Recitation 6 Worksheet 1
Complexity

1. What is big-O for each of the following?
   • \( f(x) = 17 + 3x + 21x^2 \)
   • \( g(x) = 19x + x(2x + 2) \)
   • \( h(x) = x^{n} + x^{n+2} + x^{n+4} \) for some \( n \geq 1 \)
   • \( j(x) = x^2 \log(x) \)

2. Show that \( g(x) = 19x + x(2x + 2) \) is not \( O(x) \)

3. Using \( k = 1 \), show that \( h(x) = x^{n} + x^{n+2} + x^{n+4} \) for some \( n \geq 1 \) is \( O(x^n) \) and state a witness \( C \) that can be used with \( k = 1 \) to demonstrate this fact.

4. Given an integer array \( a[] \) of size \( n \), what are the worst-case, best-case, and average-case complexity of the following?
   ```
   for (int i=0; i<n; i++) {
       for (int j=0; j<n; j++) {
           if (i!=j && a[i]==a[j]) return a[i];
       }
   }
   ```

5. Find a recurrence relation for the number of strings of digits that contain exactly one zero or one digit.

6. Use the master theorem to compute the complexity of the following:
   ```
   int foo(int[] a) {
       if (a.length == 1) return a[0];
       if (a.length == 2) return a[0] + a[1];
       int size = a.length/3;
       int extra = a.length%3;
       int[] a0 = new int[size];
       int[] a1 = new int[size];
       int[] a2 = new int[size+extra];
       for (int i=0; i<size; i++) {
           a0[i] = a[i];
           a1[i] = a[i+size];
           a2[i] = a[i+2*size];
       }
       for (int i=0; i<extra; i++) {
           a2[i+size] = a[i+3*size];
       }
       return foo(a0) + foo(a1) + foo(a2);
   }
   ```