CS250: Foundations of Computer Systems
[Data Structures for Storage]

Looking for something?

Trickle down from the root
Branching choices at the fork

A left if it's less
A right if it's more

A dead end?
What you seek
Isn't here

Shrideep Pallickara
Computer Science
Colorado State University

Computer Science Department
Colorado State University

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Frequently asked questions from the previous class survey

- Why don't we use Binary Search trees *everywhere*?

Colorado State University
Professor: Shrideep Pallickara
Computer Science Department
Data Structures for Storage
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Topics covered in this lecture

- Binary Search Trees
- B-Trees

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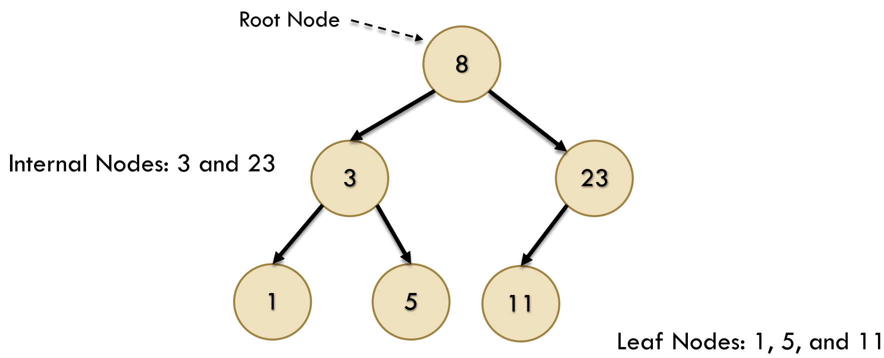
Binary Search Tree (BST)

- Binary search trees use the concepts underpinning the binary search algorithm to create a **dynamic data structure**
  - The key word here is dynamic
- Unlike sorted arrays, binary search trees support the efficient **addition and removal** of elements in addition to searches
  - Making them the perfect blend of the algorithmic efficiency of binary search and the adaptability of dynamic data structures

Tree Structures

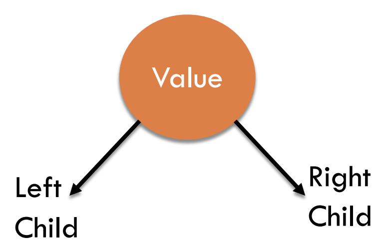
- Trees are **hierarchical** data structures
- Comprise **branching chains** of nodes
- Natural *extension* of linked lists
  - In the case of BSTs, each tree node is permitted two next pointers
    - Point to subsequent nodes in disjoint lists

Example BST



BST Nodes

- A node contains a **value** (of a given type)
  - Plus, **up to two** pointers to lower nodes in the tree
- Nodes with at least one child?
  - Internal nodes
- Nodes without any children?
  - **Leaf** nodes



Tree nodes may contain other information

- Often include a **pointer back to the node's parent**, for instance
  - Allows **bottom-up traversals** of the tree
    - In addition to the typical top down
  - Comes in handy when we consider removing nodes

The TreeNode data structure

```
TreeNode {
    Type: value
    TreeNode: left
    TreeNode: right
    TreeNode: parent
}
```

We might also want to store auxiliary data …

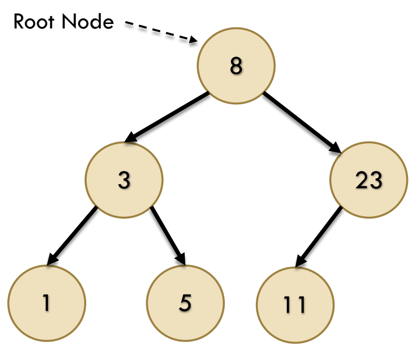
- Storing and searching for individual values are useful
- However, **using these values as keys** for looking up more detailed information greatly extends the power of the data structure
  - For e.g., coffee names as the node's values, allowing us to efficiently look up records for any coffee
    - Our auxiliary data would be a detailed record of that coffee

BST and auxiliary data

- The tree node data structure can either store this auxiliary data
  - Directly
  - Include a pointer to a composite data structure
    - Located somewhere else in memory

Binary search trees

- Start at a single root node at the top of the tree
- **Branch** into multiple paths as they descend
- Allows programs to access the binary search tree through a single pointer
  - The location of its root node



Nodes and dispersion in memory

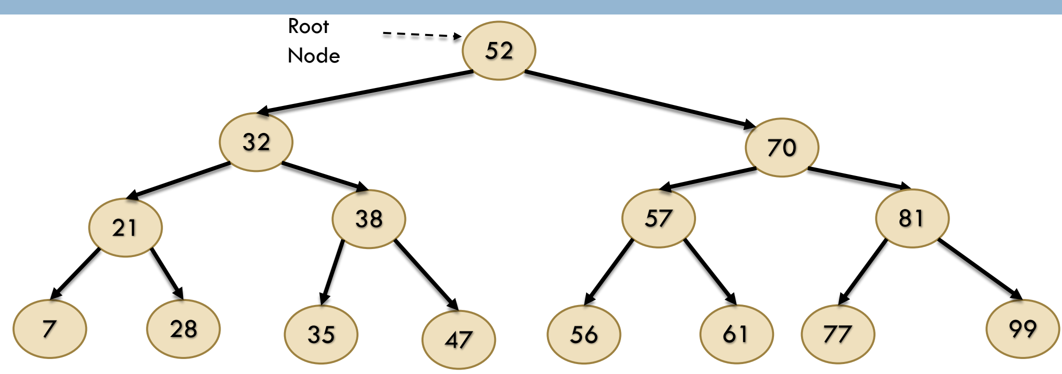
- A search tree's individual nodes can be **scattered** throughout memory
- Each node is only linked to its children and parents through?
  - The power and flexibility of **pointers**

The power of the binary search tree stems from how values are organized within the tree.

- For any node N
  - The value of *any node* in N's **left** subtree is less than N's value
    - Values in the left node and all nodes below it are less than the value of the current node
  - The value of *any node* in N's **right** subtree is greater than N's value
    - Values in the right node and all nodes below it are greater than the value of the current node
- The rule defines the tree's structure below that node
  - Partitions the subtree into two subsets

** Useful Rule: The number of right-handed people is greater than the number who are left-handed.

Values in a BST are ordered by the binary search property [1/2]



Values in a BST are ordered by the binary search property [2/2]

- This ordering of nodes might not seem like a lot of structure
  - Recall the power we got from using a similar property within binary search
- The binary search tree property is effectively keeping the data within the tree sorted with respect to its position in the tree
- As we will see, this allows us to not only efficiently **find values** in the tree but also efficiently **add and remove nodes**

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Searching Binary Search Trees

- We search the BST by **walking down** from the root node
- At each step, we determine whether to explore the left or right subtree
  - By comparing the value at the current node with the target value
    - **Left**: If the target value is *less* than the current value
    - **Right**: If the target value is *greater* than the current value
- The search ends when either the target value is found, or it reaches a node with no children in the correct direction
  - In the latter case, we can assert that the target value is not in the tree

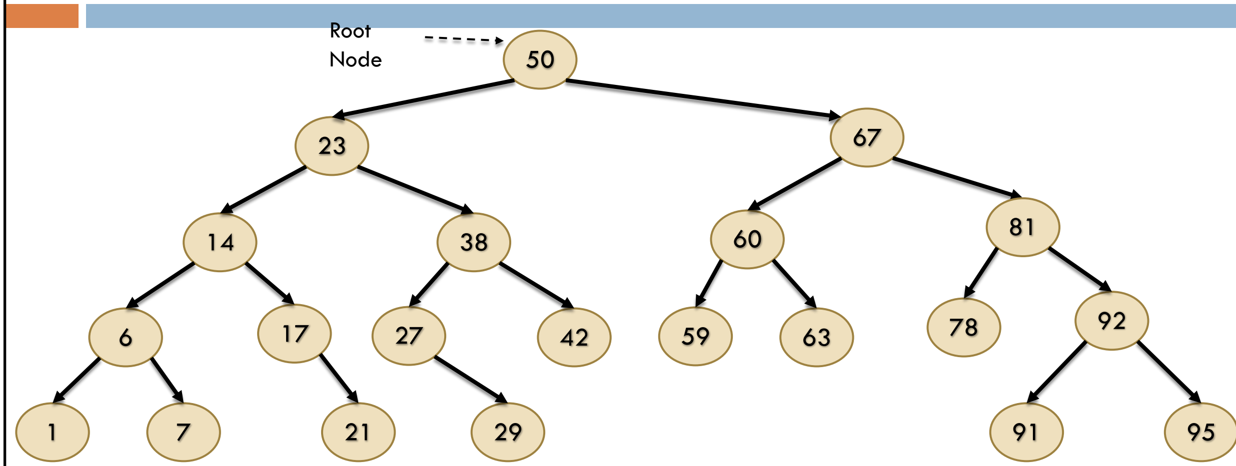
Searching Binary Search Trees: An analogy

- The node's value thus serves the same function as those helpful signs in hotels
- Signs that tell us rooms 500–519 are to the left
  - And rooms 520–545 are to the right
- With one quick check:
  - We can make the appropriate turn
    - And ignore the rooms in the other direction

