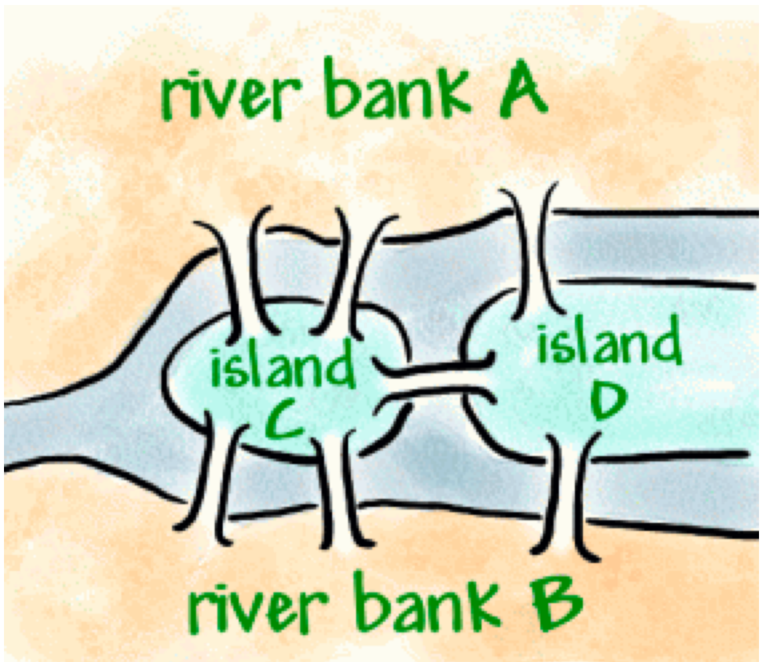
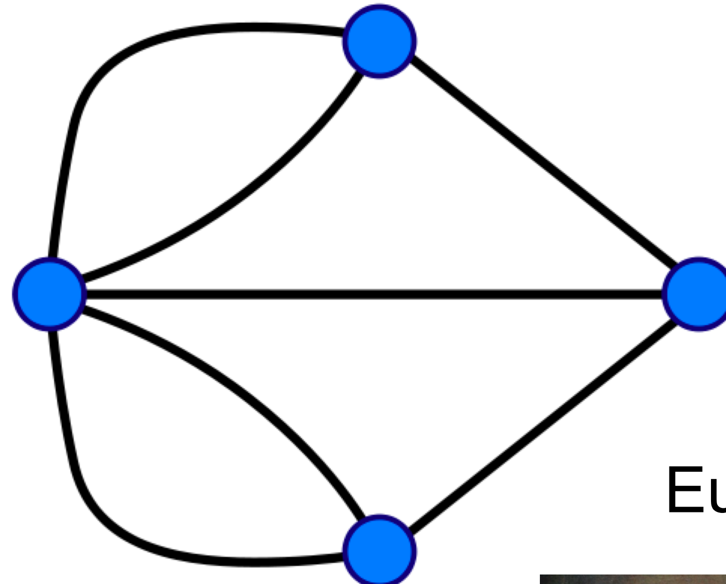
Fun and Games with Graphs

Bridges of Konigsberg Problem





Euler

Is it possible to travel across every bridge without crossing any bridge more than once?

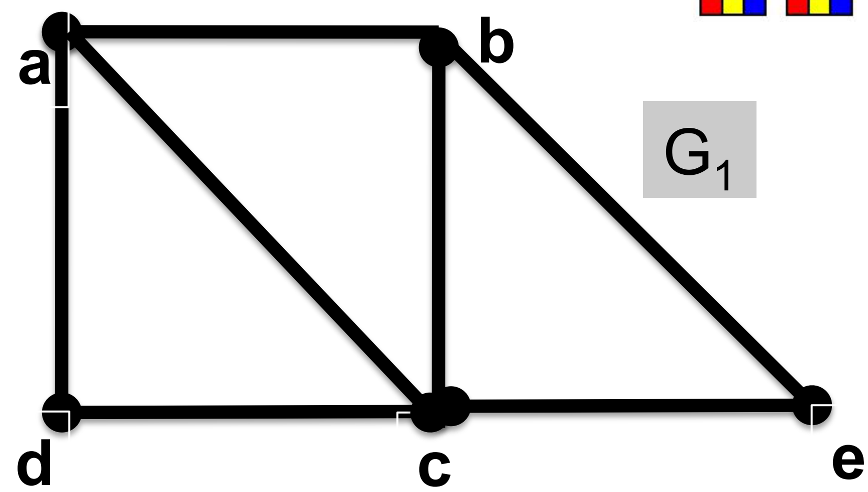
http://yeskarthi.wordpress.com/2006/07/31/euler-and-the-bridges-of-konigsberg/

Eulerian paths/circuits

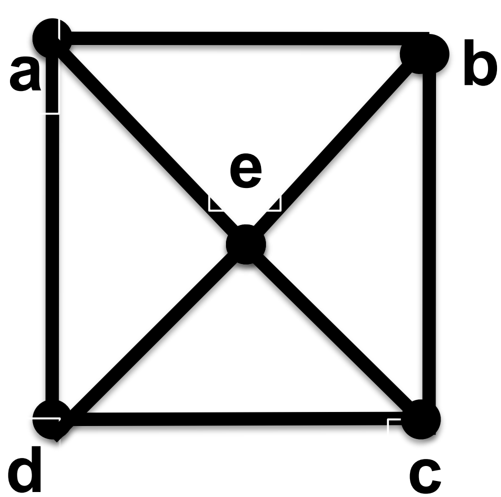
- Eulerian path: a path that visits each edge in the graph once
- Eulerian circuit: a cycle that visits each edge in the graph once

- Is there a simple criterion that allows us to determine whether a graph has an Eulerian circuit or path?

