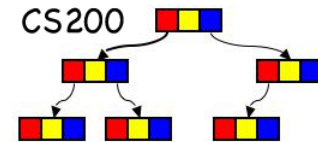
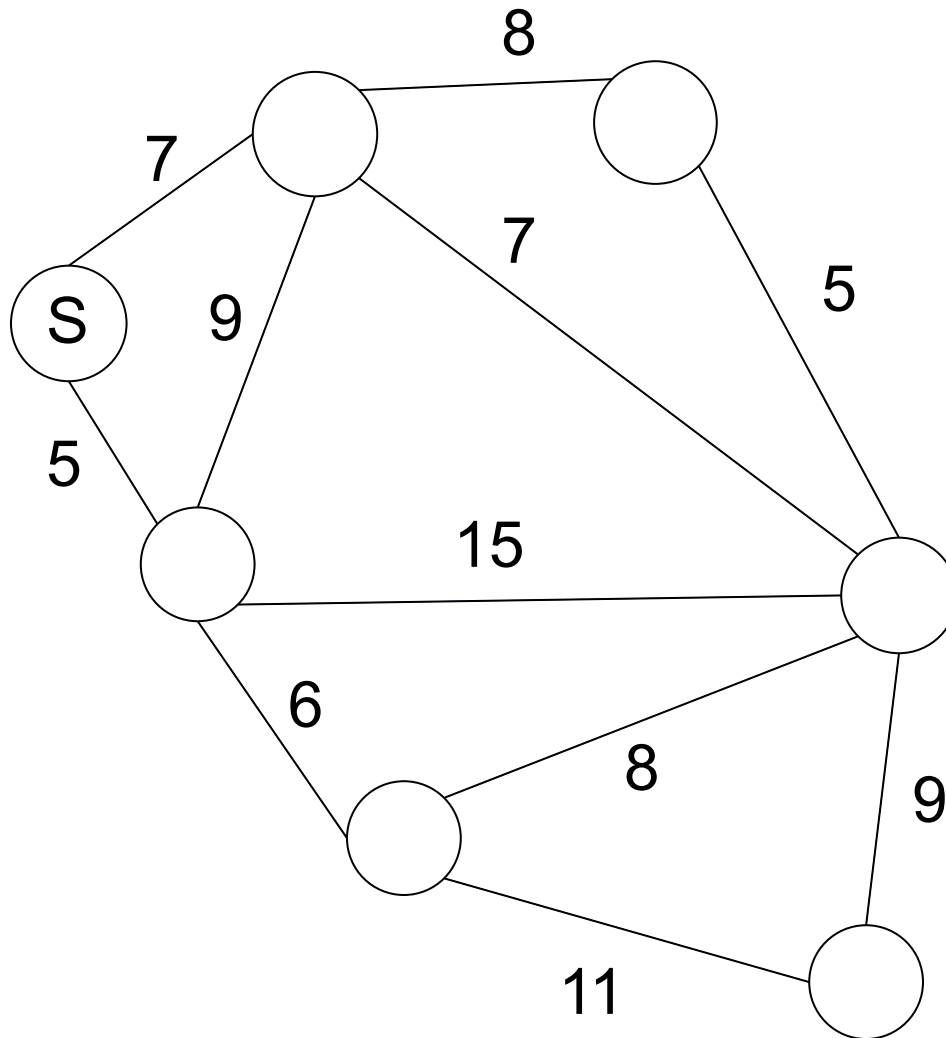
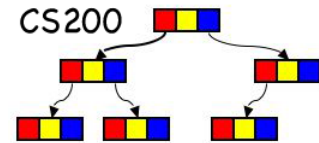
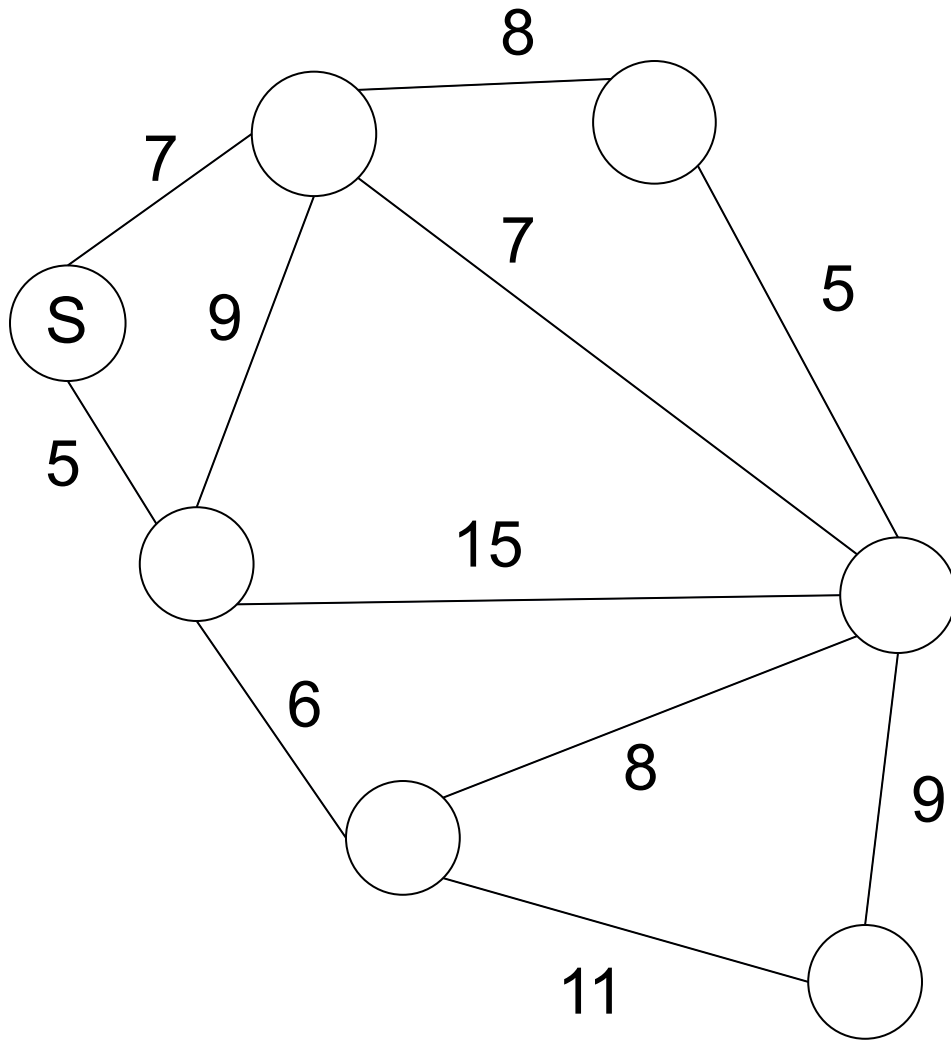