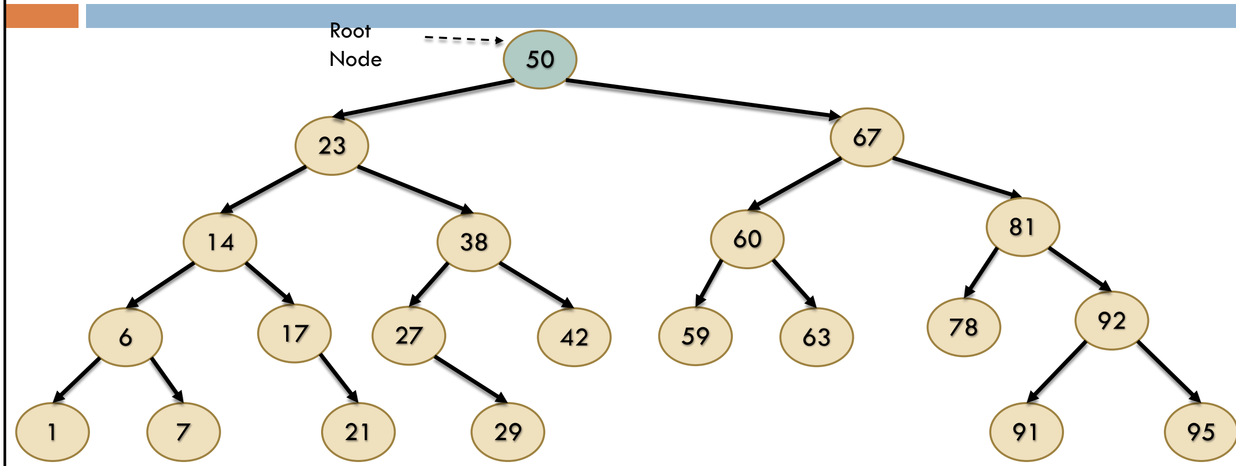
The recursive algorithm to find a value

```
FindValue(TreeNode: current, Type: target):
 1 IF current == null:
     return null
 2 IF current.value == target:
     return current
 3 IF target < current.value AND current.left != null:
     return FindValue(current.left, target)
 4 IF target > current.value AND current.right != null:
     return FindValue(current.right, target)
 5 return null
```


Suppose we used this strategy to search for 63



Suppose we used this strategy to search for 63



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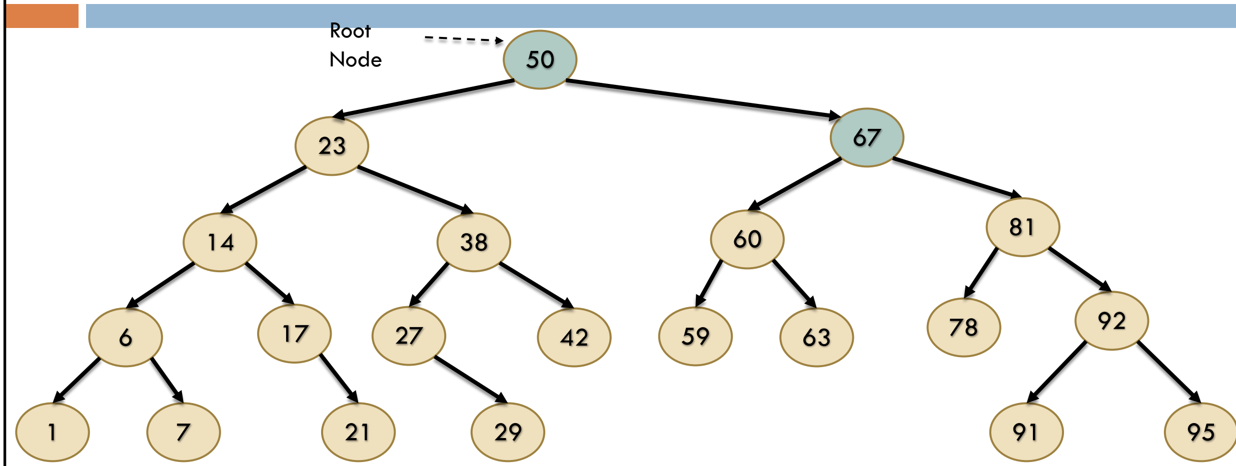
Suppose we used this strategy to search for 63



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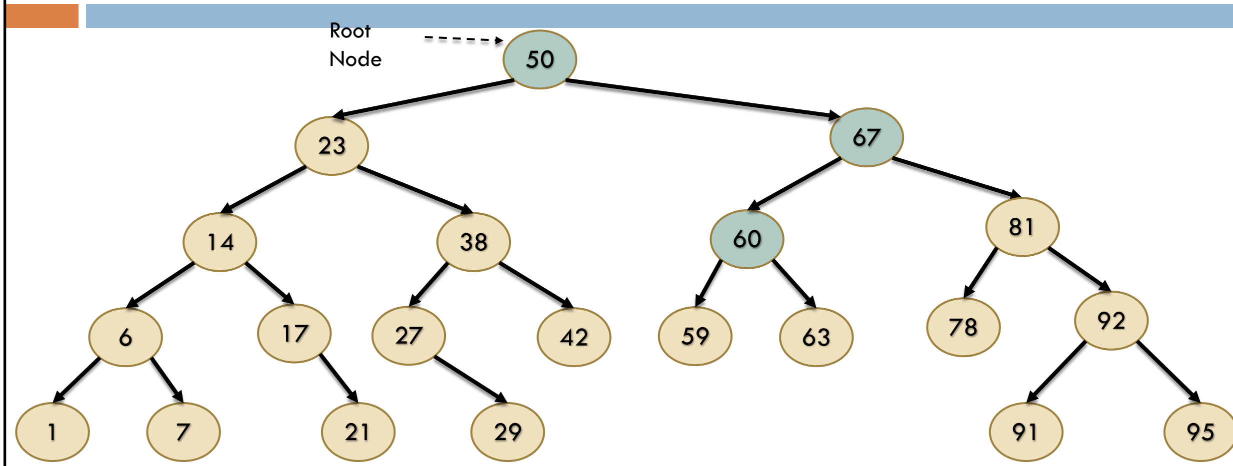
Suppose we used this strategy to search for 63



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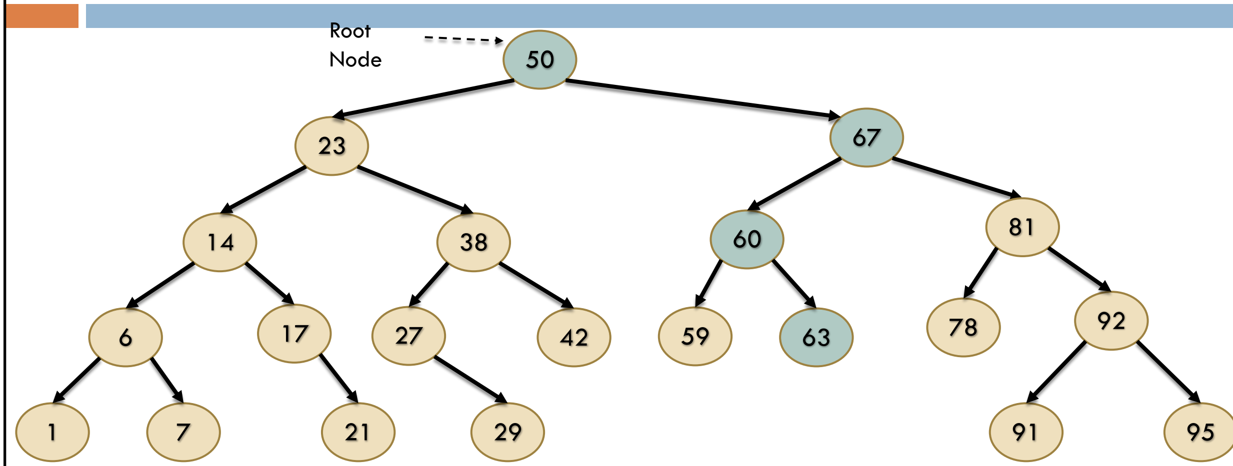
Suppose we used this strategy to search for 63



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The iterative approach to binary search

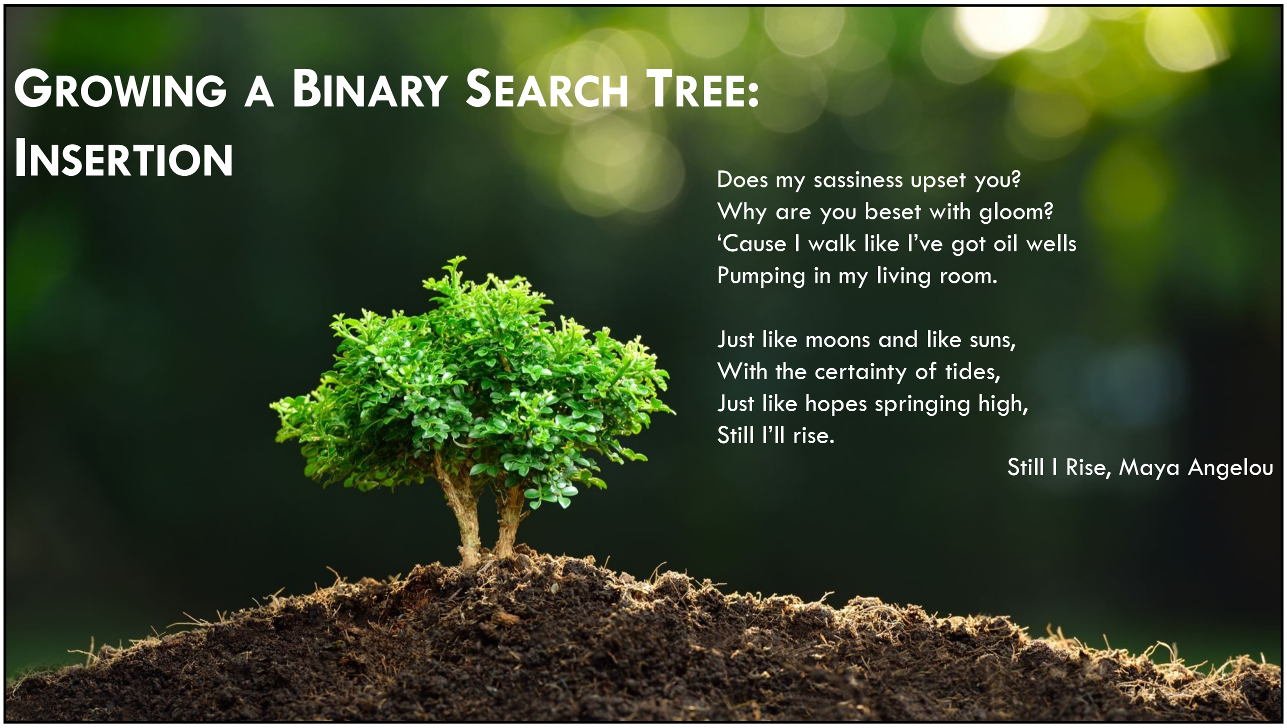
FindValueItr(TreeNode: root, Type: target):

