CS200: Priority Queues, Heaps

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Priority Queues

- Characteristics
  - Items are associated with a value: priority
  - One element at a time - the one with the highest priority
- Uses
  - Operating systems: processes and threads
  - Network management
    - Real time traffic usually gets highest priority when bandwidth is limited

Priority Queue ADT Operations

1. Create an empty priority queue

   `createPQueue()`

2. Determine whether empty

   `pqIsEmpty():boolean`

3. Insert new item

   ```
   pqInsert(in newItem:PQItemType) throws
       PQueueException
   ```

4. Retrieve and delete the item with the highest priority

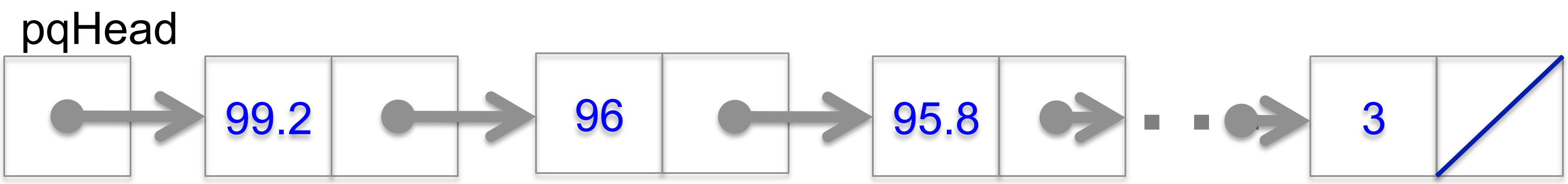
   `pqDelete():PQItemType`

PQ – ArrayList Implementation

30		3	20	…	95	95	96	99
size		0	1					29

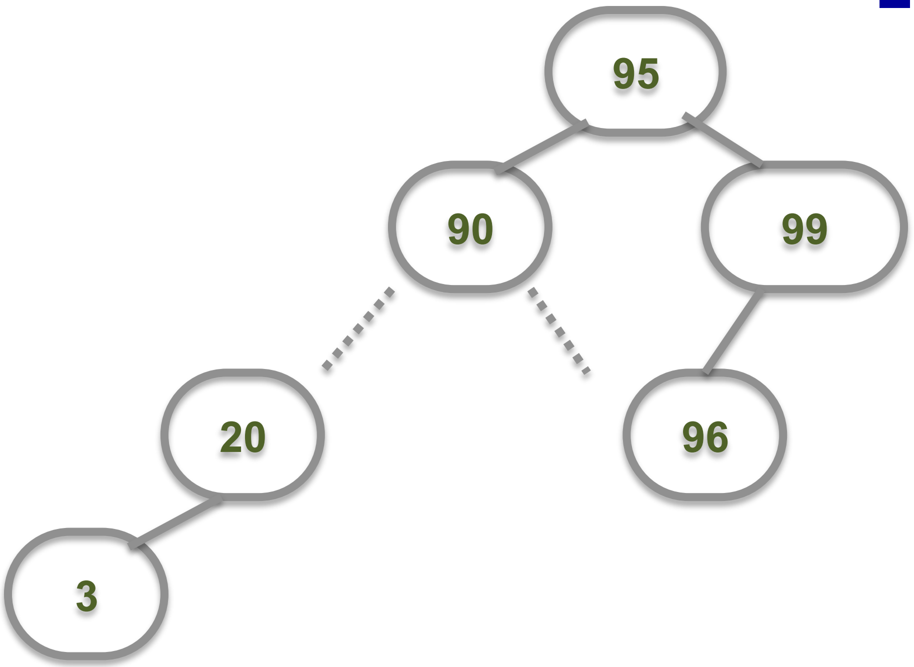
- ArrayList ordered by priority
  - pqInsert: find the correct position for add at that position, the ArrayList.add(i,item) method will shift the array elements to make room for the new item
  - pqDelete: remove last item (at size()-1)
  - **Why did we organize it in increasing order?**

PQ – Reference-based Implementation



- Reference-based implementation
  - Sorted in descending order
    - Highest priority value is at the beginning of the linked list
    - `pqDelete` returns the item that `psHead` references and changes `pqHead` to reference the next item.
    - `pqInsert` must traverse the list to find the correct position for insertion.

