

CS 250: FOUNDATIONS OF COMPUTER SYSTEMS

[BINARY REPRESENTATIONS & OPERATIONS]

A number in context

Look closely ...

a 1 or a 0 in the right place

The leftmost bit to be precise

Is also a sign

Break a sequence of bits here or there

And it gives you powers

To size up who-ville or

the universe's atoms

The only clarity that matters? Context

The interpretation that follows? Unambiguous

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Frequently asked questions from the previous class survey

- Do circuits check how big a number is prior to multiplications, etc.? For e.g., 23 is much smaller than say 9,876,543
- Because we are limited to say a few bits (e.g., 32) does it mean we are limited by how big the numbers can be?
- For binary multiplication, how do you know how many spots to shift to the left?
 - E.g.: $3,142 \times 100 = 314,200$; for binary: $1101 \times 100 = 110100$
- What's the difference between float (32-bits) and long (64 bits)



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Topics covered in this lecture

- Signed numbers
 - ▣ Two's complement
- Floating point numbers
- Hexadecimal numbers



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Summarizing two's complement for an n -bit binary system

- The system codes 2^n signed numbers, ranging from -2^{n-1} to $(2^{n-1} - 1)$
- The code of any *nonnegative* number begins with a **0**
- The code of any *negative* number begins with a **1**
- Representation of $-x \rightarrow 2^n - x$
- To obtain the binary code of $-x$ from the binary code of x ?
 - ▣ **Flip all the bits of x and add 1 to the result**



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Let's consider an 8-bit binary system and two's complement

- The system codes 2^n signed numbers, ranging from -2^{n-1} to $(2^{n-1} - 1)$
 - i.e., from -2^{8-1} to $(2^{8-1} - 1)$ or -2^7 to $(2^7 - 1)$ i.e. **-128** to **127**
- Let's look at the number -42
 - Method-1: $-42 \rightarrow 2^8 - 42 = 256 - 42 = 214 = \text{0b}11010110$
 - Method-2: Flip all bits of x and add 1 to the result
 - $42 = \text{0b}00101010$
 - Flips the bits: $\text{0b}11010101$
 - Add 1: $\text{0b}11010110$



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Converting 2s complement to decimal (or denary)

- **0b11010110**
 - The weight for the leftmost digit is negative
 - $1 \times -2^7 + 1 \times 2^6 + 0 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$
 - **-128** + 64 + 0 + 16 + 0 + 4 + 2 + 0
 - **-128** + 86
 - **-42**₁₀



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General-purpose computers are built to solve general-purpose problems

- Which involve a wide range of numbers
- You can get an idea of this range by skimming a physics textbook
 - ▣ There are **tiny numbers** such as Planck's constant (6.63×10^{-34} joule-seconds) and
 - ▣ **Huge numbers** such as Avogadro's constant (6.02×10^{23} molecules/mole)
 - ▣ This is a range of 10^{57} , which comes out to about 2^{191}
 - ▣ That's almost 200 bits!
- Bits just aren't cheap enough to use a few hundred of them to represent every number, so we need a different approach



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Look to what we have done in the past

- **Scientific notation** represents a large range of numbers by (how else?) creating a *new context* for interpretation
 - It uses a number with a **single digit** to the left of the decimal point, called the **mantissa**, multiplied by 10 raised to some power, called the **exponent**

$$\underbrace{6.63}_{\text{Mantissa}} \times \underbrace{10^{-34}}_{\text{Exponent}}$$



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Floating points

- Computers use the same system, except that the mantissa and exponent are binary numbers and 2 is used instead of 10
- This is called the **floating point representation**



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More on the scientific notation: Let's come up with a representation

\pm . $e \pm$

- A value like 9,876,543,210 would be approximated with 9.88×10^9 (or 9.88e+9 in programming language notation)



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Scientific notation complicates arithmetic somewhat

- When adding and subtracting two numbers in scientific notation, you must **adjust** the two values so that their **exponents are the same**
- For example, when adding $1.23e1$ and $4.56e0$, you could convert $4.56e0$ to $0.456e1$ and then add them
 - ▢ The result ($1.23e1 + 0.456e1 = 1.686e1$), does not fit into the three significant digits of our current format (in the previous slide)
 - ▢ So, we must either **round** or **truncate** the result to three significant digits
 - Rounding generally produces the more accurate result, so let's round to obtain $1.69e1$



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The implications of such adjustments?

- The lack of **precision** (the number of digits or bits maintained in a computation) affects the **accuracy** (the correctness of the computation)



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Floating point representation: **Binary version** of the scientific notation to represent a wide range of numbers

- The naming convention is confusing because the binary (or decimal) **point is always in the same place**
 - ▣ Between the ones and halves in binary (& tenths in decimal)
 - ▣ The “float” is just another way of saying “scientific notation,” which allows us to write 1.2×10^{-3} instead of 0.0012
- By **separating** the significant digits from the exponents
 - ▣ The floating-point system allows us to represent very small or very large numbers without having to store all those zeros



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IEEE 754 Floating standard

[1/2]

- When Intel planned to introduce a floating-point unit (FPU) for its original 8086 microprocessor?
 - ▣ Intel knew their electrical engineers and solid-state physicists didn't have the numerical analysis background to design a good floating-point representation
 - ▣ Went out and hired the best numerical analyst they could find to design a floating-point format for its 8087 FPU
- That person then hired two other experts in the field, and the three of them (Kahan, Coonen, and Stone) designed the KCS Floating-Point Standard
 - ▣ They did such a good job that the IEEE organization used this format as the basis for the **IEEE Std 754** floating-point format



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IEEE 754 Floating standard

[2/2]

- The floating-point system is the standard way to represent real numbers in computing
- There are two signs:
 - ▣ One for the **mantissa** and one for the **exponent** (*hidden*)
- There are also a lot of tricks to make sure that things like rounding work well and to minimize the number of wasted bit combinations



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The mantissa and the exponent

- The mantissa is a base value that usually falls within a **limited range** (for example, between 0 and 1)
- The exponent is a **multiplier** that, when applied to the mantissa, produces values outside this range
- The big advantage of the mantissa/exponent configuration
 - ▢ Floating-point format can represent values across a **wide range**



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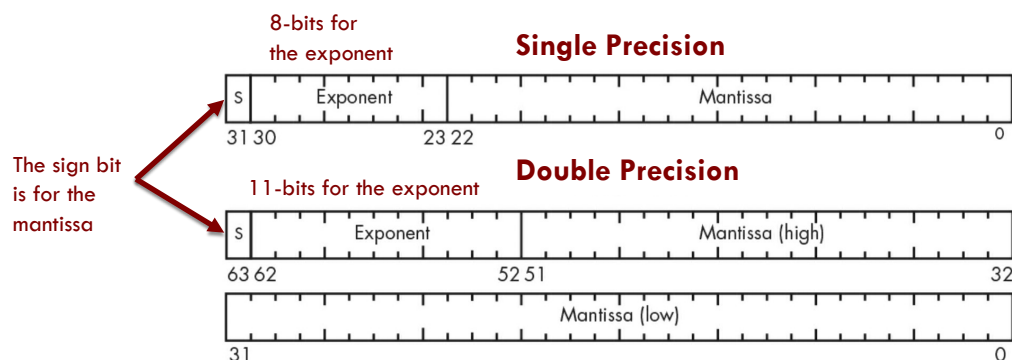
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IEEE Floating point number formats



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A couple of the tricks that are used in the standard

- **Normalization**, which adjusts the mantissa so that there are no leading (that is, on the left) zeros
- A second trick, from Digital Equipment Corporation (DEC), doubles the accuracy
 - ▣ By throwing away the leftmost bit of the mantissa since we know that it will always be 1, which makes room for one more bit
- Exponent values of all 0s and all 1s would have special meaning



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Two types of floating point numbers: Single and double-precision

- **Single-precision** numbers use 32 bits and can represent numbers approximately in the range $\pm 10^{\pm 38}$ with about 7 digits of accuracy
 - ▣ Although there is an infinite number of values between 1 and 2, we can represent only 8 million (2^{23}) of them because we use a 23-bit mantissa
 - Therefore, have only 23 bits of precision
- **Double-precision** numbers use 64 bits and can represent a wider range of numbers, approximately $\pm 10^{\pm 308}$, with about 15 digits of accuracy [*Nota Bene*: $2^{1024} \approx 10^{308}$]
 - ▣ Number of atoms in the known universe is between $10^{78} \sim 10^{82}$



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IEEE 754 also uses some special bit patterns

- To represent things like **division by zero**, which evaluates to positive or negative infinity
- It also specifies a special value called **NaN**, which stands for “not a number”
 - ▣ If you find yourself in the NaN state, it probably means that you did some illegal arithmetic operation



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Let's look at a number with a decimal point

- 0.15625
- We will compute its binary representation

	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Place-value	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125



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Unlike the division (by 2) that we did for number left of the decimal point

- During conversions, for numbers to the right of the decimal, we will **multiply 2**
- 0.15625_{10}

	2^4	2^3	2^2	2^1	2^0	2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}
Place-value	16	8	4	2	1	0.5	0.25	0.125	0.0625	0.03125
						0	0	1	0	1

$0.15625 \times 2 = 0.3125$	0
$0.3125 \times 2 = 0.625$	0
$0.625 \times 2 = 1.250$	1
$0.250 \times 2 = 0.500$	0
$0.500 \times 2 = 1.000$	1

Top to bottom is left-to-right



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$$0.15625_{10} = 0.00101_2$$

- $0.00101_2 = 1.01 \times 2^{-3}$
- Fraction is .01 and the exponent is -3
- The number is positive
- In the IEEE-754 standard, the exponent is written as a **biased exponent**
 - ▣ In single-precision, you need to add 127
 - -3 would be written as $-3 + 127 = 124_{10} = 01111100_2$
 - ▣ In double-precision you need to add 1023



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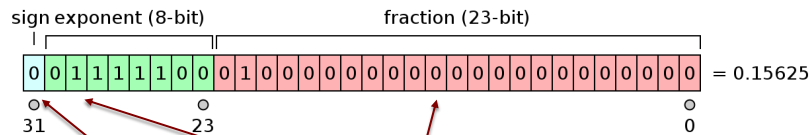
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Representation of $0.15625_{10} = 0.00101_2$ in IEEE-754 single precision



- $0.00101_2 = 1.01 \times 2^{-3}$
- The number is positive
- Exponent is written as a **biased exponent**: 01111100_2
- We only write the **fractional part** (the 1 in 1.01 is implied) i.e., 01 and then pad zeros all the way to the right



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NaN and Infinity

- The biased-exponent field is filled with all 1 bits to indicate either
 - ▣ Infinity mantissa field = 0
 - ▣ NaN mantissa field $\neq 0$



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Let's look at π up to a 1000 decimal points ...

□ 3.1415926535 8979323846 2643383279 5028841971 6939937510
5820974944 5923078164 0628620899 8628034825 3421170679 8214808651
3282306647 0938446095 5058223172 5359408128 4811174502 8410270193
8521105559 6446229489 5493038196 4428810975 6659334461 2847564823
3786783165 2712019091 4564856692 3460348610 4543266482 1339360726
0249141273 7245870066 0631558817 4881520920 9628292540 9171536436
7892590360 0113305305 4882046652 1384146951 9415116094 3305727036
5759591953 0921861173 8193261179 3105118548 0744623799 6274956735
1885752724 8912279381 8301194912 9833673362 4406566430 8602139494
6395224737 1907021798 6094370277 0539217176 2931767523 8467481846
7669405132 0005681271 4526356082 7785771342 7577896091 7363717872
1468440901 2249534301 4654958537 1050792279 6892589235 4201995611
2129021960 8640344181 5981362977 4771309960 5187072113 4999999837
2978049951 0597317328 1609631859 5024459455 3469083026 4252230825
3344685035 2619311881 7101000313 7838752886 5875332083 8142061717
7669147303 5982534904 2875546873 1159562863 8823537875 9375195778
1857780532 1712268066 1300192787 6611195909 2164201989



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How many digits of π can be represented using single-precision (i.e., 32-bits) IEEE 754?

- The single-precision format uses 24 bits for the mantissa, *including an implicit leading bit*
- This binary precision translates to roughly 7.22 decimal digits of accuracy; this is calculated as $\log_{10}(2^{24})$
- Consequently, when represented in this format, π can be accurately expressed up to the seventh significant digit, such as 3.141592



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How many digits of π can be represented using single-precision (i.e., 64-bits) IEEE 754?

- The double-precision format uses 53 bits for the mantissa, including an *implicit leading bit*
- This binary precision translates to roughly 15.95 decimal digits of accuracy; this is calculated as $\log_{10}(2^{53})$
- Consequently, when represented in this format, π can be accurately expressed up to the seventh significant digit, such as 3.141592653589793
 - ▣ Any digits beyond the 16th in a 64-bit float are considered machine *epsilon* noise or rounding artifacts, as the format lacks the bits to define them further



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HEXADECIMAL

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Hexadecimal

- Hexadecimal is **base 16**
- Given what we've already seen so far, you probably know what that means
- Hexadecimal, or just **hex** for short, is a place-value system where
 - ▣ Each place represents a power of 16
 - ▣ and each place can be one of 16 symbols



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Hexadecimal number representation

- As in all place-value systems, the rightmost place will still be the ones place
- The next place to the left will be the sixteens place, then the 256s (16×16) place, then the 4,096s ($16 \times 16 \times 16$) place, and so on
- Simple enough!



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But what about the other requirement that each place can be one of 16 symbols? [1/2]

- We usually have ten symbols to use to represent numbers, 0 through 9
- We need to add **six more** symbols to represent the other values



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But what about the other requirement that each place can be one of 16 symbols? [2/2]

- We could pick some random symbols like & @ #, but these symbols have no obvious order
- Instead, the standard is to use A, B, C, D, E, and F
 - ▣ Either uppercase or lowercase is fine!
- In this scheme, A represents ten, B represents eleven, and so on, up to F, which represents fifteen



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Symbols in the hexadecimal or hex number system

- **A** represents ten, **B** represents eleven, and so on, up to **F**, which represents fifteen ...

Hexadecimal	Decimal	Binary (4-bit)
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111



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Consider the number **0x1A5** in hexadecimal

- What's the value of this number in decimal?
- The rightmost place is worth 5
- The next place has a weight of 16, and there's an **A** there, which is 10 in decimal, so the middle place is worth $16 \times 10 = 160$
- The leftmost place has a weight of 256, and there's a **1** in that place, so that place is worth 256
- The total value then is $256 + 160 + 5 = 421$ in decimal



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Another view of the number 0x1A5 in hexadecimal

1	A	5
256s place	16s place	1s place
16^2	16^1	16^0
$16 \times 16 = 256$	16	1

$$\begin{aligned} &= 1 \times 256 + 10 \times 16 + 1 \times 5 \\ &= 256 + 160 + 5 \\ &= 421 \text{ in decimal} \end{aligned}$$



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Conversion from decimal to hex

- The hex number is constructed from **right to left**
- Step 1: Divide the decimal number by 16 and note down the remainder
- Step 2: Divide the obtained **quotient** by 16, and note remainder again
- Step 3: Repeat the above steps until you get **0** as the quotient
 - ▣ **Stopping criteria**



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Example: Decimal number 421

- $421 \div 16 = 26$ (Remainder 5)
- $26 \div 16 = 1$ (Remainder 10 or A)
- $1 \div 16 = 0$ (Remainder 1)

□ Hex representation: 0x1A5

- $= 1 \cdot 256 + 10 \cdot 16 + 1 \cdot 5$
- $= 256 + 160 + 5$
- $= 421$ in decimal

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is
Right to Left
{LSB to MSB}



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Some conversions across number systems

	Example 1	Example 2
Binary	1111 0000 0000 1111	1000 1000 1000 0001
Hexadecimal	F00F	8881
Decimal	61,455	34,945