Fun and Games with Graphs

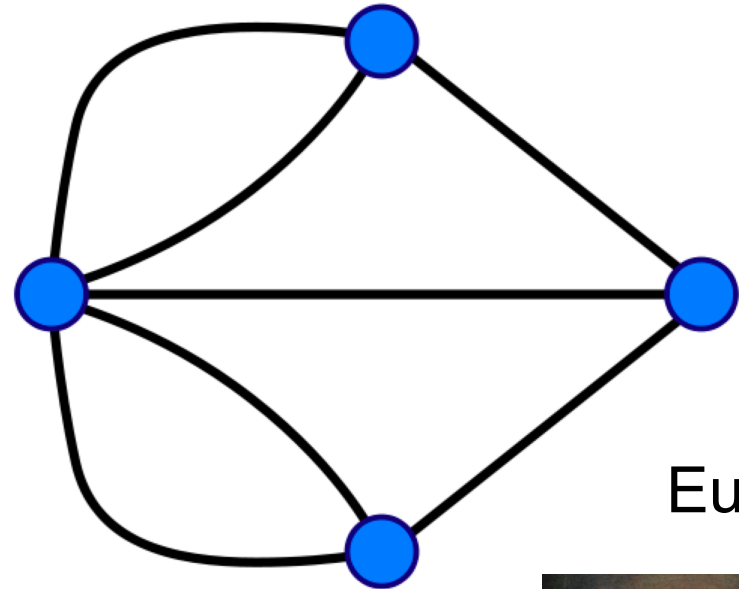
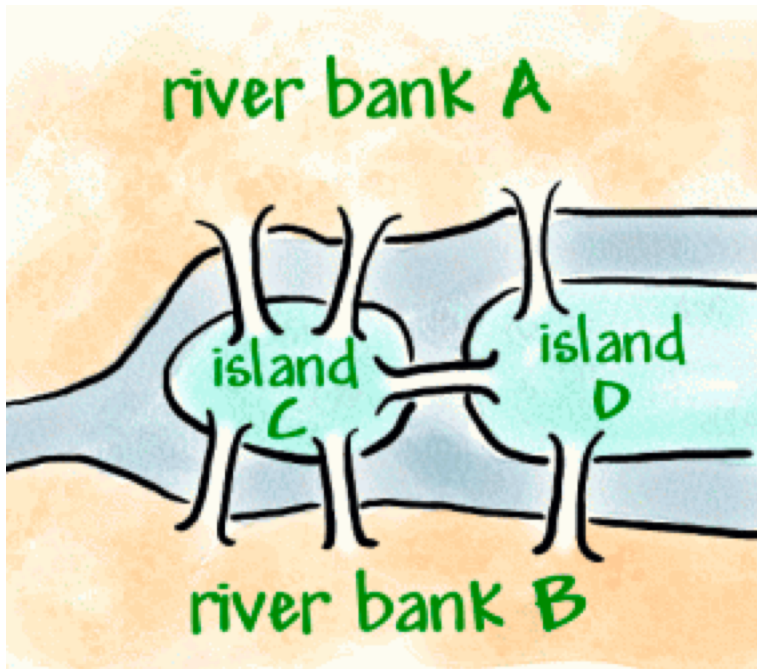
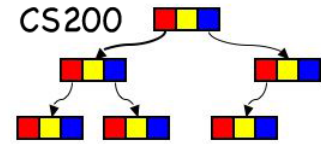


Do Dijkstra's Shortest Paths Algorithm, Source: S



Do Prim's Minimum Spanning Tree Algorithm, Source: S

Bridges of Königsberg Problem



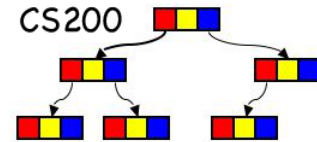
Euler

Is it possible to travel across every bridge without crossing any bridge more than once?



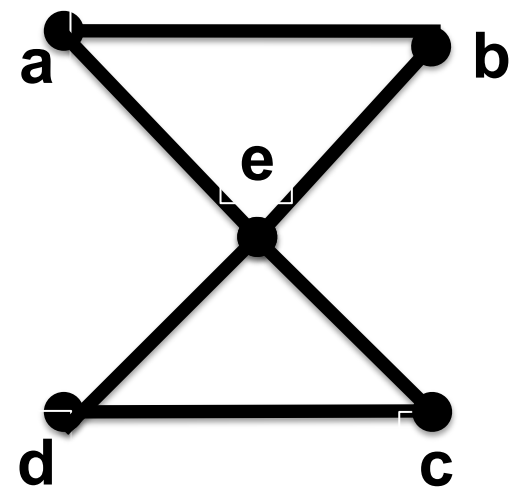
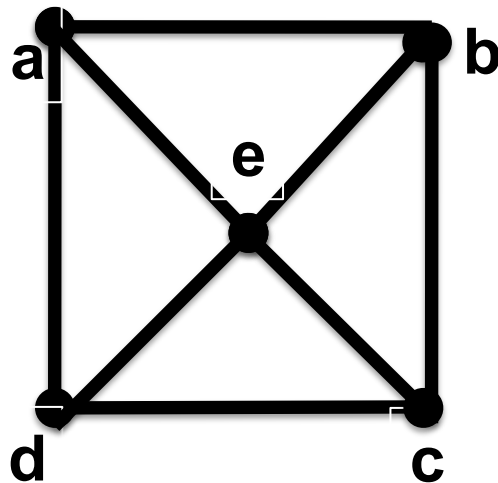
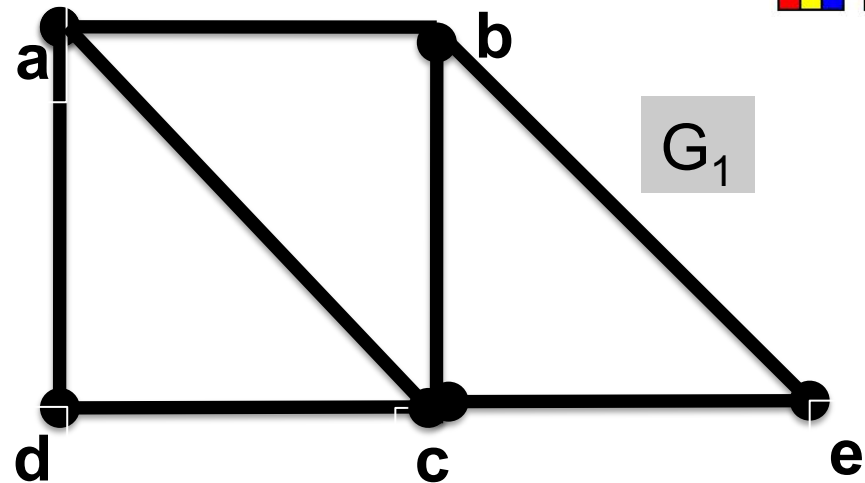
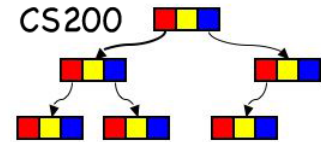
<http://yeskarthi.wordpress.com/2006/07/31/euler-and-the-bridges-of-konigsberg/>