PQ – BST Implementation



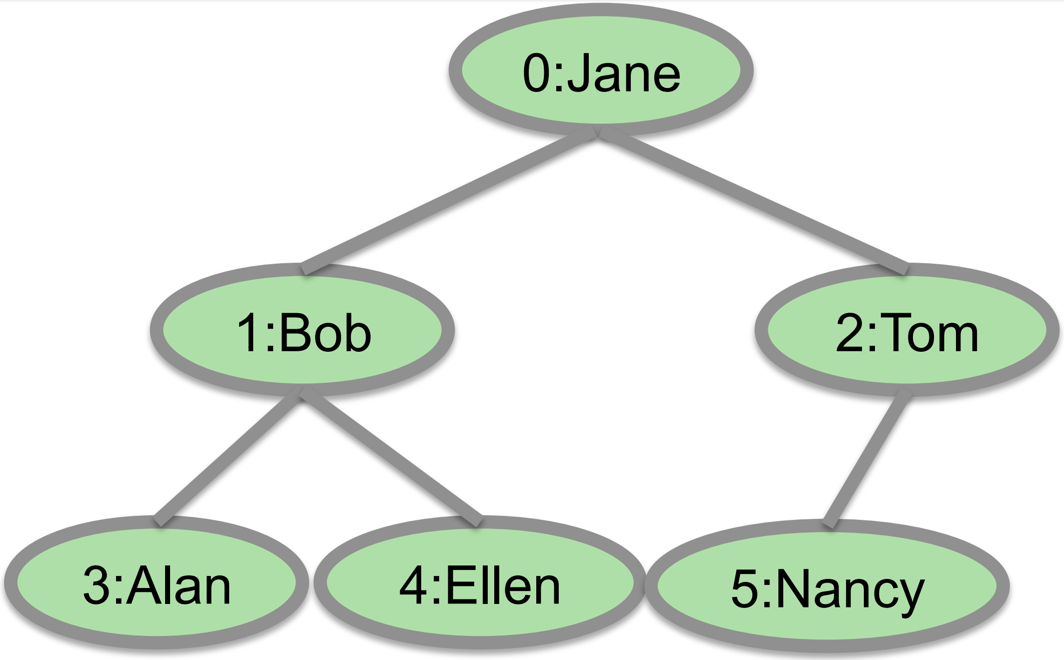
- Binary search tree
  - Where is the highest value of the nodes?
  - pqInsert O(height)
    - at a new leaf, e.g.30
  - pqDelete O(height)
    - need to remove the max
    - max has at most one child

The problem with BST

- BST can get unbalanced (height = O(n)) so in the worst case pqInsert and pqDelete can get O(n).
- A more balanced tree structure would be better.
- What is a **balanced** binary tree structure?
  - Height of any node's right sub-tree differs from left sub-tree by 0 or 1
- A complete binary tree is balanced, and it is easy to put the nodes in an array.

Complete Binary Tree

Level-by-level numbering of a complete binary tree, NOTE 0 based!



What is the parent child index relationship?

*left child i: at 2*i+1*

right child i: at 2(i+1)*

lparent i: at (i-1)/2

Heap - Definition

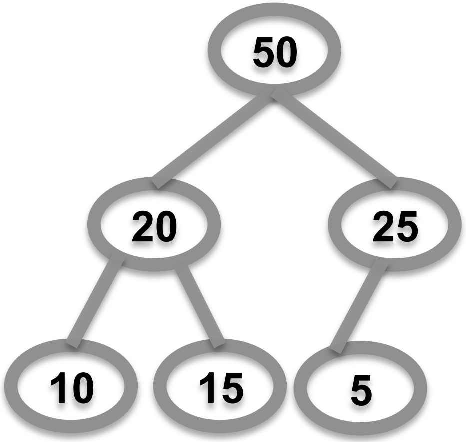
- A maximum heap (maxheap) is a ***complete* binary tree** that satisfies the following:
  - It is an empty tree

    or it has the heap property:
    - Its root contains a key greater or equal to the keys of its children
    - Its left and right sub-trees are also maxheaps
  - A minheap has the root less or equal children, and left and right sub trees are also minheaps

