Grammar Worksheet

Warm Up Exercises:
This exercise pertains to the tortoise and the hare from Aesop’s Fables.
The next three exercises refer to the grammar with:
Start symbol (S) = sentence
Set of terminals (T) = {the, sleepy, happy, tortoise, hare, passes, runs, quickly, slowly}
Set of nonterminals (N) = {noun, phrase, transitive verb phrase, intransitive verb phrase, article, adjective, noun, verb, adverb}
Productions (P) these are the rules associated with the grammar =

- sentence → noun phrase, transitive verb phrase, noun phrase
- sentence → noun phrase, intransitive, verb phrase
- noun phrase → article, adjective, noun
- noun phrase → article, noun
- transitive verb phrase → transitive verb
- intransitive verb phrase → intransitive verb, adverb
- intransitive verb phrase → intransitive verb

These are the valid words associated with the grammar:

- article → the
- adjective → sleepy
- adjective → happy
- noun → tortoise
- noun → hare
- transitive verb → passes
- intransitive verb → runs
- adverb → quickly
- adverb → slowly

(1) Use the set of productions to show that these are valid sentences:
Example: The happy hare runs

```
  sentence → noun phrase, transitive verb phrase, noun phrase
  noun phrase → article, adjective, noun
  noun phrase → article, noun
  transitive verb phrase → transitive verb
  intransitive verb phrase → intransitive verb, adverb
  intransitive verb phrase → intransitive verb
```

- a) The sleepy tortoise runs quickly
- b) The tortoise passes the hare
- c) The sleepy hare passes the happy tortoise
(2) Find three additional valid sentences (remember you must use the grammar associated above):

a)

b)

c)

(3) Explain why “the hare runs the sleepy tortoise” is not a valid sentence:

Additional Exercises:

(4) Let $G = (V, T, S, P)$ be the grammar with $V = \{0, 1, A, B, S\}$, $T = \{0, 1\}$, and set of productions $P$ consisting of $S \rightarrow 0A$, $S \rightarrow 1A$, $A \rightarrow 0B$, $B \rightarrow 1A$, $B \rightarrow 1$

a) Describe the language generated by $G$ - you can answer this problem by explaining types of sentences can be legally created - think about what HAS to be true (ex: how does each sentence start? Is there a pattern? How does each sentence have to terminate?):

b) Derivation Trees: a derivation tree has a specific format; the root represents the start symbol, internal nodes are labeled with nonterminal symbols and the leaves are labeled with terminal symbols. For each sentence there exists at least one derivation tree but you always need a single grammatically correct sentence to create a derivation tree.

The example to the right shows a derivation tree for the sentence 001 that uses the grammar in question 4:

Draw the derivation tree associated with the sentence 101:
Draw the derivation tree associated with the sentence 1011:

(5) Let $V = \{S, A, B, a, b\}$ and $T = \{a, b\}$. Find the language generated by the grammar $(V, T, S, P)$ when the set $P$ of productions consists of:

a) $S \rightarrow AB$, $S \rightarrow aA$, $A \rightarrow a$, $B \rightarrow ba$

b) $S \rightarrow AA$, $S \rightarrow B$, $A \rightarrow aaA$, $A \rightarrow aa$, $B \rightarrow bB$, $B \rightarrow b$

(6) Find the grammar for the language with the set consisting of each of the following:

Example: the set of all bit strings containing an even number of 0s and no 1s:

$$<S> = 00<S> \mid \lambda$$

(Note: the lambda, $\lambda$, represents an empty string)

Alternatively:

$$<S> = 00<A>$$

$$<A> = 00<A> \mid \lambda$$

a) The set consisting of the strings 0, 11, and 010

b) The set of strings of three 0s followed by two or more 0s
c) The set of strings that contain exactly one 1

d) The set of odd-length strings whose first, middle, and last characters are all the same, over the alphabet \{0,1\}. (Some examples include: 000, 01000, 10111, 1011)

(7) A palindrome is a string that reads the same backward as it does forward, that is, a string \(w\), where \(w = w^R\), where \(w^R\) is the reversal of the string \(w\). Find that grammar that generates the set of odd length palindromes over the alphabet \{a,b\}: