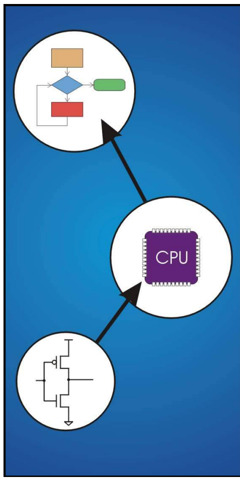


Chapter 2
Bits, Data Types, and Operations

Original slides from Gregory Byrd, North Carolina State University

Modified slides by Chris Wilcox, Colorado State University

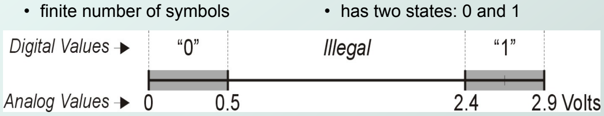
How do we represent data in a computer?

- At the lowest level, a computer is an electronic machine.
  - works by controlling the flow of electrons
- Easy to recognize two conditions:
  1. presence of a voltage – we'll call this state "1"
  2. absence of a voltage – we'll call this state "0"
- Could base state on *value* of voltage, but control and detection circuits more complex.
  - compare turning on a light switch to measuring or regulating voltage

Computer is a binary digital system.

Digital system:
- finite number of symbols

Binary (base two) system:
- has two states: 0 and 1



- Basic unit of information is the *binary digit*, or *bit*.
- Values with >2 states require multiple bits.
  - A collection of **two** bits has **four** possible states: **00, 01, 10, 11**
  - A collection of **three** bits has **eight** possible states: **000, 001, 010, 011, 100, 101, 110, 111**
  - *A collection of n bits has 2^n possible states.*

What kinds of data do we need to represent?

- **Numbers** – signed, unsigned, integers, floating point, complex, rational, irrational, …
- **Text** – characters, strings, …
- **Logical** – true, false
- **Images** – pixels, colors, shapes, …
- **Sound** – wave forms
- **Instructions**
- **…**

- Data type:
  - *representation* and *operations* within the computer
- We'll start with numbers…

Unsigned Integers

- Non-positional notation
  - could represent a number ("5") with a string of ones ("11111")
  - problems?
- Weighted positional notation
  - like decimal numbers: "329"
  - "3" is worth 300, because of its position, while "9" is only worth 9

329: 10^2 10^1 10^0

3x100 + 2x10 + 9x1 = 329

most significant — 101 — least significant: 2^2 2^1 2^0

1x4 + 0x2 + 1x1 = 5

Unsigned Integers (cont.)

- An *n*-bit unsigned integer represents 2^n values: from 0 to 2^n-1.

2^2	2^1	2^0	
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

Unsigned Binary Arithmetic

- Base-2 addition – just like base-10!
  - add from right to left, propagating carry

```
  10010        carry          carry
+  1001      10010           1111
  11011    +  1011         +    1
              11101         10000

              10111
           +    111
```

Subtraction, multiplication, division: remember integer math!

Signed Integers

- With n bits, we have 2^n distinct values.
  - assign about half to positive integers (1 through 2^{n-1})
  - assign about half to negative (- 2^{n-1} through -1)
  - that leaves two values: one for 0, and one extra
- Positive integers
  - just like unsigned – zero in *most significant* (MS) bit
    00101 = 5
- Negative integers
  - sign-magnitude – set sign bit to show negative
    10101 = -5
  - One's complement – flip every bit to represent negative
    11010 = -5
  - in either case, MS bit indicates sign: 0=pos., 1=neg.

Two's Complement

- Problems with sign-magnitude, 1's complement
  - two representations of zero (+0 and –0)
  - arithmetic circuits are complex
    - How to add two sign-magnitude numbers?
      - e.g., try 2 + (-3)
    - How to add to one's complement numbers?
      - e.g., try 4 + (-3)

Two's Complement

- ***Two's complement*** representation developed to make circuits easy for arithmetic.
  - for each positive number (X), assign value to its negative (-X),
    such that X + (-X) = 0 with "normal" addition, ignoring carry out

```
  00101  (5)        01001  (9)
+ 11011  (-5)     + _____  (-9)
  00000  (0)        00000  (0)
```


Two's Complement Representation

- If number is positive or zero,
  - normal binary representation, zeroes in upper bit(s)
- If number is negative,
  - start with positive number
  - flip every bit (i.e., take the one's complement)
  - then add one

```
  00101  (5)          01001  (9)
  11010  (1's comp)          (1's comp)
+     1             +     1
  11011  (-5)                (-9)
```


Two's Complement Shortcut

- To take the two's complement of a number:
  - copy bits from right to left until (and including) first "1"
  - flip remaining bits to the left

```
  011010000               011010000
  100101111  (1's comp)   (flip) | (copy)
+         1
  100110000               100110000
```

Two's Complement Signed Integers

- MS bit is sign bit – it has weight -2^{n-1}.
- Range of an n-bit number: -2^{n-1} through $2^{n-1} - 1$.
  - The most negative number has no positive counterpart.

-2^3	2^2	2^1	2^0	
0	0	0	0	0
0	0	0	1	1
0	0	1	0	2
0	0	1	1	3
0	1	0	0	4
0	1	0	1	5
0	1	1	0	6
0	1	1	1	7

-2^3	2^2	2^1	2^0	
1	0	0	0	-8
1	0	0	1	-7
1	0	1	0	-6
1	0	1	1	-5
1	1	0	0	-4
1	1	0	1	-3
1	1	1	0	-2
1	1	1	1	-1

Converting Binary (2's C) to Decimal

1. If leading bit is one, take two's complement to get a positive number.
2. Add powers of 2 that have "1" in the corresponding bit positions.
3. If original number was negative, add a minus sign.

$X = 01101000_{two}$
$= 2^6+2^5+2^3 = 64+32+8$
$= 104_{ten}$

Assuming 8-bit 2's complement numbers.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

More Examples

$X = 00100111_{two}$
$= 2^5+2^2+2^1+2^0 = 32+4+2+1$
$= 39_{ten}$

$X = 11100110_{two}$
$-X = 00011010$
$= 2^4+2^3+2^1 = 16+8+2$
$= 26_{ten}$
$X = -26_{ten}$

Assuming 8-bit 2's complement numbers.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Converting Decimal to Binary (2's C)

- First Method: ***Division***

1. Find magnitude of decimal number
2. Divide by two – remainder is least significant bit.
3. Keep dividing by two until answer is zero, writing remainders from right to left.
4. Append a zero as the MS bit; for negative, take two's complement.

$X = 104_{ten}$

104 - 64	= 40	*bit 6*
40 - 32	= 8	*bit 5*
8 - 8	= 0	*bit 3*

$X = 01101000_{two}$

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

Converting Decimal to Binary (2's C)

- Second Method: ***Subtract Powers of Two***

1. Find magnitude of decimal number.
2. Subtract largest power of two less than or equal to number.
3. Put a one in the corresponding bit position.
4. Keep subtracting until result is zero.
5. Append a zero as MS bit; if original was negative, take two's complement.

n	2^n
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512
10	1024

$X = 104_{ten}$

104 - 64 =	40	*bit 6*
40 - 32 =	8	*bit 5*
8 - 8 =	0	*bit 3*

$X = 01101000_{two}$

Operations: Arithmetic and Logical

- Recall: data types include *representation* and *operations*.
- 2's complement is a good representation for signed integers, now we need arithmetic operations:
  - Addition (including overflow)
  - Subtraction
  - Sign Extension
- Multiplication and division can be built from these basic operations.
- Logical operations are also useful:
  - AND
  - OR
  - NOT

Addition

- As we've discussed, 2's comp. addition is just binary addition.
  - assume all integers have the same number of bits
  - ignore carry out
  - for now, assume that sum fits in n-bit 2's comp. representation

01101000	(104)	**11110110**	(-10)
+ **11110000**	(-16)	+ ________	(-9)
01011000	(98)		(-19)

Assuming 8-bit 2's complement numbers.