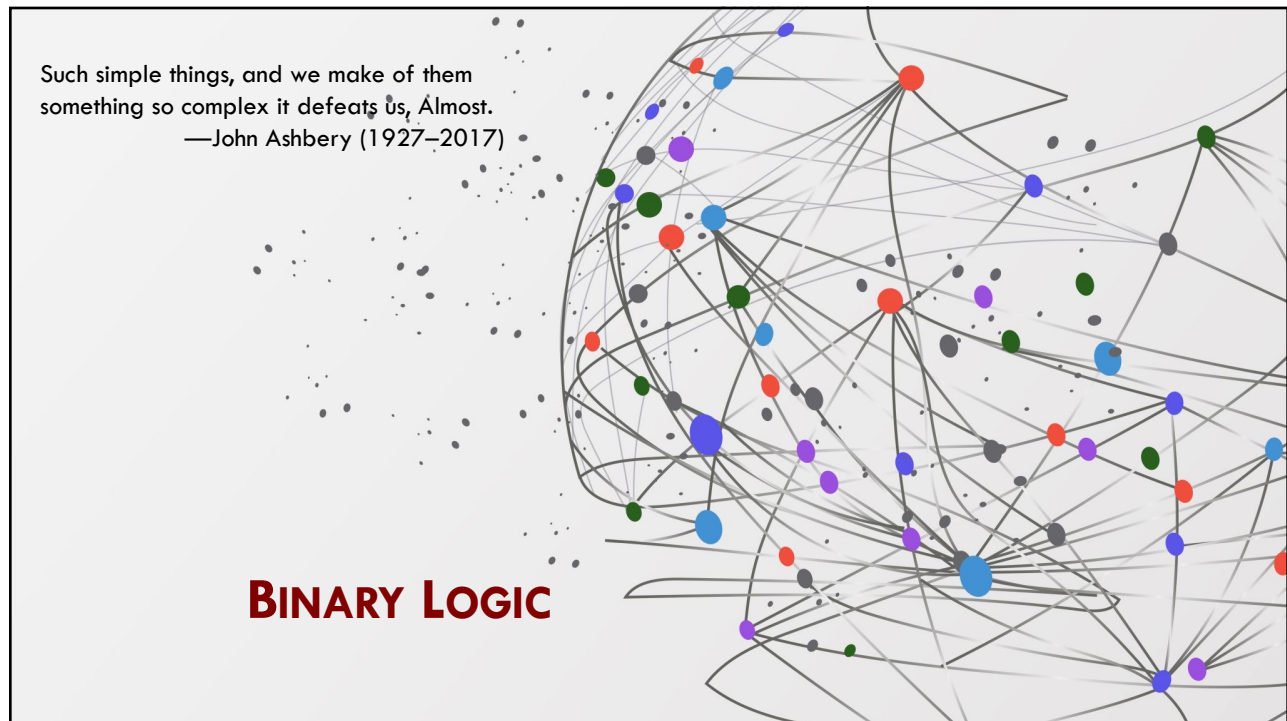
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Why binary logic matters

- Every digital device (be it a PC, a cell phone, or a network router) is based on chips designed to store and process binary information
- Although these chips come in different shapes and forms, they are all made of the same building blocks: **elementary logic gates**
- The gates can be physically realized using many different hardware technologies
 - ▣ But their logical behavior, or abstraction, is consistent across all implementations



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We've looked at using binary to represent data, but computers do more than simply store data

- Binary allows us to work with data as well
- Computers give us the capability to process data using hardware that we can program to execute a sequence of simple instructions
 - ▣ Instructions like “add two numbers together” or “check if two values are equal”
- Computer processors that **implement these instructions** are fundamentally based on **binary logic**
 - ▣ A system for describing logical statements where variables can only be one of two values: `true` or `false`



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Why is binary a natural fit for logic?

- Typically, when someone speaks of logic, they mean **reasoning**, or thinking through *what is known* in order to **arrive at a valid conclusion**
- When presented with a set of facts, logic allows us to determine whether *another related statement* is also **factual**
- Logic is all about **truth** that is true, and what is false
- Likewise, a bit can only be one of two values, 1 or 0
 - ▣ Therefore, a single bit can be used to represent a logical state of true (1) or false (0)



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Let's consider the logical statements for a rectangle

- If the shape does not have four sides **and** does not have four right angles
 - ▣ It is not a rectangle
- If the shape does not have four sides **and** does have four right angles
 - ▣ It is not a rectangle
- If the shape does have four sides **and** does not have four right angles
 - ▣ It is not a rectangle
- If the shape does have four sides **and** does have four right angles
 - ▣ It is a rectangle!



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Let's put those statements in table

Four sides	Four right angles	Is a rectangle
False	False	False
False	True	False
True	False	False
True	True	True



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What's so special about this table?

- This type of table is known as a **truth table**
- A truth table shows **all the possible combinations** of
 - ▣ **Conditions** (inputs) and
 - ▣ Their **logical conclusions** (outputs)
- Our previous table was written specifically for our statement about a rectangle, but ...
 - ▣ The same table **applies to any logical statement joined with AND**



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Let's represent false as 0 and true as a 1

x	y	$x \text{ And } y$
0	0	0
0	1	0
1	0	0
1	1	1



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Let's look at another example: **OR**

- Say you work at a shop that gives a discount to only two types of customers:
 - Children
 - People wearing sunglasses
- No one else is eligible for a discount
- If you wanted to state the store's policy as a logical expression, you could say the following:
 - GIVEN the customer is a child
 - OR GIVEN the customer is wearing sunglasses
 - I CONCLUDE that the customer is eligible for a discount



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The OR Truth Table

x	y	$x \text{ Or } y$
0	0	0
0	1	1
1	0	1
1	1	1



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The contents of this slide-set are based on the following references

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