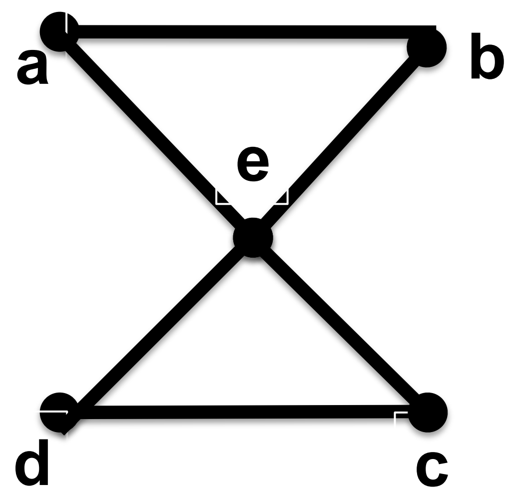
Example: Does any graph have an Eulerian path?



G_1

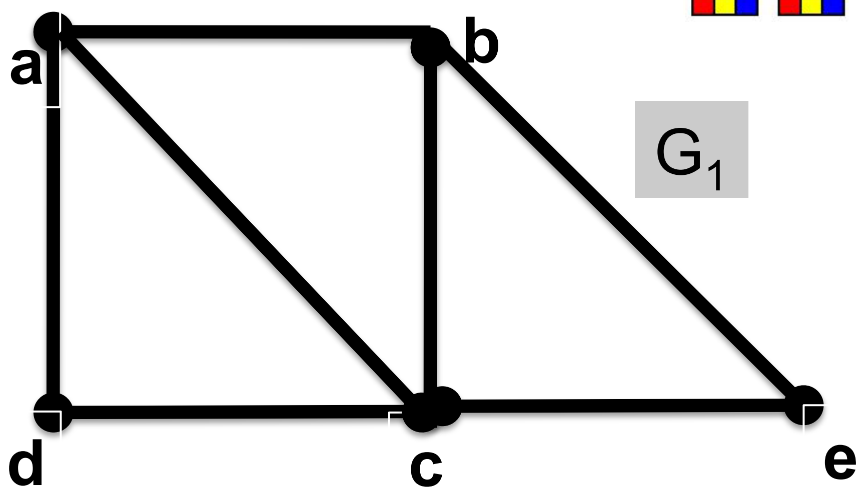


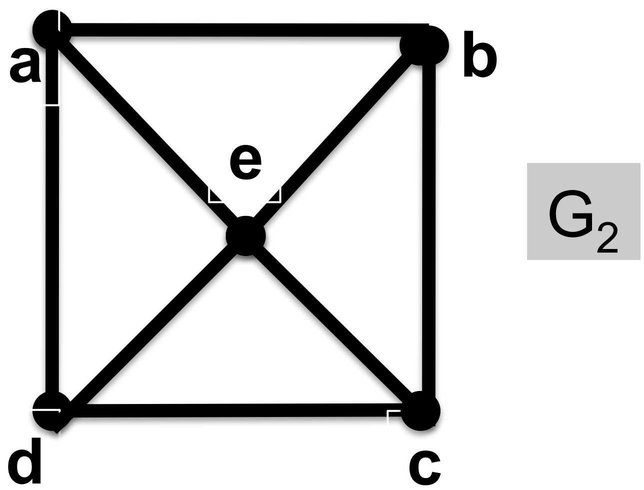
G_2

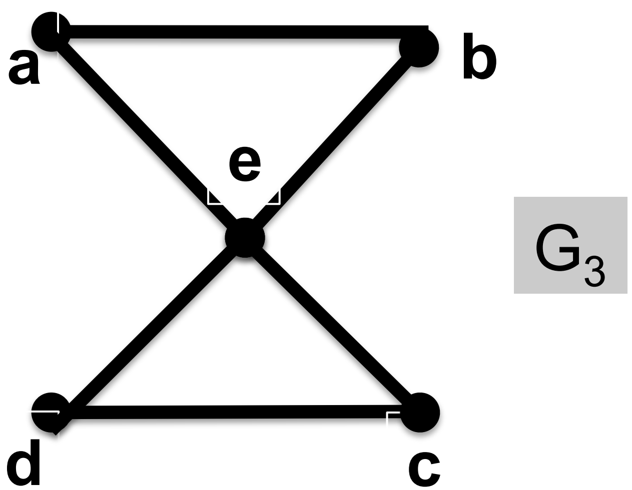


G_3

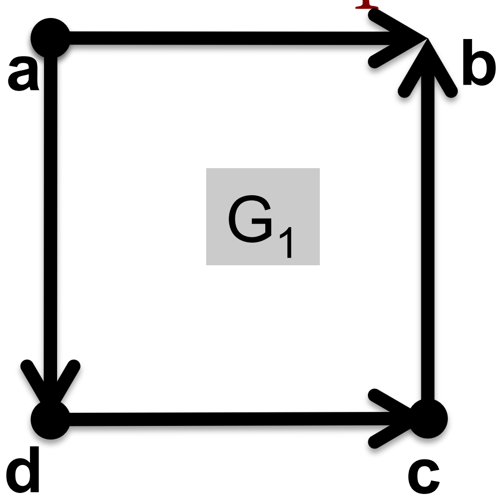
Example: Does any graph have an Eulerian circuit?

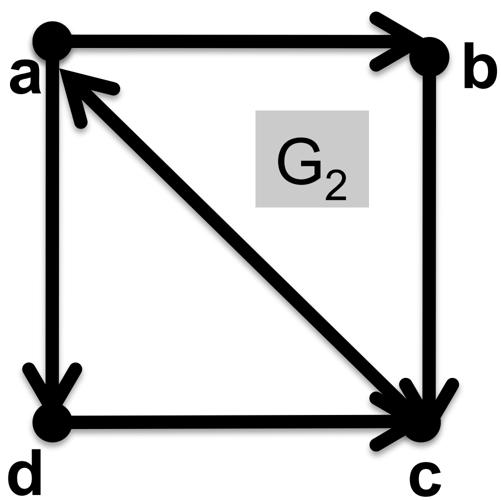


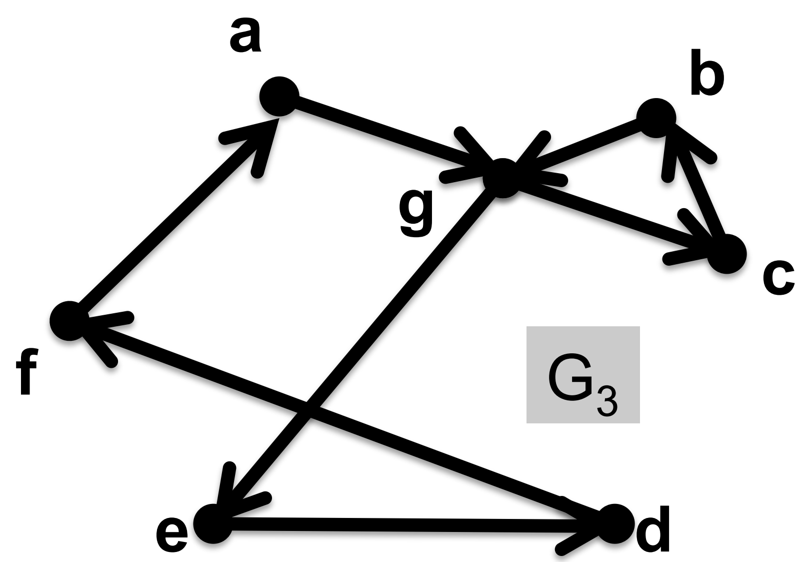




Example: Does any graph have an Eulerian circuit or path?







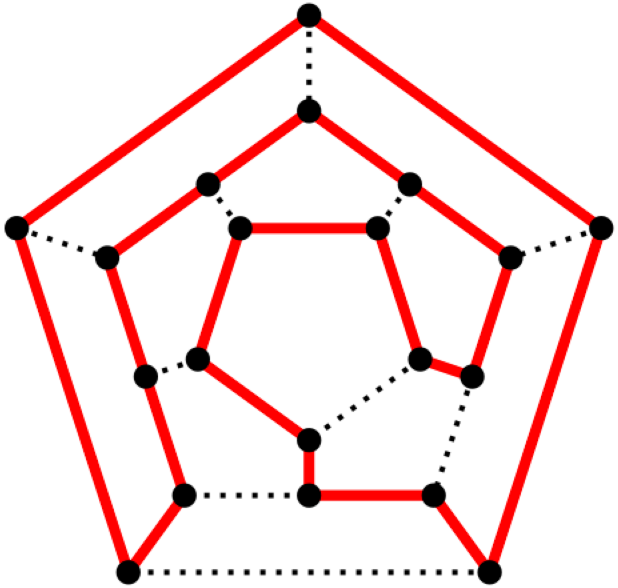
Theorems about Eulerian Paths & Circuits

- Theorem: A connected multigraph has an Euler path iff it has exactly two vertices of odd degree.