Subtraction

- Negate second operand, then add.
  - assume all integers have the same number of bits
  - ignore carry out
  - for now, assume that difference fits in n-bit 2's comp. representation

01101000	(104)	**11110110**	(-10)
- **00010000**	(16)	- ________	(-9)
01101000	(104)	**11110110**	(-10)
+ **11110000**	(-16)	+ ________	(9)
01011000	(88)		(-1)

Assuming 8-bit 2's complement numbers.

Sign Extension

- To add two numbers, we must represent them with the same number of bits.
- If we just pad with zeroes on the left:

4-bit		8-bit	
0100	(4)	00000100	(still 4)
1100	(-4)	00001100	(12, not -4)

- Instead, replicate the MS bit -- the sign bit:

4-bit		8-bit	
0100	(4)	00000100	(still 4)
1100	(-4)	11111100	(still -4)

Overflow

- If operands are too big, then sum cannot be represented as an *n*-bit 2's comp number.

01000	(8)		11000	(-8)
+ 01001	(9)		+ 10111	(-9)
10001	(-15)		01111	(+15)

- We have overflow if:
  - signs of both operands are the same, and
  - sign of sum is different.
- Another test -- easy for hardware:
  - carry into MS bit does not equal carry out

Logical Operations

- Operations on logical TRUE or FALSE
  - two states -- takes one bit to represent: TRUE=1, FALSE=0

A	B	A AND B
0	0	0
0	1	0
1	0	0
1	1	1

A	B	A OR B
0	0	0
0	1	1
1	0	1
1	1	1

A	NOT A
0	1
1	0

- View *n*-bit number as a collection of *n* logical values
  - operation applied to each bit independently

Examples of Logical Operations

- AND
  - useful for clearing bits
    - AND with zero = 0
    - AND with one = no change

```
     11000101
AND  00001111
     00000101
```

- OR
  - useful for setting bits
    - OR with zero = no change
    - OR with one = 1

```
     11000101
OR   00001111
     11001111
```

- NOT
  - unary operation -- one argument
  - flips every bit

```
NOT  11000101
     00111010
```

Hexadecimal Notation

- It is often convenient to write binary (base-2) numbers in hexadecimal (base-16) instead.
  - fewer digits - four bits per hex digit
  - less error prone - no long string of 1's and 0's

Binary	Hex	Decimal	Binary	Hex	Decimal
0000	0	0	1000	8	8
0001	1	1	1001	9	9
0010	2	2	1010	A	10
0011	3	3	1011	B	11
0100	4	4	1100	C	12
0101	5	5	1101	D	13
0110	6	6	1110	E	14
0111	7	7	1111	F	15

Converting from Binary to Hexadecimal

- Every four bits is a hex digit.
  - start grouping from right-hand side

011 1010 1000 1111 0100 1101 0111

⇓ ⇓ ⇓ ⇓ ⇓ ⇓ ⇓

3 A 8 F 4 D 7

This is not a new machine representation, just a convenient way to write the number.

Fractions: Fixed-Point

- How can we represent fractions?
  - Use a "binary point" to separate positive from negative powers of two -- just like "decimal point."
  - 2's comp addition and subtraction still work (if binary points are aligned)

$2^{-1} = 0.5$
$2^{-2} = 0.25$
$2^{-3} = 0.125$

00101000.101 (40.625)
+ 11111110.110 (-1.25)
00100111.011 (39.375)

No new operations -- same as integer arithmetic.

Very Large and Very Small: Floating-Point

- Large values: 6.023×10^{23} -- requires 79 bits
- Small values: 6.626×10^{-34} -- requires >110 bits
- Use equivalent of "scientific notation": $F \times 2^E$
- Must have F (*fraction*), E (*exponent*), and sign.
- IEEE 754 Floating-Point Standard (32-bits):

1b	8b	23b
S	Exponent	Fraction

$$N = (-1)^S \times 1.\text{fraction} \times 2^{\text{exponent}-127},\ 1 \le \text{exponent} \le 254$$
$$N = (-1)^S \times 0.\text{fraction} \times 2^{-126},\ \text{exponent} = 0$$

Floating Point Example

- Single-precision IEEE floating point number:
- 1 01111110 10000000000000000000000

  sign exponent fraction

  - Sign is 1 – number is negative.
  - Exponent field is 01111110 = 126 (decimal).
  - Fraction is 1.100000000000… = 1.5 (decimal).