1 TreeNode: current = root
2 WHILE current != null AND current.value != target:
  3 IF target < current.value:
      current = current.left
    ELSE:
      current = current.right
4 return current

Simplifying the logic for using binary search trees

- We can wrap the entire tree in a **thin** data structure that contains the root node:

```
BinarySearchTree {
    TreeNode: root
}
```



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Adding a node to a BST [1/2]

- We use the same basic algorithm to add values to a binary search tree as we do to search it
- Start at the root node, progress down the tree **as if searching for the new value**, and
  - Terminate once we hit a dead end:
    - Either a leaf node or an internal node with a single child

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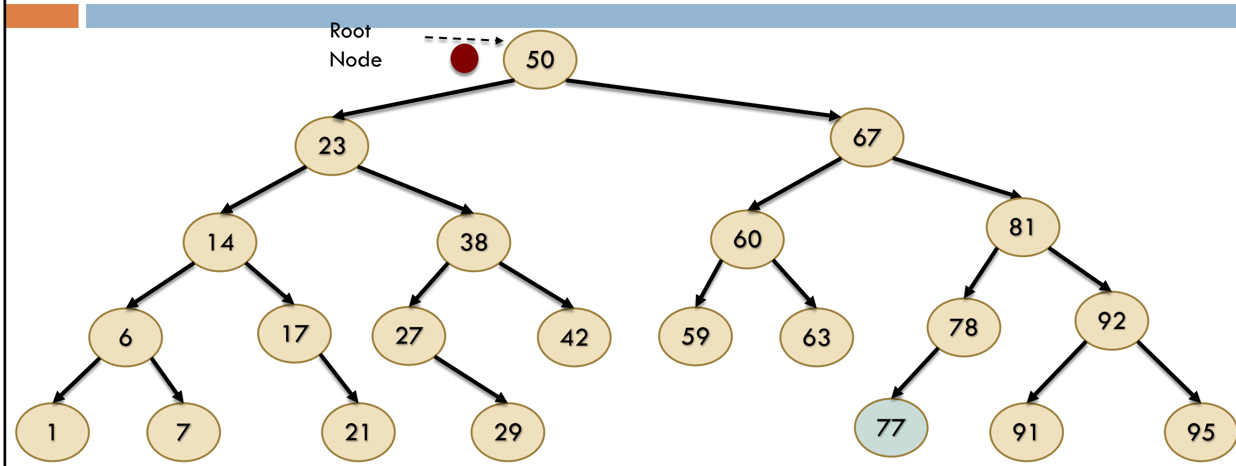
Adding a node to a BST [2/2]

- The primary **difference** between our search and insertion algorithms comes **after we hit the dead end**
- The insertion algorithm creates a new node as a child of the current node:
  - A left-hand child if the new value is less than that of the current node
  - A right-hand child if the new value is greater than that of the current node

For example, if we want to add 77



Cost of inserting a node?

- Proportional to the **depth of the branch** along which we insert the new node

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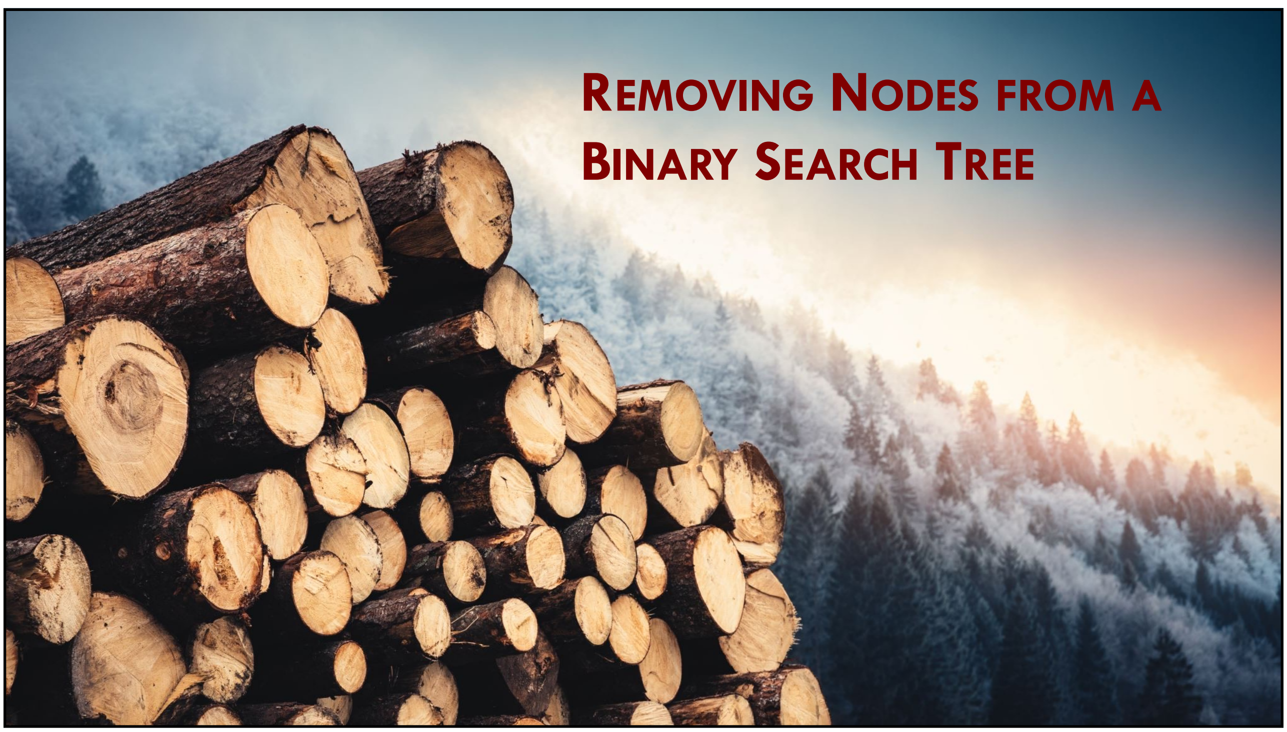

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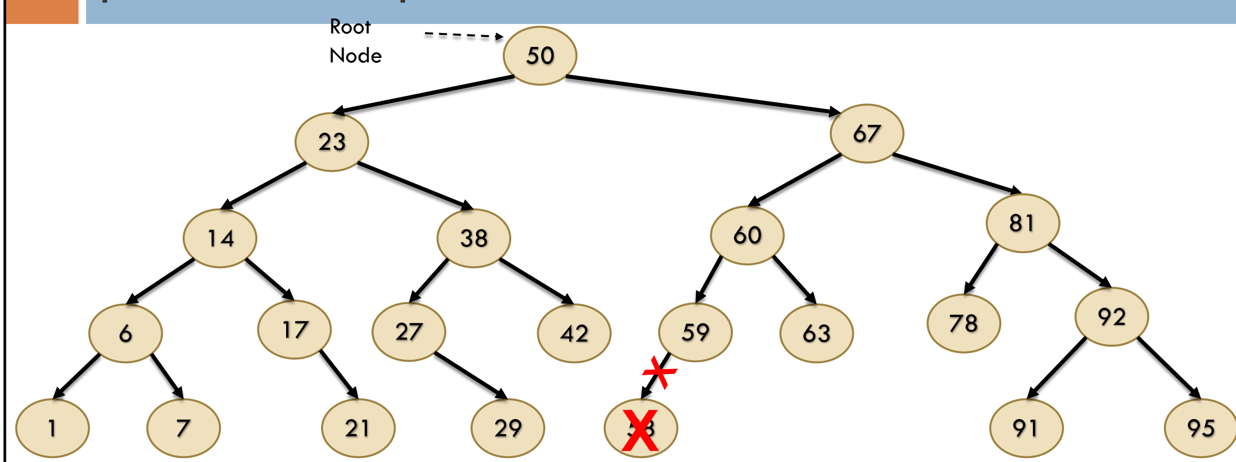
Removing Nodes is more complicated than adding them

- There are three cases of node removals to consider:
  1. Removing a leaf node (with no children)
  2. Removing an internal node with a single child
  3. Removing an internal node with two children

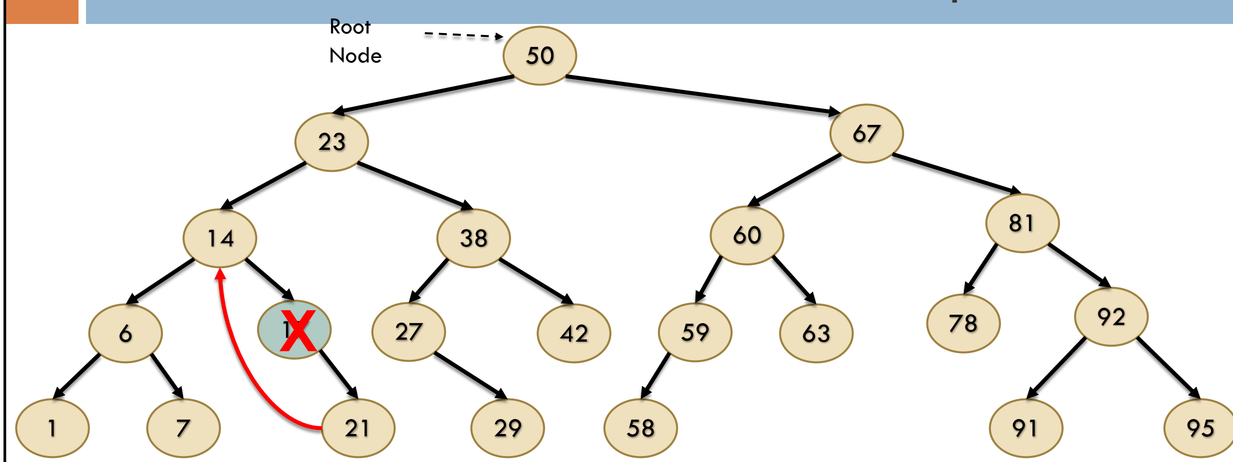
Remove a leaf node: Delete that node & update its parent's child pointer to reflect that it doesn't exist