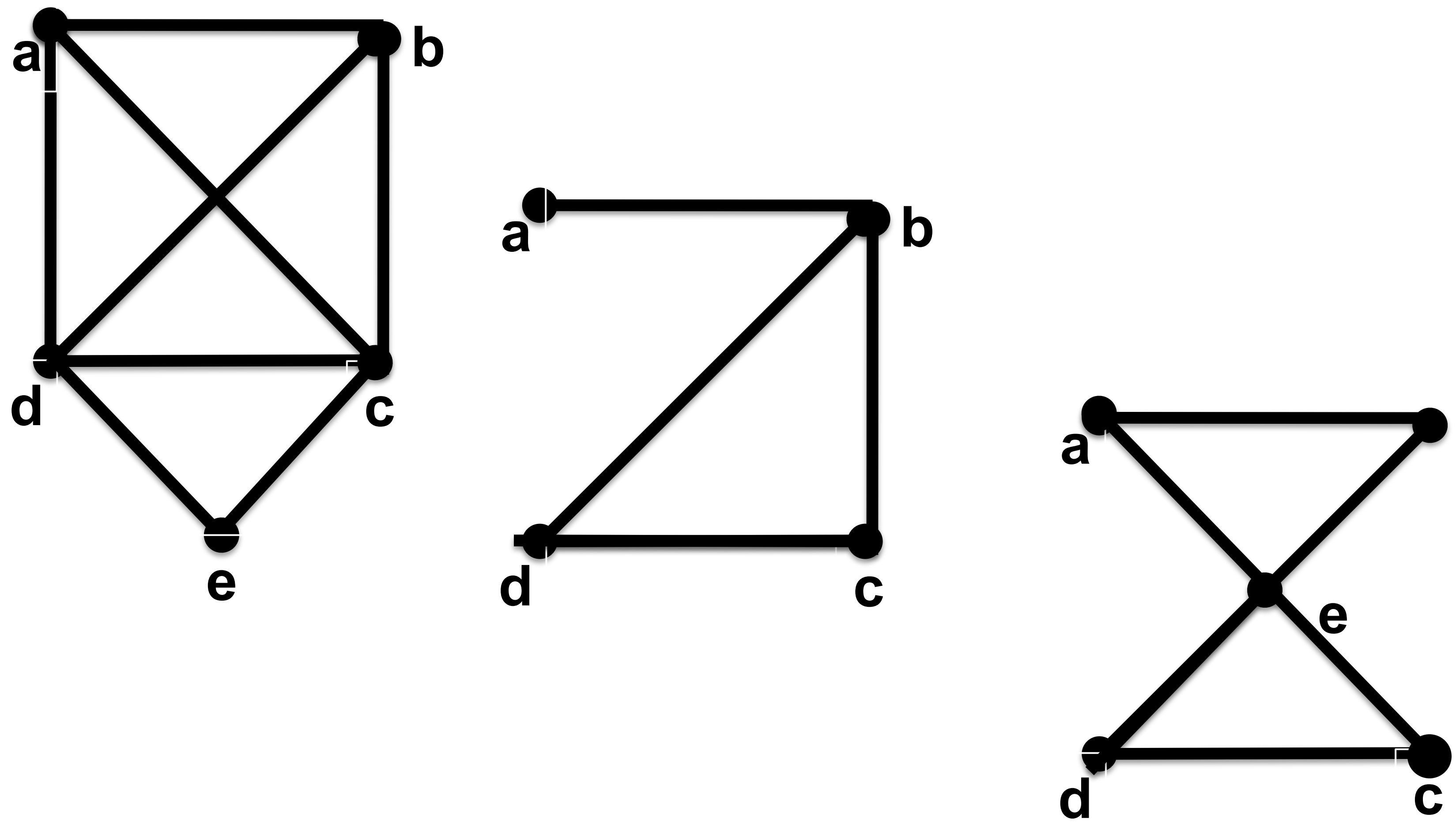
- Theorem: A connected multigraph with at least two vertices has an Euler circuit iff each vertex has an even degree.

Hamiltonian Paths/Circuits

- A Hamiltonian path/circuit: path/circuit that visits every vertex exactly once.
- Defined for directed and undirected graphs



Does any graph have a Hamiltonian circuit or a Hamiltonian path?



Hamiltonian Paths/Circuits

- Is there an efficient way to determine whether a graph has a Hamiltonian circuit?
  - NO!
  - This problem belongs to a class of problems for which it is believed there is no efficient (polynomial running time) algorithm.
  - What is an algorithm for doing this?
  - What is its complexity?

The Traveling Salesman Problem

TSP: Given a list of cities and their pairwise distances, find a shortest possible tour that visits each city exactly once.

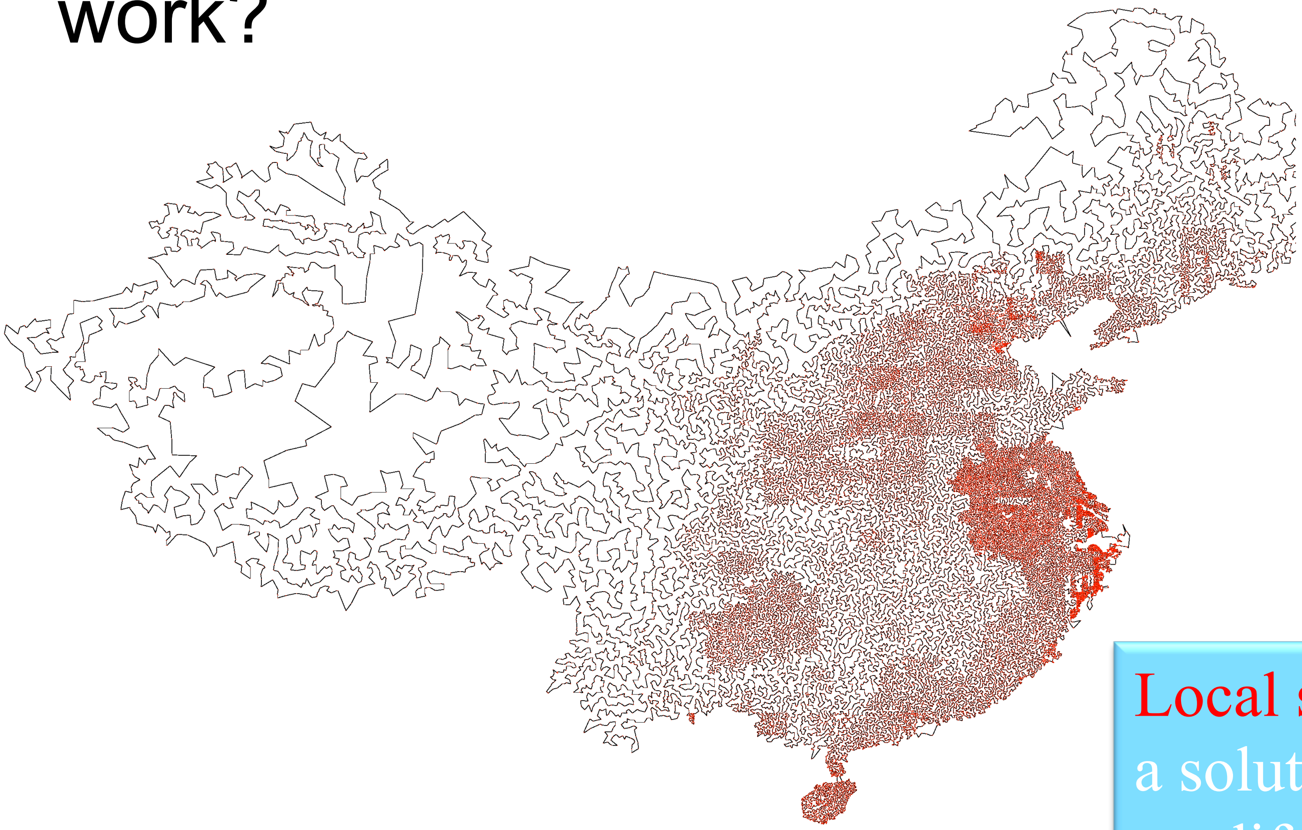
13,509 cities and towns in the US that have more than 500 residents

Using Hamiltonian Circuits

- Examine all possible Hamiltonian circuits and select one of minimum total length
- With n cities..
  - (n-1)! Different Hamiltonian circuits
  - Ignore the reverse ordered circuits
  - (n-1)!/2
- With 50 cities
- 12,413,915,592,536,072,670,862,289,047,373,375,038,521,486,354,677,760,000,000,000 routes

TSP

- How would a approximating algorithm for TSP work?