- Value = -1.5 x $2^{(126-127)}$ = -1.5 x 2^{-1} = **-0.75**

Floating-Point Operations

- Will regular 2's complement arithmetic work for Floating Point numbers?
- (*Hint*: In decimal, how do we compute 3.07 x 10^{12} + 9.11 x 10^{8}?)

Text: ASCII Characters

- ASCII: Maps 128 characters to 7-bit code.
  - printable and non-printable (ESC, DEL, …) characters

00	nul	10	dle	20	sp	30	0	40	@	50	P	60	`	70	p
01	soh	11	dc1	21	!	31	1	41	A	51	Q	61	a	71	q
02	stx	12	dc2	22	"	32	2	42	B	52	R	62	b	72	r
03	etx	13	dc3	23	#	33	3	43	C	53	S	63	c	73	s
04	eot	14	dc4	24	$	34	4	44	D	54	T	64	d	74	t
05	enq	15	nak	25	%	35	5	45	E	55	U	65	e	75	u
06	ack	16	syn	26	&	36	6	46	F	56	V	66	f	76	v
07	bel	17	etb	27	'	37	7	47	G	57	W	67	g	77	w
08	bs	18	can	28	(	38	8	48	H	58	X	68	h	78	x
09	ht	19	em	29	)	39	9	49	I	59	Y	69	i	79	y
0a	nl	1a	sub	2a	*	3a	:	4a	J	5a	Z	6a	j	7a	z
0b	vt	1b	esc	2b	+	3b	;	4b	K	5b	[	6b	k	7b	{
0c	np	1c	fs	2c	,	3c	<	4c	L	5c	\	6c	l	7c	|
0d	cr	1d	gs	2d	-	3d	=	4d	M	5d	]	6d	m	7d	}
0e	so	1e	rs	2e	.	3e	>	4e	N	5e	^	6e	n	7e	~
0f	si	1f	us	2f	/	3f	?	4f	O	5f	_	6f	o	7f	del

Text: ASCII Characters

- ASCII is a seven-bit code. "Eight-bit ASCII" makes as sense as a square circle.
- There is no need to memorize the ASCII chart.
- There is no need to insert ASCII values into a program.
  - if (c >= 65 && c <= 90) … // just showing off
  - if (c >= 'A' && c <= 'Z') … // easy to understand
  - if ('A' <= c && c <= 'Z') … // I like this even more

Interesting Properties of ASCII Code

- What is relationship between a decimal digit ('0', '1', …) and its ASCII code?
- What is the difference between an upper-case letter ('A', 'B', …) and its lower-case equivalent ('a', 'b', …)?
- Given two ASCII characters, how do we tell which comes first in alphabetical order?
- Are 128 characters enough? (http://www.unicode.org/)

No new operations needed for ASCII codes – integer arithmetic and logic are sufficient.

Other Data Types

- Text strings
  - array of characters, terminated with null character ('\0')
  - typically, no hardware support
- Image
  - array of pixels
    - monochrome: one bit (0/1 = black/white)
    - color: red, green, blue (RGB) components
    - other properties: transparency
  - hardware support:
    - typically none, in general-purpose processors
    - MMX -- multiple 8-bit operations on 32-bit word
- Sound
  - sequence of fixed-point numbers

LC-3 Data Types

- Some data types are supported directly by the instruction set architecture.
- For LC-3, there is only one hardware-supported data type:
  - 16-bit 2's complement signed integer
  - Operations: ADD, AND, NOT
- Other data types are supported by interpreting 16-bit values as logical, text, fixed-point, floating-point, etc., in the software that we write.