Eulerian paths/circuits

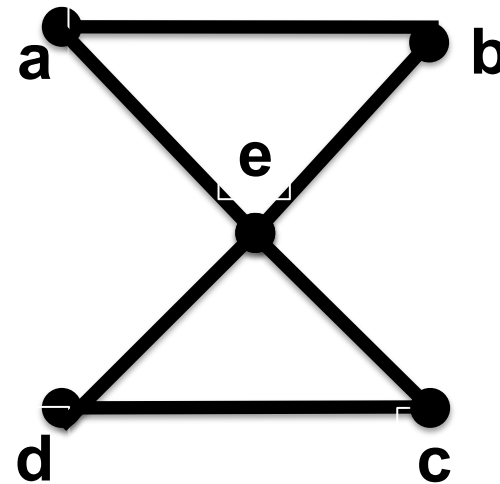
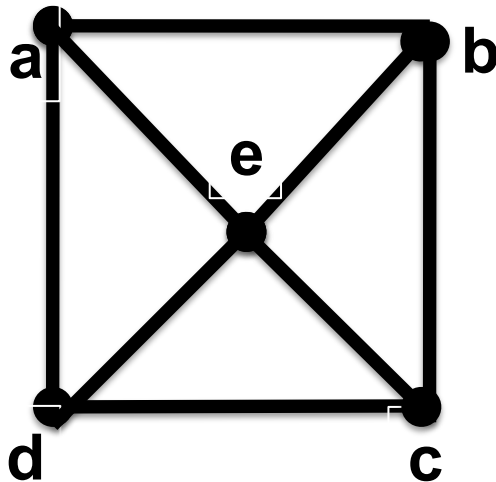
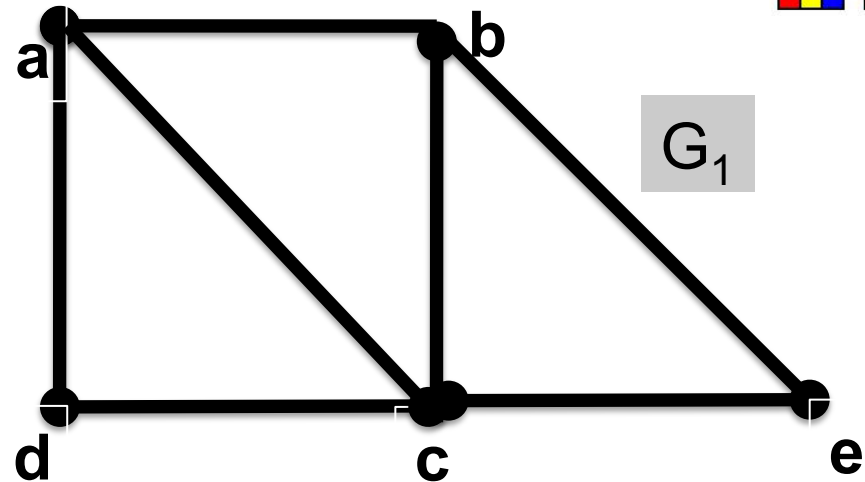
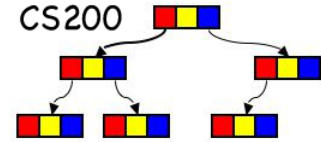


- Eulerian path: a path that visits each edge in the graph once
- Eulerian circuit: a cycle that visits each edge in the graph once
- Is there a simple criterion that allows us to determine whether a graph has an Eulerian circuit or path?

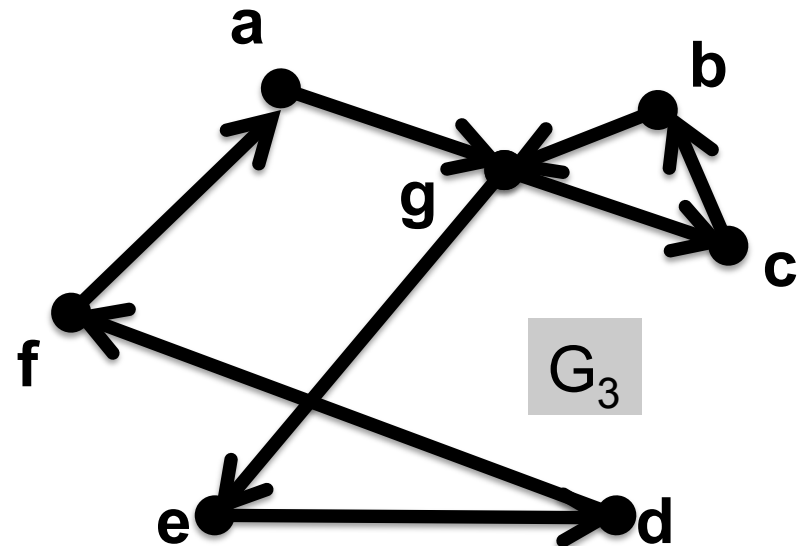
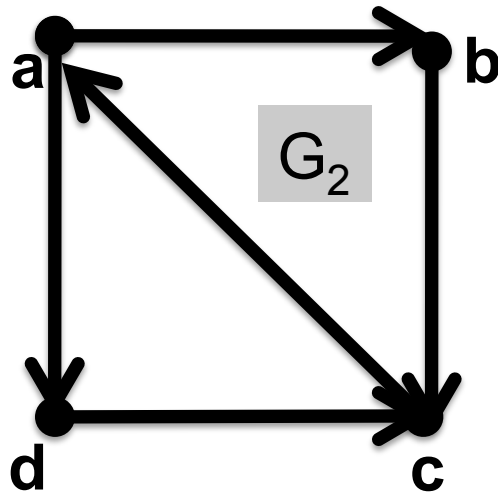
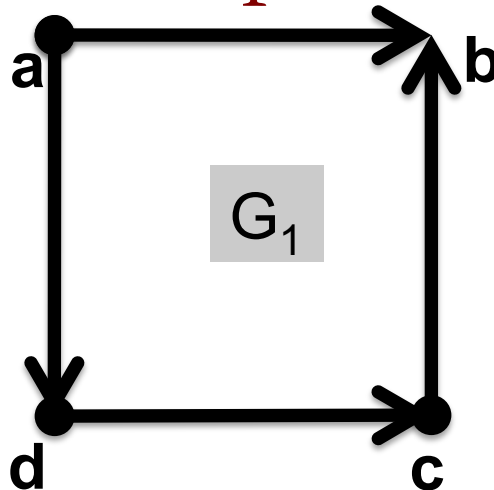
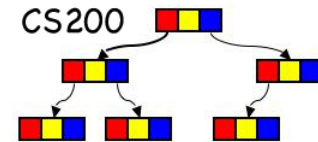
Example: Does any graph have an Euler path?