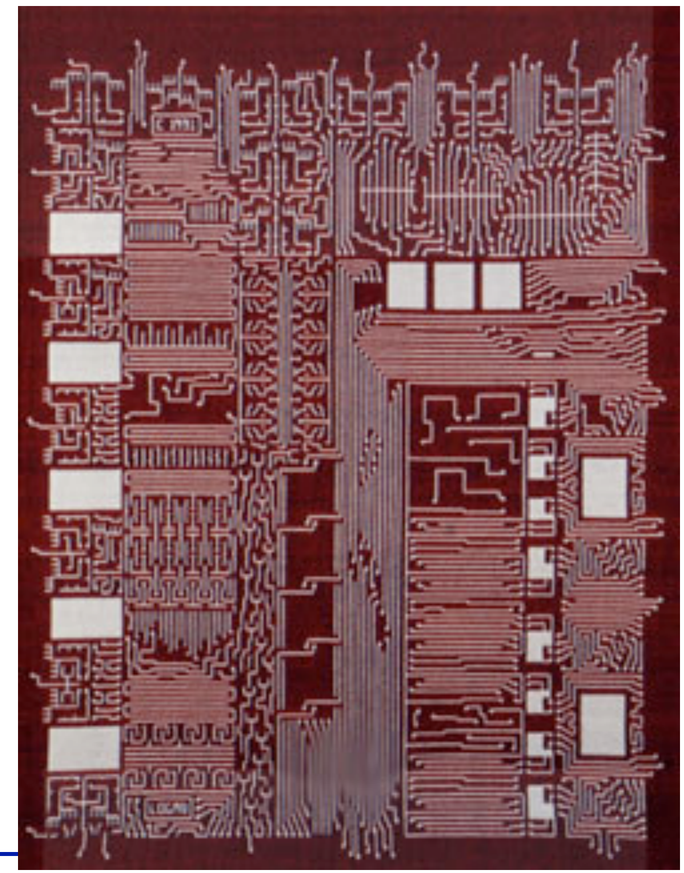
71,009 Cities in China

Local search: construct a solution and then modify it to improve it

Planar Graphs

- You are designing a chip

connections between any two units cannot cross

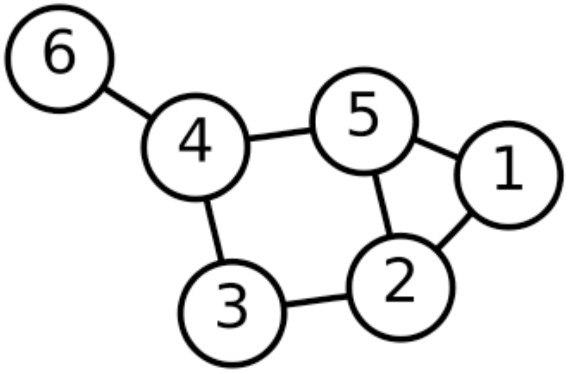


http://www.dmoma.org/

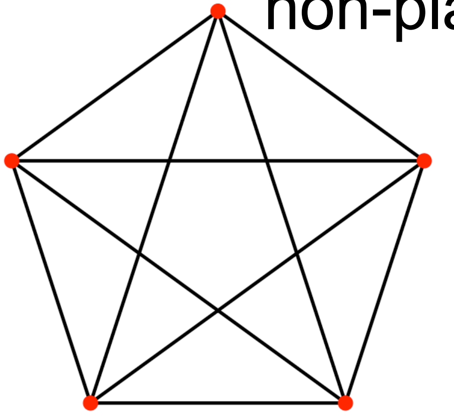
Planar Graphs

- You are designing a chip connections between any two units cannot cross
- The graph describing the chip must be planar

planar

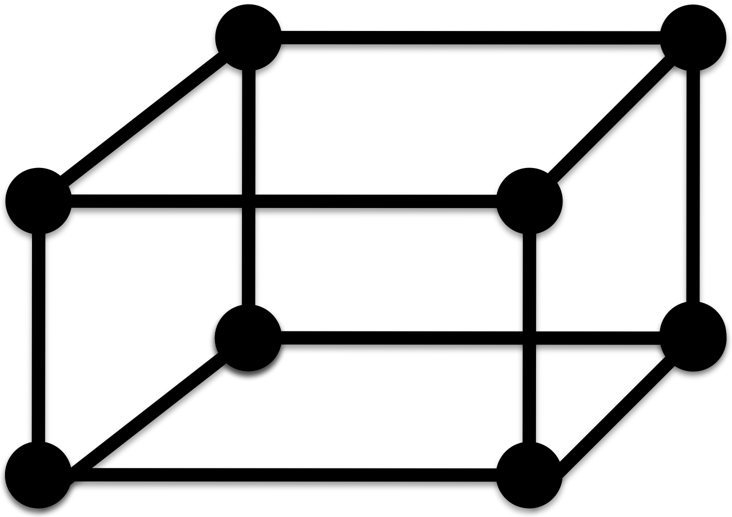


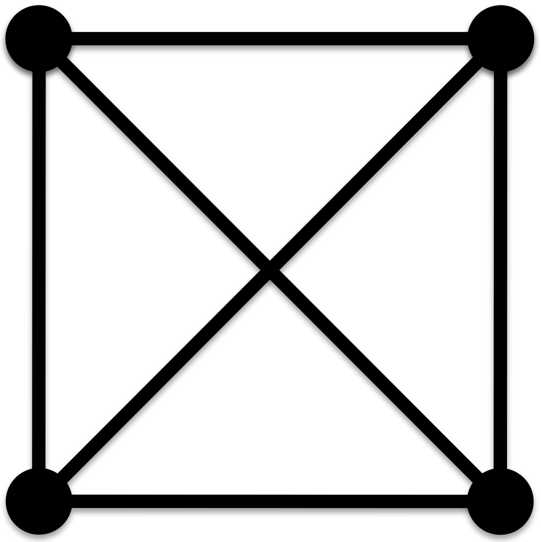
non-planar



http://en.wikipedia.org/wiki/Planar_graph

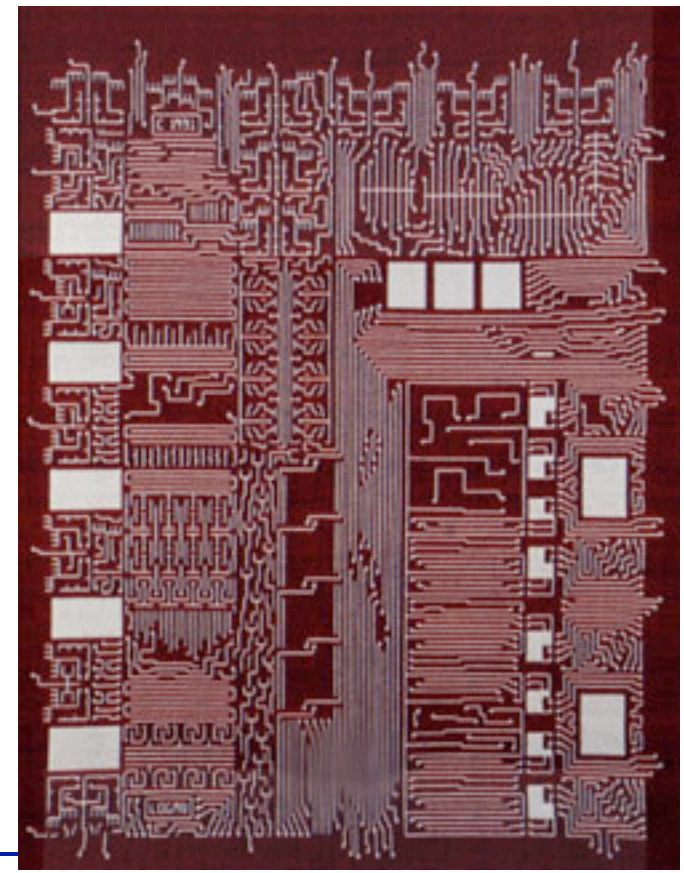
Are these graphs planar?





Chip Design

- You want **more** than planarity: the lengths of the connections need to be as short as possible (faster, and less heat is generated)
- We are now designing 3D chips, less constraint w.r.t. planarity, and shorter distances, but harder to build.



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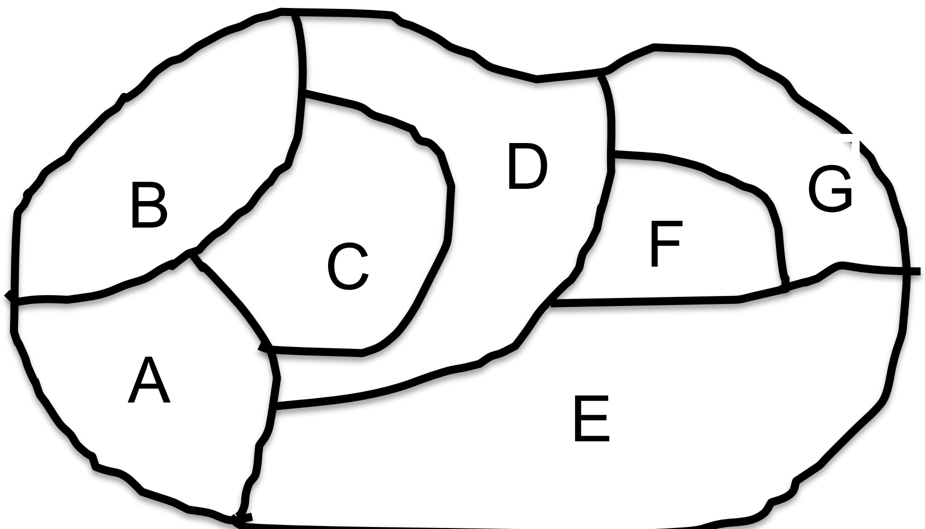
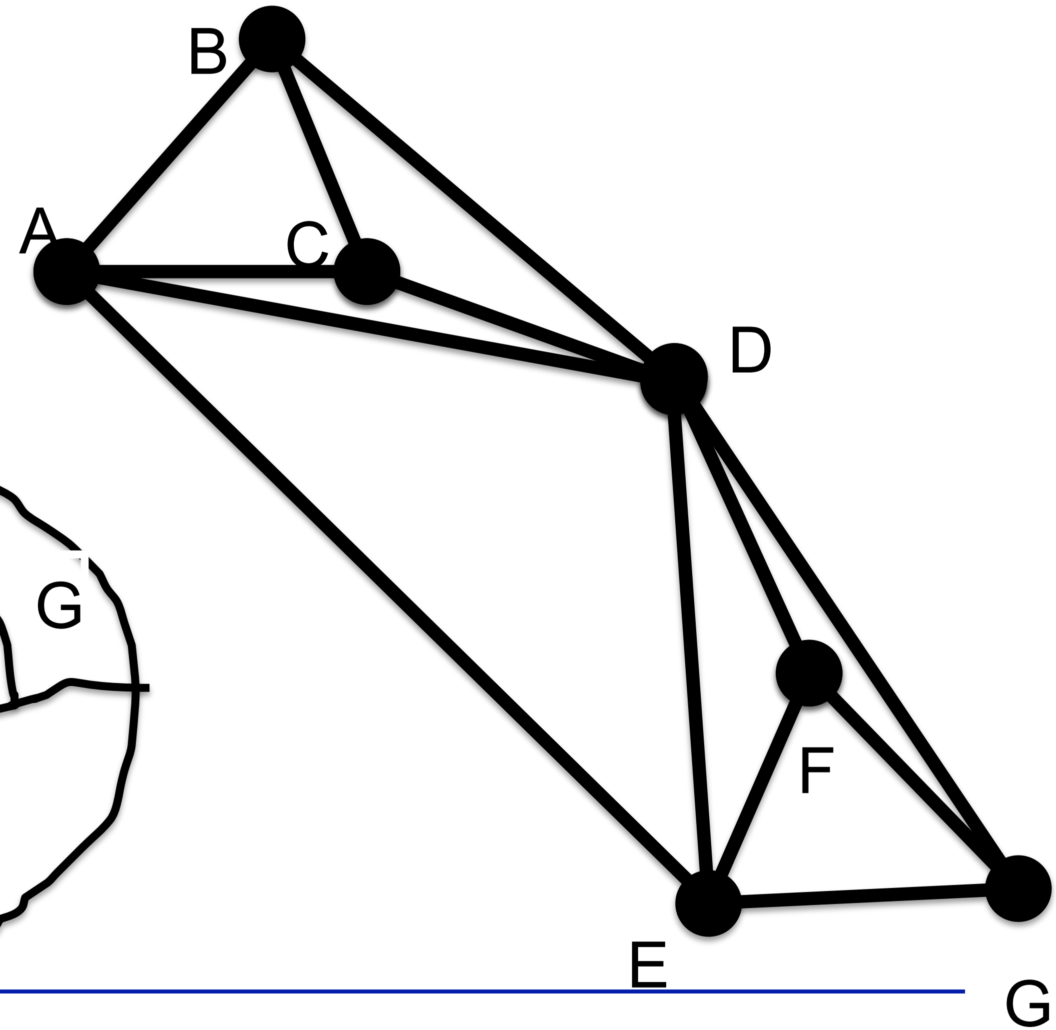
Graph Coloring

- A coloring of a simple graph is the assignment of a color to each vertex of the graph so that **no two adjacent vertices** are assigned the same color

Map and graph





Chromatic number

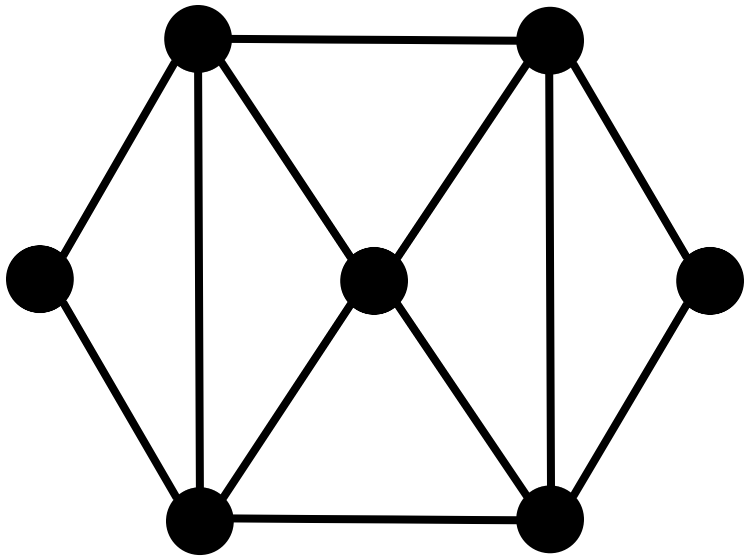
- The least number of colors needed for a coloring of this graph.
- The chromatic number of a graph G is denoted by $\chi(G)$

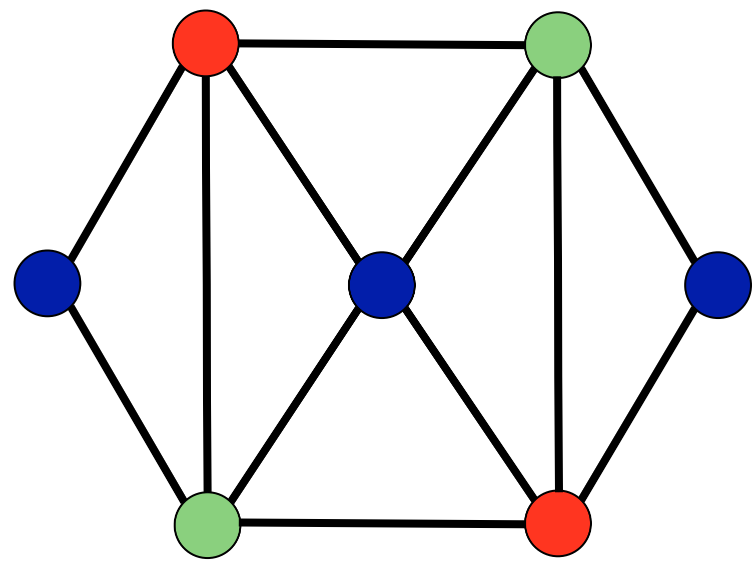
The four color theorem

- The chromatic number of a planar graph is no greater than four

- This theorem was proved by a (theorem prover) program!

Example





Example