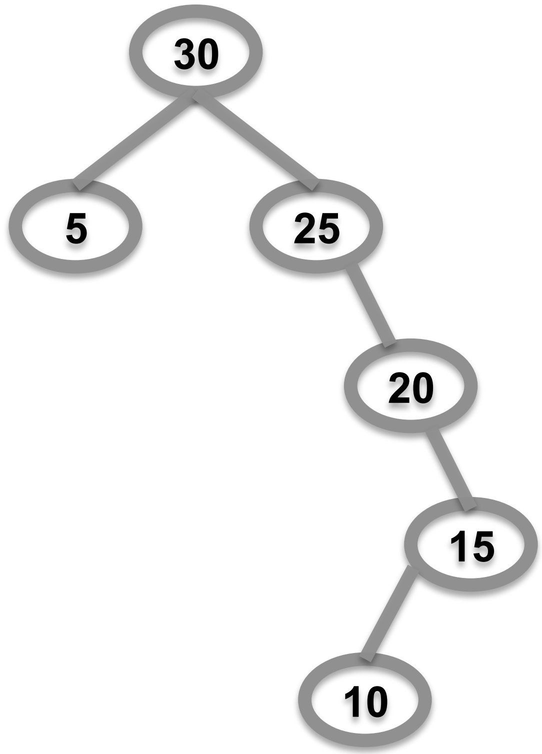
maxHeap Property Implications

- Implications of the heap property:
  - **The root holds the maximum value (global property)**
  - **Values in descending order on every path from root to leaf**
- Heap is NOT a binary search tree, as in a BST the nodes in the right sub tree of the root are larger than the root

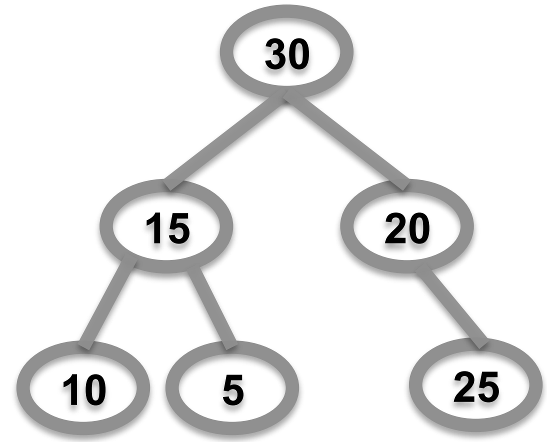
Examples



Satisfies heap property AND Complete



Satisfies heap property BUT Not complete



Does not satisfy heap property AND Not complete

Heap ADT

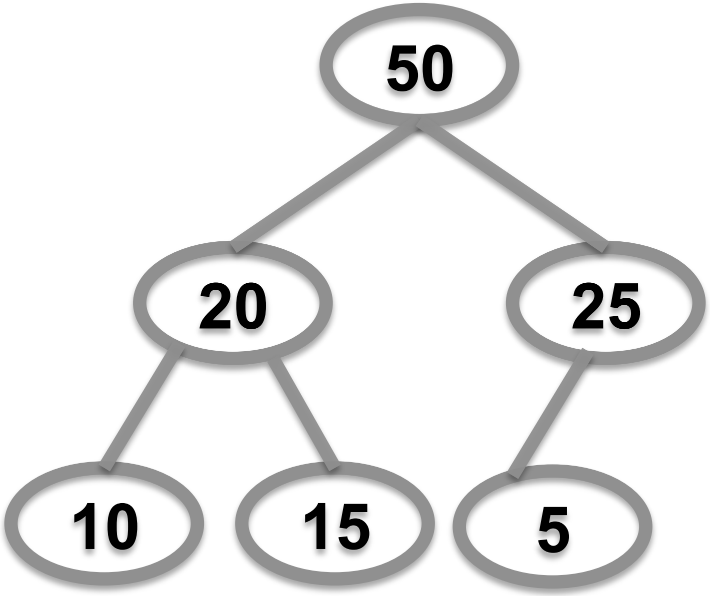
```
createHeap()  // create empty heap

heapIsEmpty():boolean
  // determines if empty

heapInsert(in newItem:HeapItemType)
  throws HeapException
  /* inserts newItem based on its search key.
    Throws exception if heap full
    This may not happen if e.g.implemented
    with an ArrayList */

heapDelete():HeapItemType
  // retrieves and then deletes heap's root
  // item which has largest search key
```

Array(List) Implementation



0	50
1	20
2	25
3	10
4	15
5	5

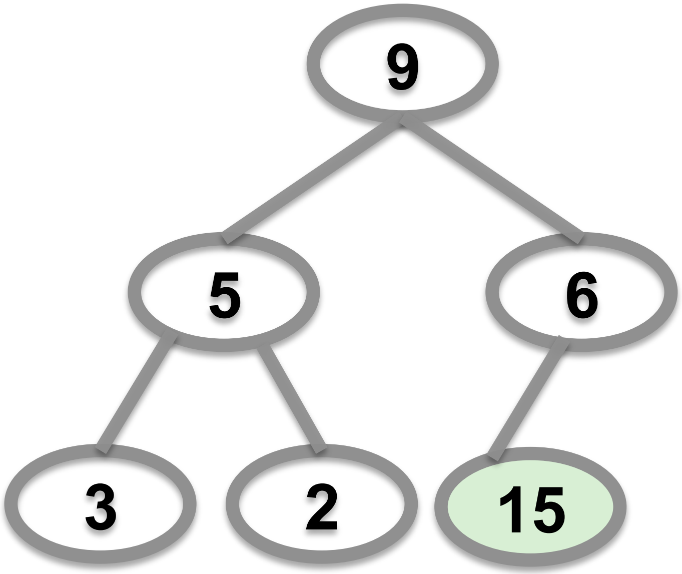
Array(List) Implementation

- Traversal items:
  - Root at position 0
  - Left child of position i at position 2*i+1
  - Right child of position i at position 2*(i+1)
  - Parent of position i at position (i-1)/2
    (integer division)

Heap Operations - `heapInsert`

- **Step 1**: put a new value into first open position (maintaining completeness), i.e. at the end
- but now we potentially violated the heap property, so:
- **Step 2**: bubble values up
  - **Re-enforcing the heap property**
  - Swap with parent until in the right place

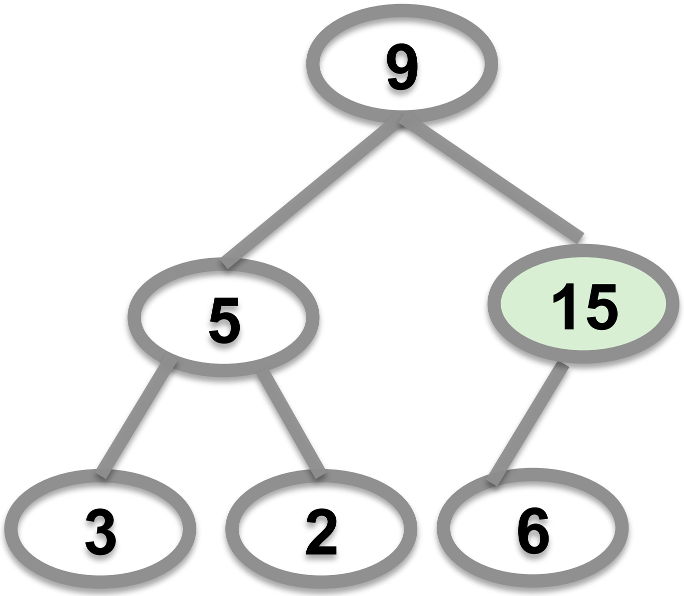
Insertion into a heap (Insert 15)



Insert 15

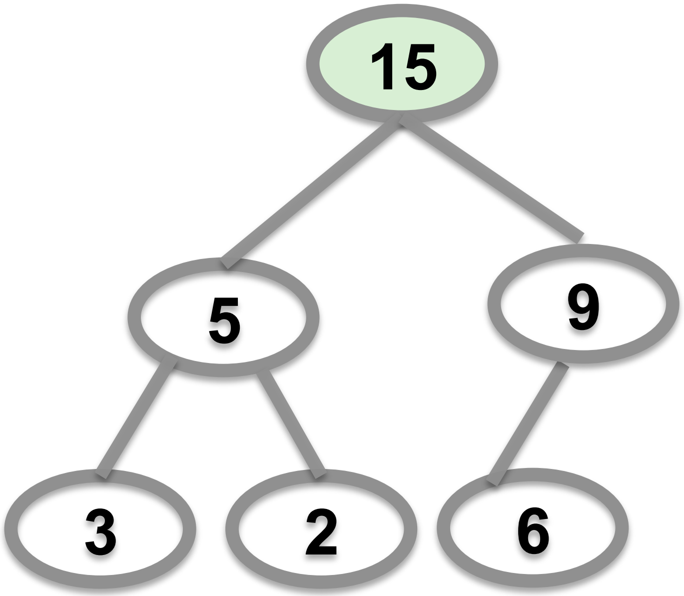
bubble up

Insertion into a heap (Insert 15)



bubble up

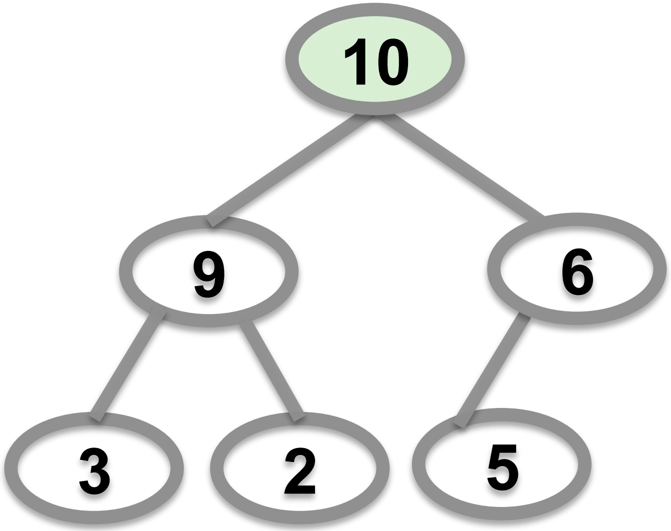
Insertion into a heap (Insert 15)



Heap operations – `heapDelete`

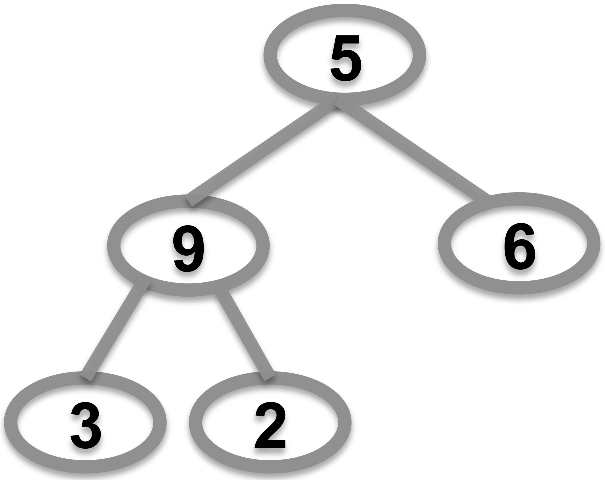
- **Step 1**: always remove value at root, the max/min is at root.
- **Step 2**: substitute with rightmost leaf of bottom level to fill the void by removing the very last element in the array.
- **Step 3**: percolate / bubble down to satisfy heap property.
  - Swap with maximum child as necessary, until in place
  - this is called **HEAPIFY**

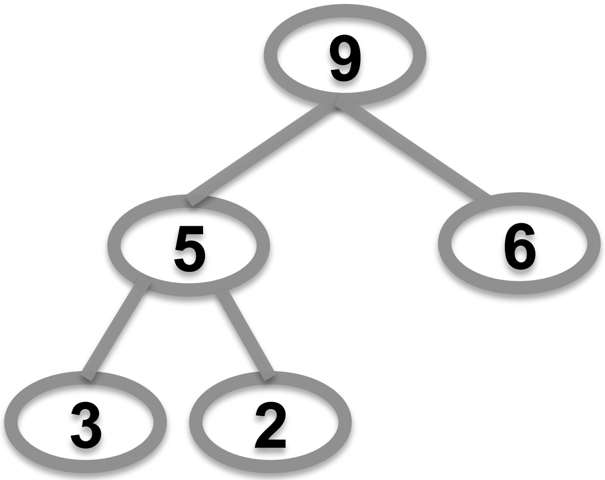
Deletion from a heap



Delete 10
Place last node in root

bubble down
heapify
draw the heap




9
5
6
3
2

Array-based Heaps: Complexity

	Average	Worst Case
insert	O(log n)	O(log n)
delete	O(log n)	O(log n)

Heap versus BST for PriorityQueue

- BST can also be used to implement a priority queue
- How does worst case complexity compare?
  - BST: O(n) - Heap: O(log n)
- How does average case complexity compare?
  - BST: O(log n) if balanced - Heap: O(log n)

Small number of priorities

- A heap of queues with a queue for each priority value.

- This is more efficient for a large number of items and small number of priorities.

- Notice the connection to Radix sort.

HeapSort

- Algorithm
  - Insert all elements (one at a time) to a heap
  - Iteratively delete them
    - Removes minimum/maximum value at each step
- Computational complexity?

HeapSort

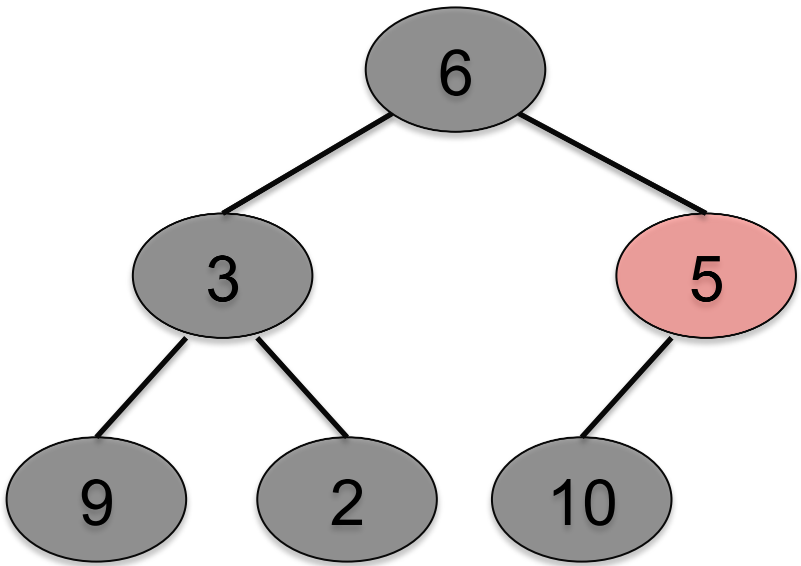
- Alternative method (in-place):
  - **buildHeap:** create a heap out of the input array:
    - Consider the input array as a complete binary tree
    - Create a heap by iteratively expanding the portion of the tree that is a heap
      - Leaves are already heaps
      - Start at last internal node
      - **Go backwards** calling **`heapify`** with each internal node
  - Iteratively swap the root item with last item in unsorted portion and rebuild

Building the heap

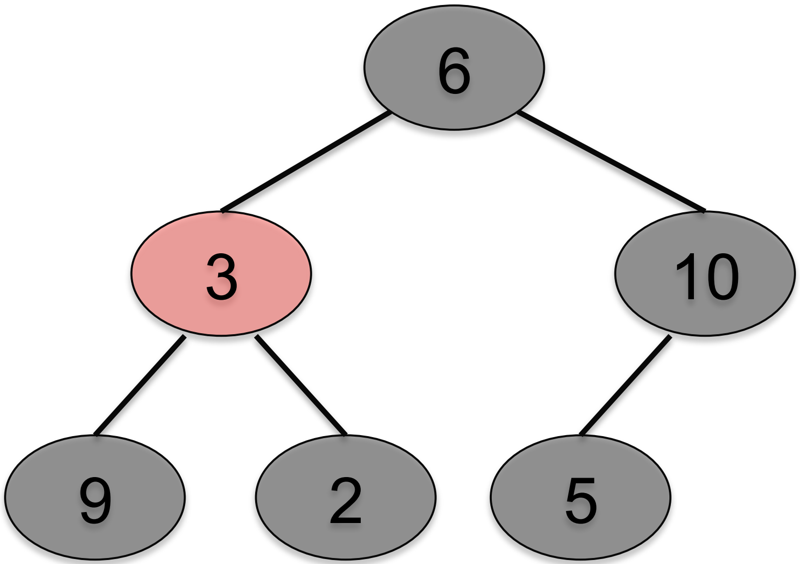
```
for (i = (n-2)/2 down to 0)
    //pre: the tree rooted at index is a semiheap
    //i.e., the sub trees are heaps
    heapify(i); // bubble down
    //post: the tree rooted at index is a heap
```

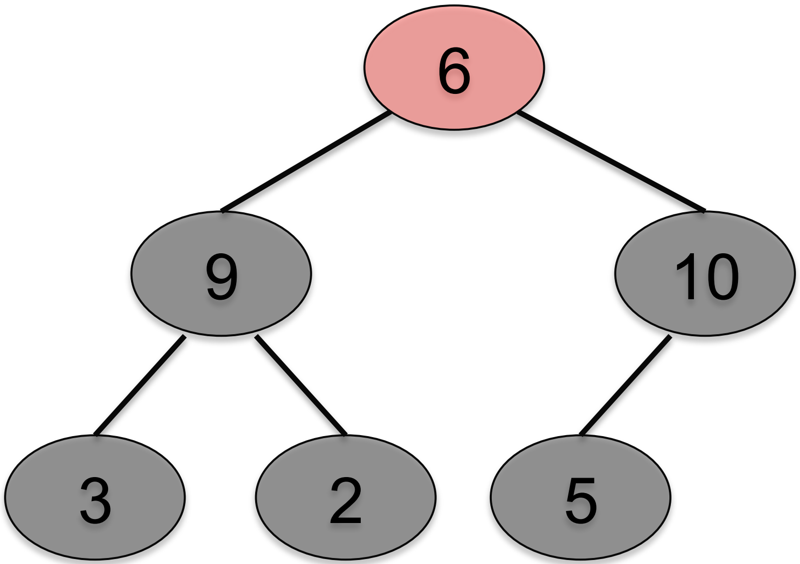
- WHY start at (n-2)/2?
- WHY go backwards?

- The whole method is called **buildHeap**
- One bubble down is called **heapify**



6	3	5	9	2	10


6
3
10
9
2
5


6
9
10
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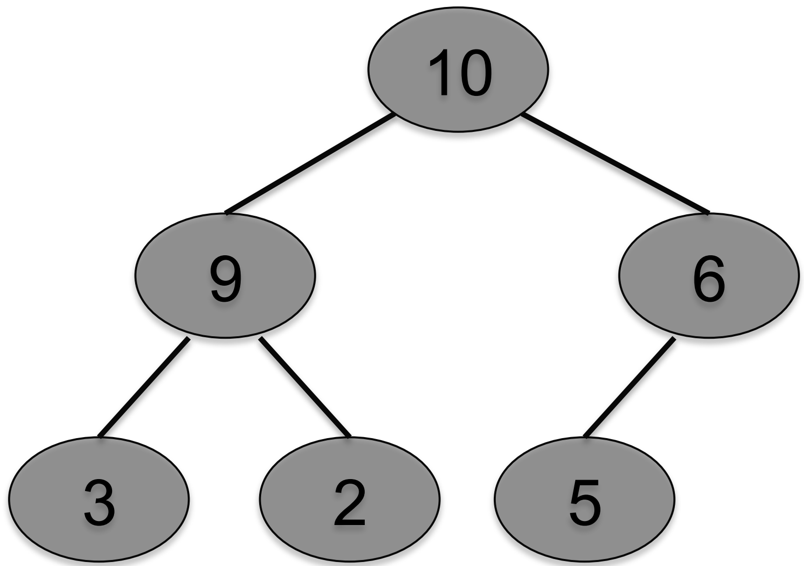
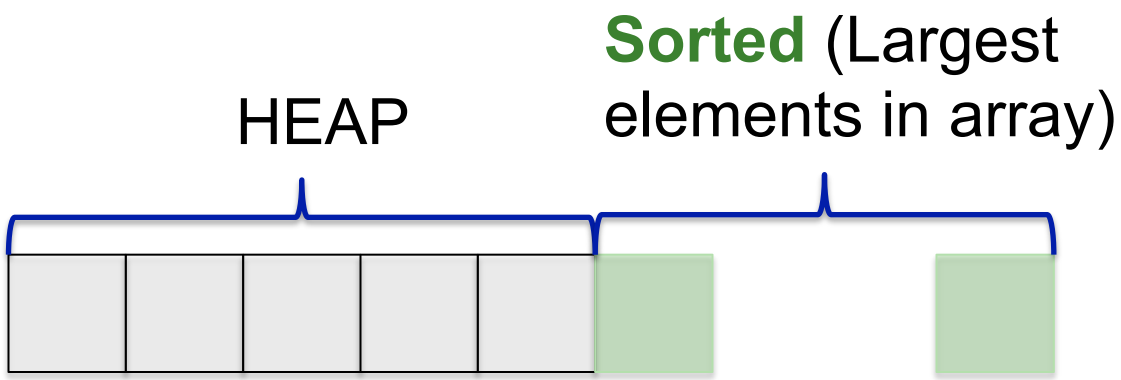
5



10	9	6	3	2	5

In place heapsort using an array

- First build a heap out of an input array using buildHeap()
- Then partition the array into two regions; starting with the full heap and an empty sorted and stepwise growing sorted and shrinking heap.



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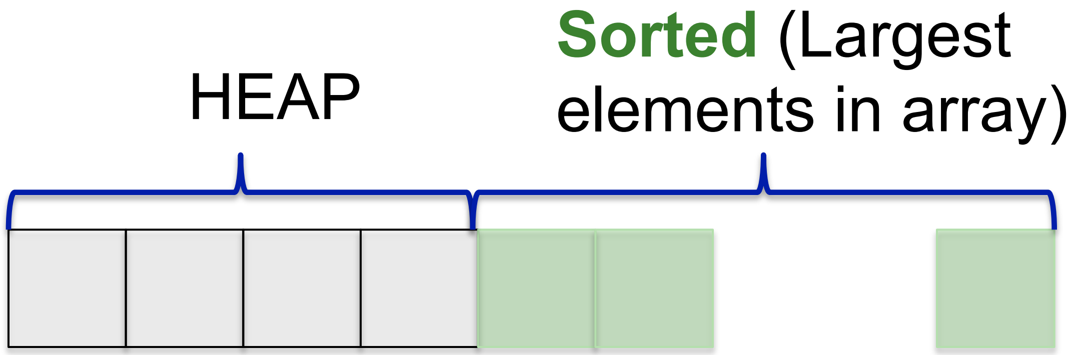

Do it, do it

HEAP

10	9	6	3	2	5
9	5	6	3	2	10
6	5	2	3	9	10
5	3	2	6	9	10
3	2	5	6	9	10
2	3	5	6	9	10
2	3	5	6	9	10

SORTED