Example: Does any graph have an Euler circuit?

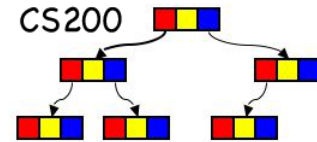


Example: Does any graph have an Euler circuit or path?

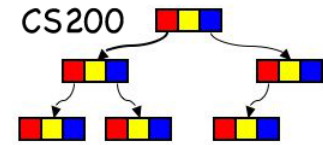


Which has an Euler Circuit?
A. G_1 B. G_2 C. G_3
D. None E. All

Theorems about Eulerian Paths & Circuits

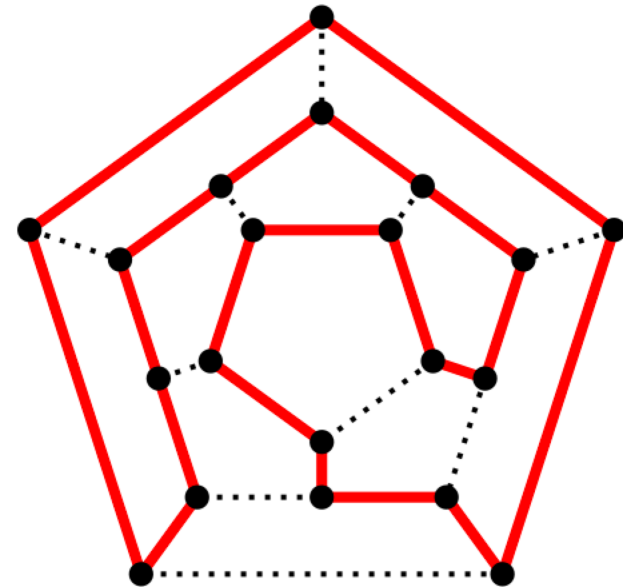


- **Theorem:** A connected multigraph has an Euler path iff it has exactly two vertices of odd degree.
- **Theorem:** A connected multigraph with at least two vertices has an Euler circuit iff each vertex has an even degree.
- Demo:
<http://www.mathcove.net/petersen/lessons/get-lesson?les=23>

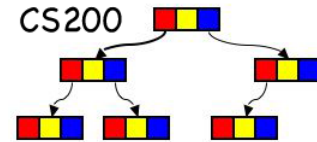


Hamiltonian Paths/Circuits

- A Hamiltonian path/circuit: path/circuit that visits every vertex exactly once.
- Defined for directed and undirected graphs

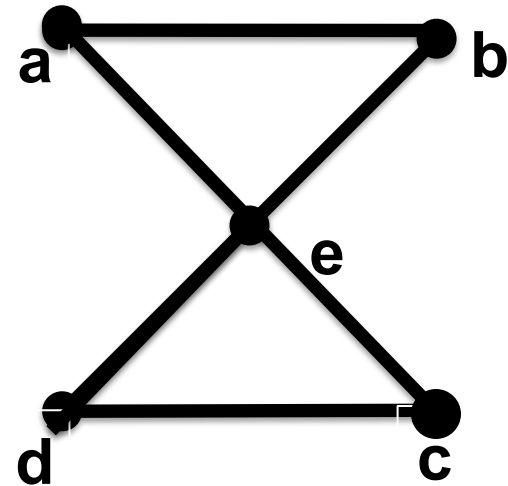
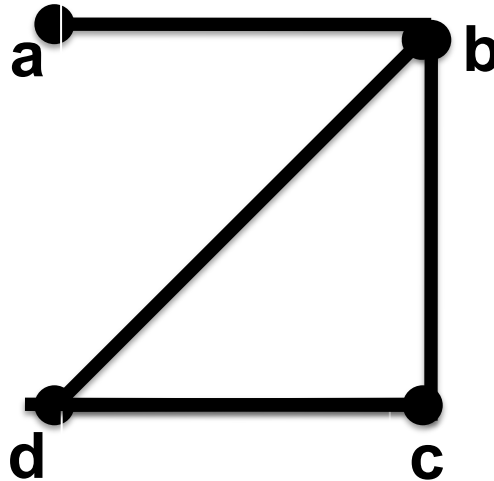
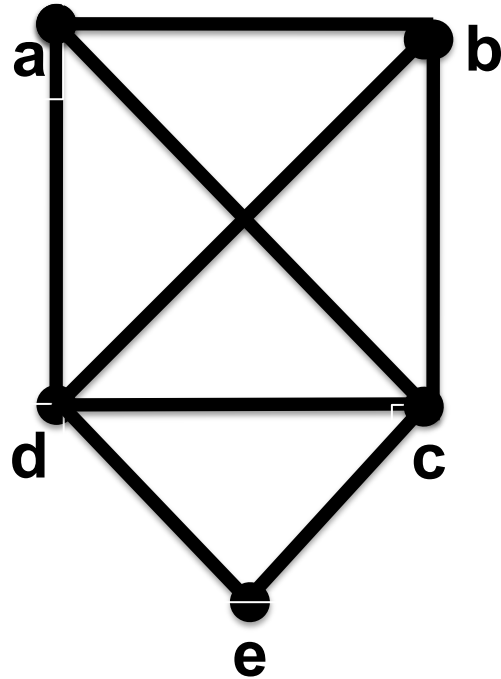
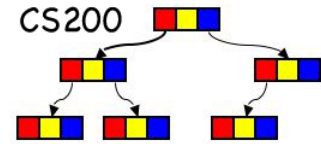


Circuits (cont.)

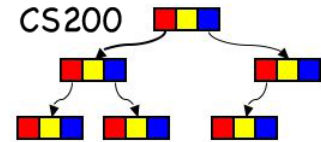


- **Hamiltonian Circuit:** path that begins at vertex v , passes through every *vertex* in the graph exactly once, and ends at v .
 - <http://www.mathcove.net/petersen/lessons/get-lesson?les=24>

Does any graph have a Hamiltonian circuit or a Hamiltonian path?

