manner of Examples 1 and 2 from section 3.2 of the Rosen text, show that $4x^3 + 12x + 8$ is $O(x^4)$. Give an explanation and values of $C$ and $k$.

4. [15 points] For the following equation: $32x^4 + ((5x^2 + x)(13x^3 + 1))$
   a. State the closest $g(x)$ for the equation.
   b. Explain how the Big-O is computed given Theorems 1, 2 and 3 in Section 3.2 in Rosen.

5. [20 points] Assuming you use $k=1$ as a witness, state whether the following functions are $O(x^3)$, and if so, state for what value of $C$.
   a. $3^x$
   b. $10x + 42$
   c. $2 + 4x + 8x^2$
   d. $(\log x + 1)(2x + 3)$
   e. $2x + x!$

6. [18 points] Given the following algorithm, what is its Big-O? Justify your answer using the three theorems presented in the Complexity notes (and found in section 3.2 of Rosen). Assume the line “d” takes $O(1)$ or $O(c)$.
   a. for (int $x = 1$; $x <= n$; ++$x$) {
   b. for (int $y = 0$; $y < n$; ++$y$) {
   c. for (int $z = n$; $z > n-10$; --$z$) {
   d. $m = m * aArray[x] * bArray[y] * cArray[z]$;
   e. }
   f. }
   g. }