CS200 Fall 2016 written homework 2

name:

id:

1. Using the Master Theorem:

Let f be an increasing function that satisfies
$f(n) = a \cdot f(n/b) + c \cdot n^d$
whenever $n = b^k$, where k is a positive integer, $a \geq 1$, b is an integer > 1, and c and d are real numbers with c positive and d nonnegative. Then

$$f(n) = \begin{Bmatrix} O(n^d) & \text{if } a < b^d \\ O(n^d \log n) & \text{if } a = b^d \\ O(n^{\log_b a}) & \text{if } a > b^d \end{Bmatrix}$$

What are the big-O bounds recurrence relations? (Simplify logs and exponents.)

a) f(n) = 4 f(n/2) + n

b) f(n) = 2 f(n/4) + n

c) f(n) = 4 f(n/4) + n^2

d) f(n) = 2 f(n/2) + n

e) f(n) = 2 f(n/2) + 1

f) f(n) = f(n/2) + 1

2. Which of the above describes the complexity of

a) Binary Search

b) Merge Sort

3. Given the following method:

```
public int recMax (int[] A){
     return recMax(A,0,A.length-1);
}
private int recMax(int[]A, int lo, int hi){
     if(lo==hi) return A[lo];
     else{
          int mid = (lo+hi)/2;
          int m1 = recMax(A,lo,mid);
          int m2 = recMax(A,mid+1,hi);
          return Math.max(m1, m2);
     }
}
```

a) Derive a recurrence rM(n) relation for recMax(A, lo, hi), where n = hi-lo+1.

rM(n) = 1 for n = 1

rM(n) = for n > 1

b) Use the Master Theorem to solve the recurrence and obtain the big O complexity of recMax.

```
rM(n) = O(      )
```

4. Find a solution to the following recurrence relation, using repeated substitution:

```
f(1) = 2000
f(n) = 1.1 f(n-1)  for n>1
```