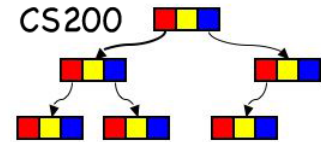


Hamiltonian Paths/Circuits

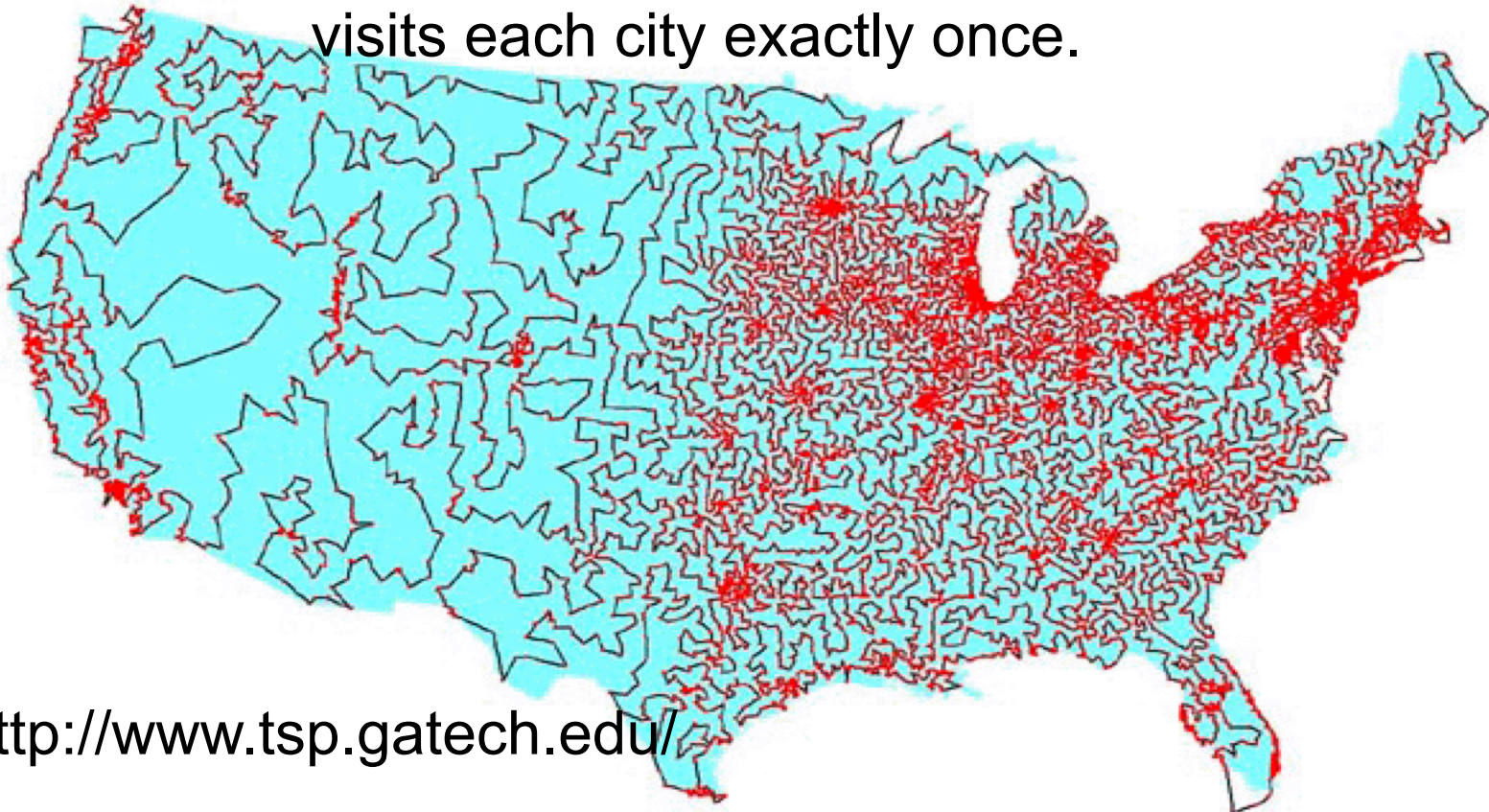


- Is there an efficient way to determine whether a graph has a Hamiltonian circuit?
 - NO!
 - This problem belongs to a class of problems for which it is believed there is no efficient (polynomial running time) algorithm.

The Traveling Salesman Problem



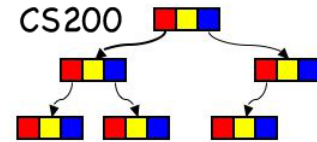
TSP: Given a list of cities and their pairwise distances, find a shortest possible tour that visits each city exactly once.



<http://www.tsp.gatech.edu/>

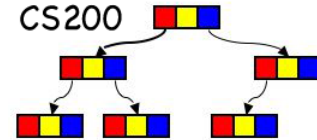
13,509 cities and towns in the US that have more than 500 residents

Using Hamiltonian Circuits

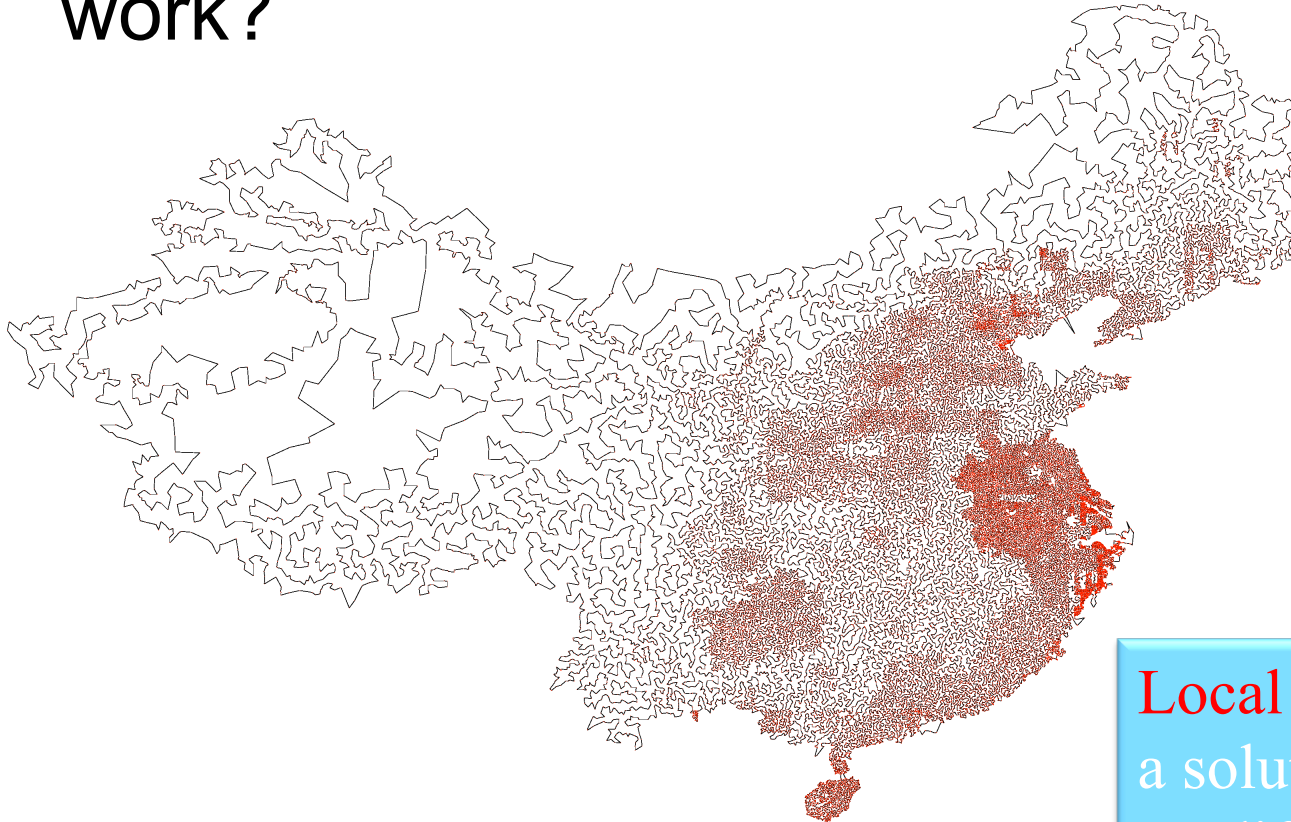


- Examine all possible Hamiltonian circuits and select one of minimum total length
- With n cities..
 - $(n-1)!$ Different Hamiltonian circuits
 - Ignore the reverse ordered circuits
 - $(n-1)!/2$
- With 50 cities
- 12,413,915,592,536,072,670,862,289,047,373,375,038,521,486,354,677,760,000,000,000 routes

TSP



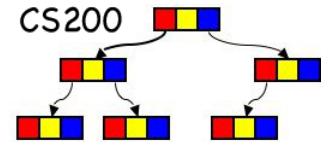
- How would a approximating algorithm for TSP work?



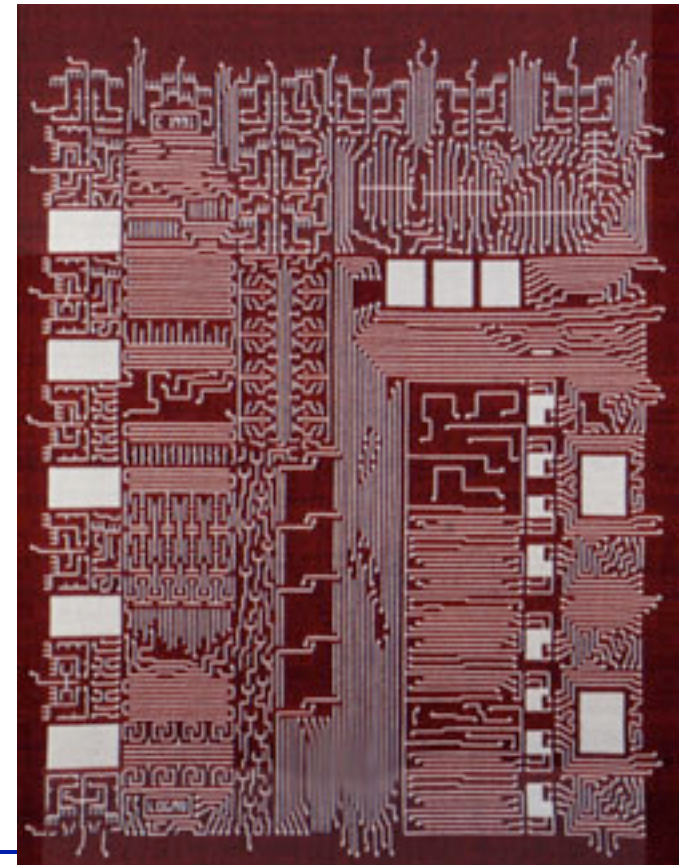
Local search: construct a solution and then modify it to improve it

71,009 Cities in China CS200 - Graphs

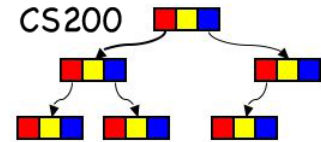
Planar Graphs



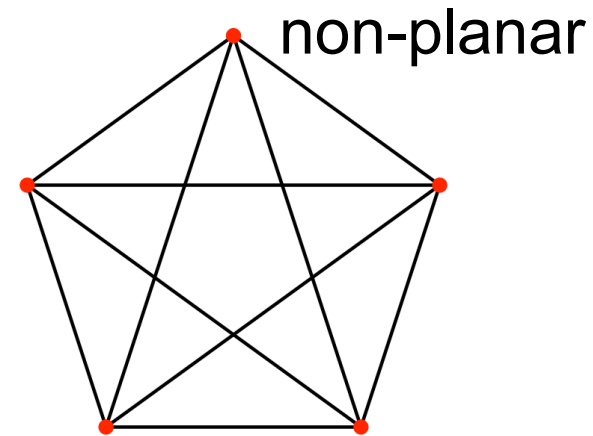
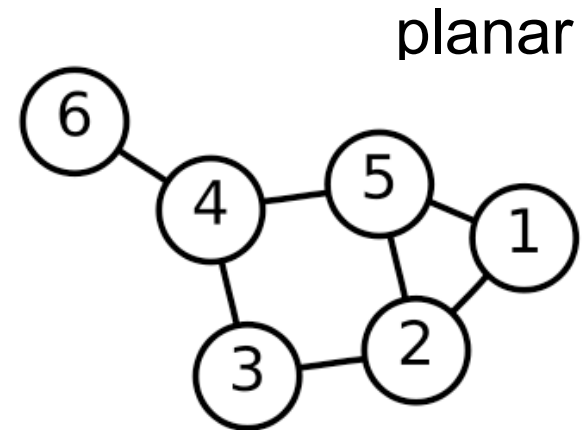
- You are designing a microchip – connections between any two units cannot cross



Planar Graphs

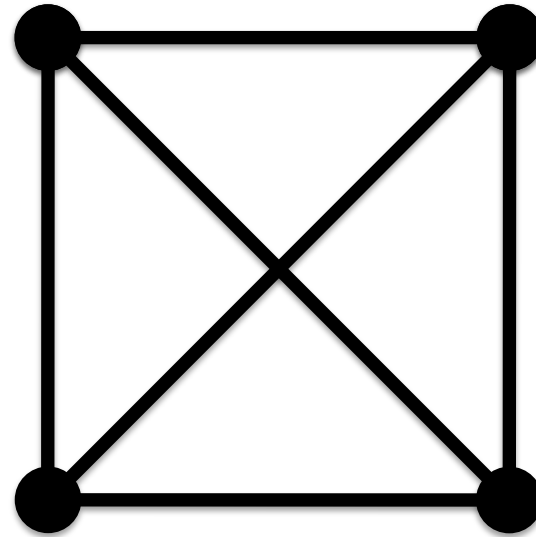
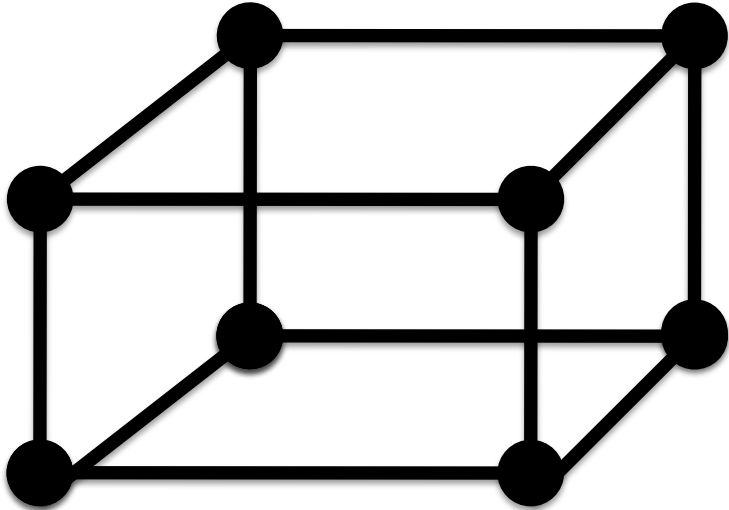
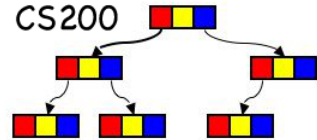


- You are designing a microchip – connections between any two units cannot cross
- The graph describing the chip must be **planar**

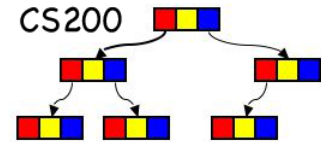


http://en.wikipedia.org/wiki/Planar_graph

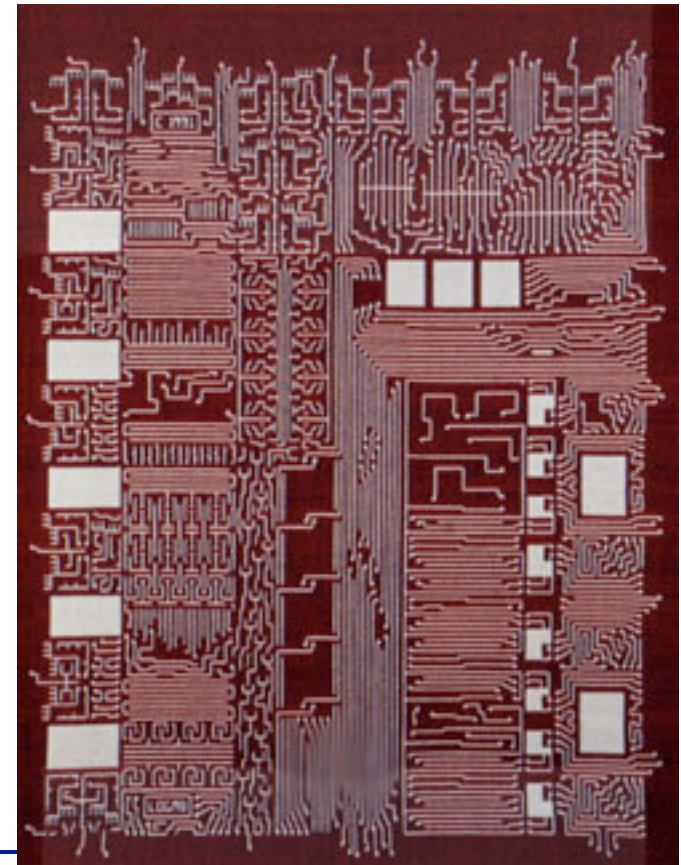
Are these graphs planar?



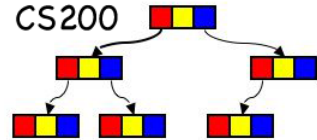
Chip Design



- You want more than planarity: the lengths of the connections need to be as short as possible (faster, and less heat is generated)
- We are now designing 3D chips, less constraint w.r.t. planarity, and shorter distances, but harder to build.

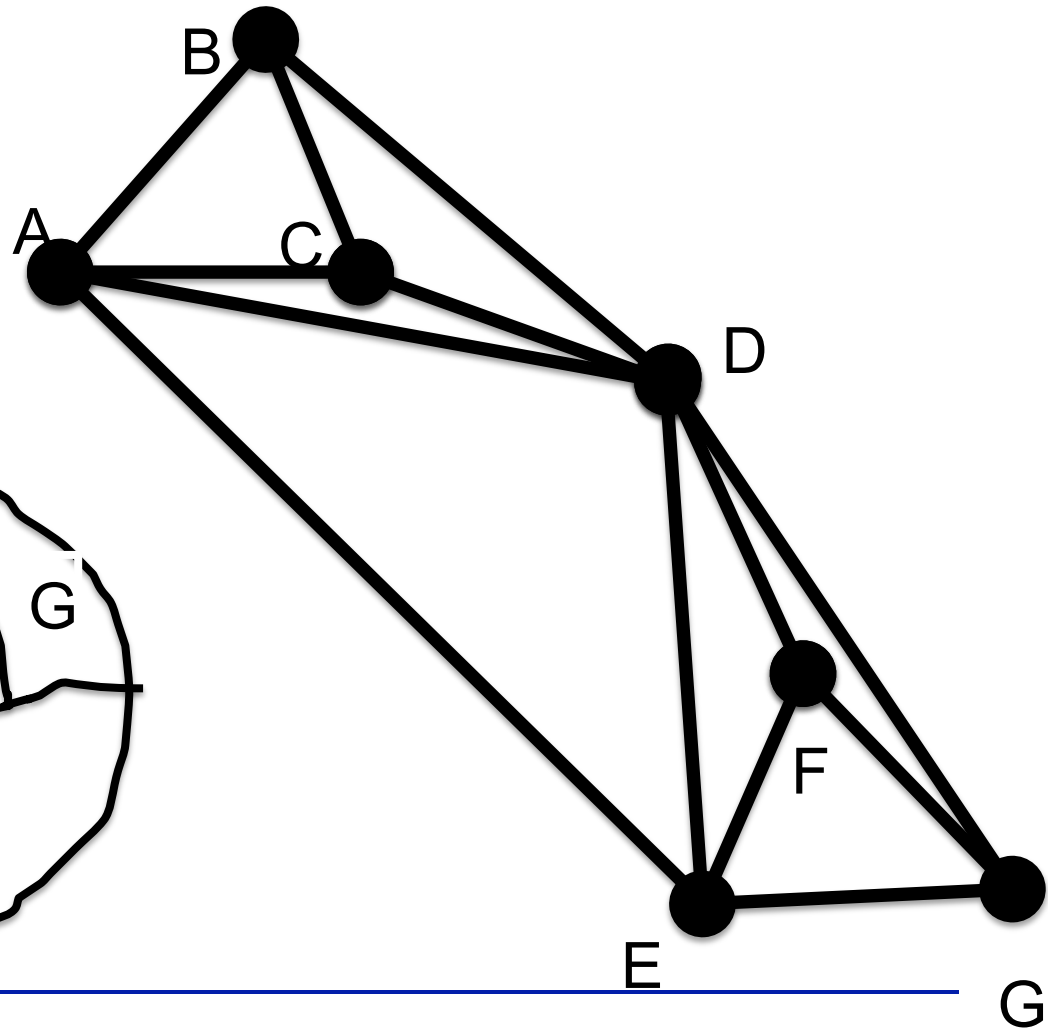
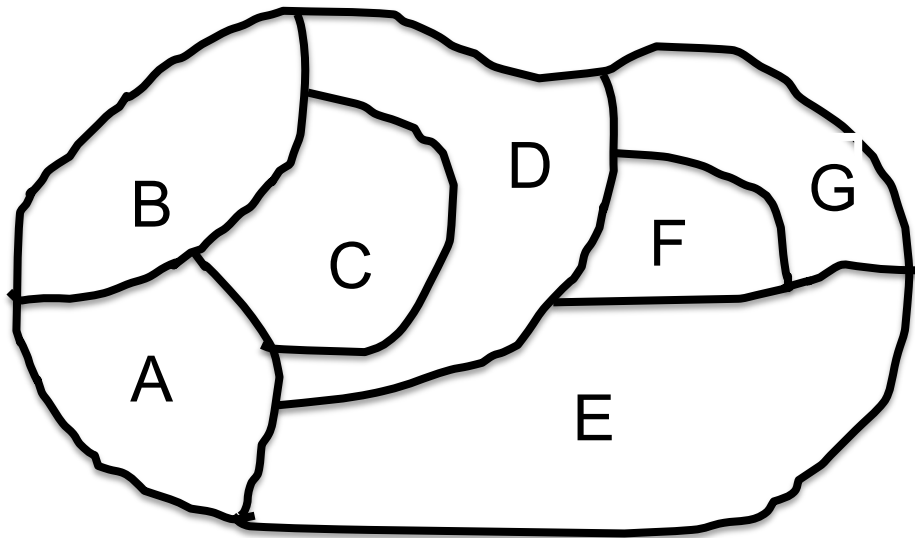
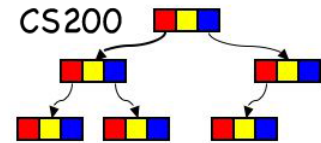


Graph Coloring

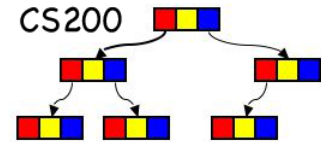


- A coloring of a simple graph is the assignment of a color to each vertex of the graph so that no two adjacent vertices are assigned the same color

Map and graph

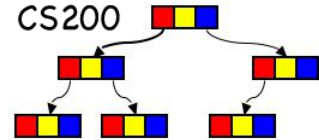


Chromatic number



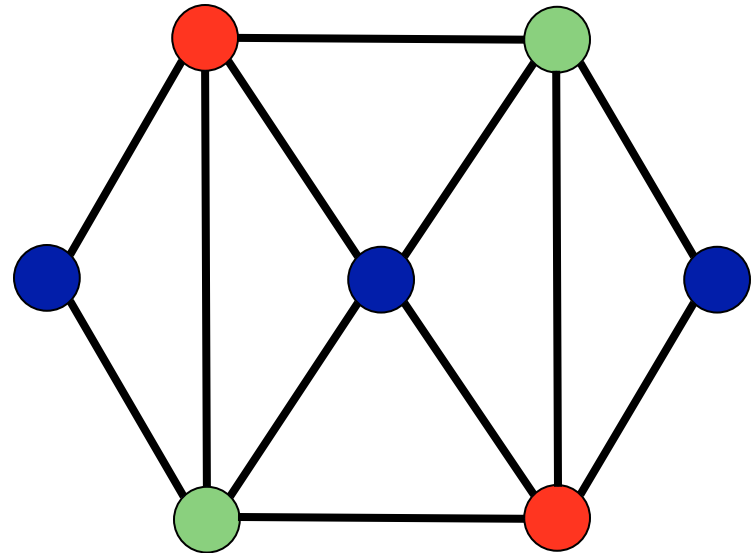
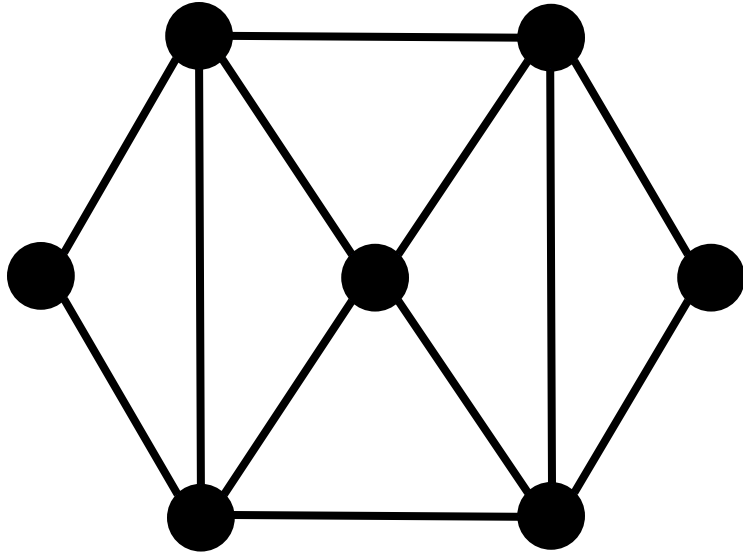
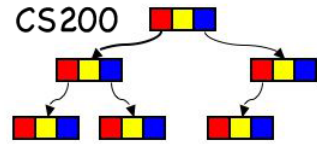
- The least number of colors needed for a coloring of this graph.
- The chromatic number of a graph G is denoted by $\chi(G)$

The four color theorem



- The chromatic number of a planar graph is no greater than four
- This theorem was proved by a (theorem prover) program!

Example



Example