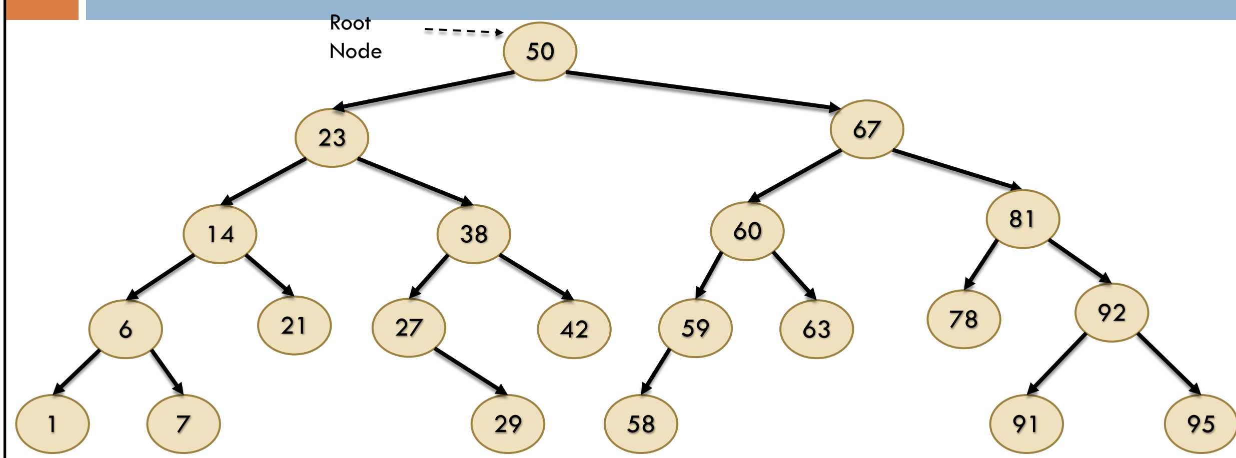
Removal of a leaf node

- Might make the parent node into a leaf
- The usefulness of parent pointers
  - Allows us to follow the parent pointer back to that node's parent, and set the corresponding child pointer to null
  - Storing this single piece of additional data makes deletion much simpler

Removal: If target node has a single child, **promote that child** to be child of deleted node's parent



Removal: If target node has a single child, **promote that child** to be child of deleted node's parent



What about deleting a node with two children [1/2]

- The complexity ramps up substantially
- No longer sufficient to just
  - Delete the node (leaf)
  - Shift a single child up (node with single child)

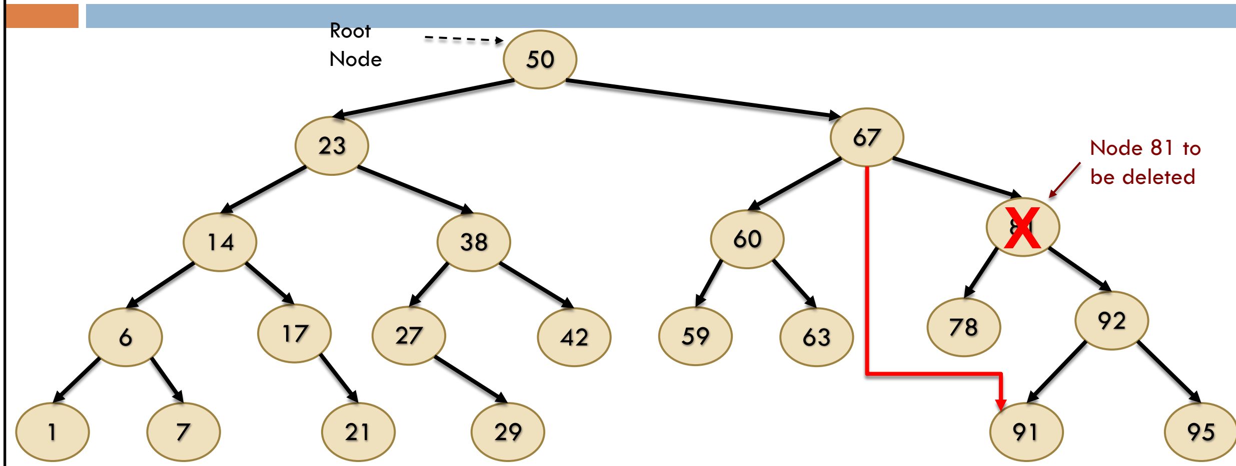
What about deleting a node with two children [2/2]

- Efficiently **find that node's successor**
- While this might seem like a daunting task, it isn't
- We can always find the successor **in the node's right-hand subtree**
  - Specifically, the successor will be the minimum (or leftmost) node in the right-hand subtree

BST: Predecessor and successor of a node

- Predecessor
  - Maximum value in its left subtree
- Successor
  - Minimum value in its right subtree

Deleting node 81



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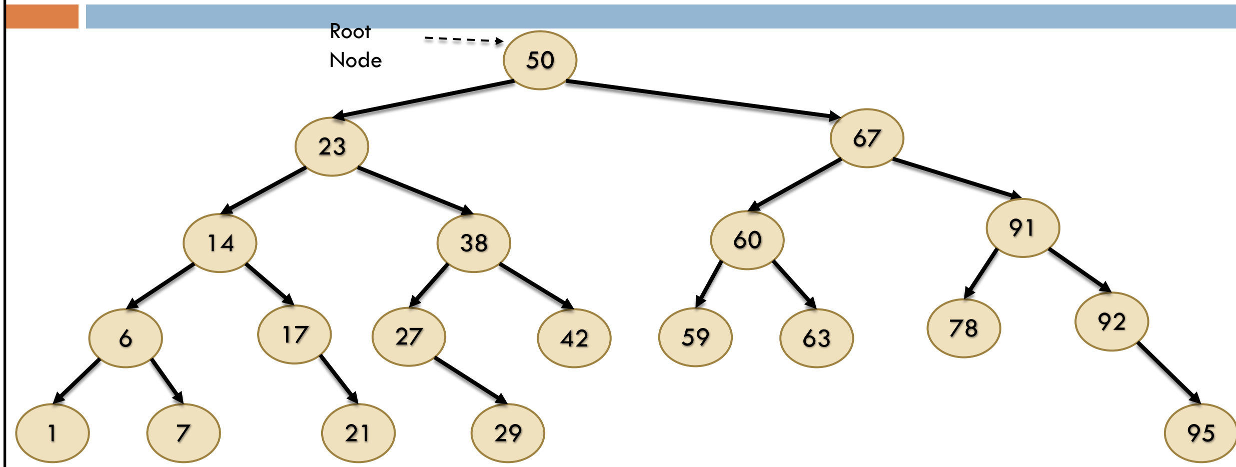
Deleting node 81



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UNBALANCED BINARY SEARCH TREES

Life is like riding a bicycle. To keep your balance, you must keep moving.

Albert Einstein

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The perils of unbalanced trees

- Binary search trees are still efficient as long as the trees are **mostly, if not perfectly, balanced**
- But if the tree becomes highly unbalanced?
  - Its depth could grow linearly with the number of elements
- In fact, in the *extreme case*, our splendid BST becomes nothing more than a **sorted linked list**
  - All the nodes have a single child in the same direction

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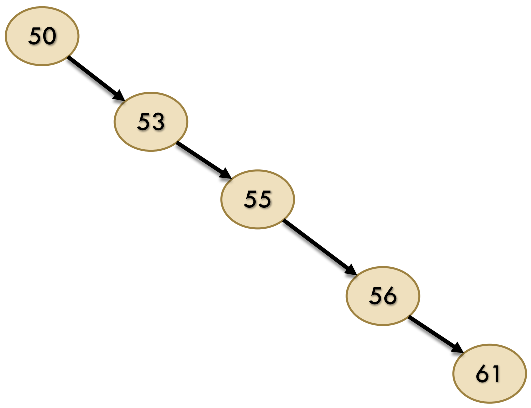
Example of a BST devolving into a sorted linked list



Highly unbalanced trees can easily occur in many real-world applications

- Imagine storing coffee log in a BST indexed by timestamp
- Every time we drink a cup of coffee, we insert the relevant information into our tree
- Things go bad quickly
  - Due to the **monotonically increasing timestamps**, we insert every entry in sorted order
  - We end up creating a linked list using only the **right-hand child pointers**

Operations on an unbalanced tree can be extremely inefficient

- Consider a tree with N nodes
- If our tree is balanced?
  - Our operations take time logarithmic in N or O(log N)
- In the opposite case, where our tree is a list?
  - Our operations can take time linearly proportional to N or O(N)
- We can use red-black trees, 2-3 trees, and B-trees, to preserve balance
  - While undergoing dynamic insertions and deletions
  - The tradeoff? Increased complexity in the tree operations

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PIVOTING TO BALANCE BSTs

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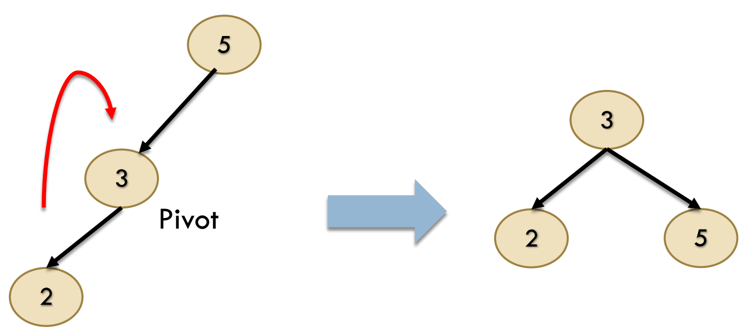
Balancing BSTs

- Perform a **rotation step** *after* nodes are added or removed
- If the insert operation leaves a branch unbalanced?
  - i.e., two consecutive nodes in the branch have only one child
  - **Rotate** nodes around the middle one

Balancing BSTs: Rotation



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B-Trees

- B-Trees combine ideas such as:
  - Increase node fanout
  - Reduce tree height
  - Reduce the number of node pointers
  - Reduce frequency of balancing operations

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The contents of this slide-set are based on the following references

- Jeremy Kubica. *Data Structures the Fun Way: An Amusing Adventure with Coffee-Filled Examples*. No Starch Press. ISBN-10. /13: 1718502605/978-1718502604. [Chapter 5]