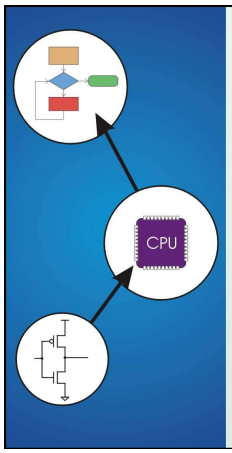


Chapter 17
Recursion

Original slides from Gregory Byrd, North Carolina State University

Modified slides by Chris Wilcox, Colorado State University

What is Recursion?

- A **recursive function** is one that solves its task by **calling itself** on smaller pieces of data.
  - Similar to recurrence function in mathematics.
  - Like iteration -- can be used interchangeably; sometimes recursion results in a simpler solution.

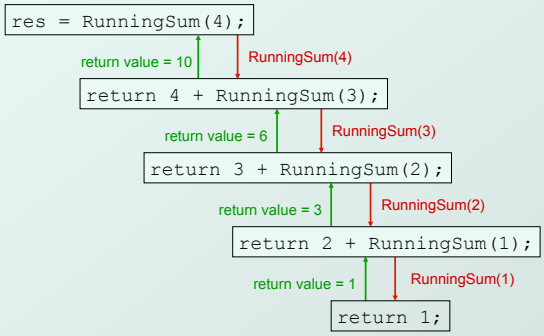
Example: Running sum ($\sum_{1}^{n} i$)

Mathematical Definition:
RunningSum(1) = 1
RunningSum(n) =
n + RunningSum(n-1)

Recursive Function:

```
int RunningSum(int n) {
  if (n == 1)
    return 1;
  else
    return n + RunningSum(n-1);
}
```


Executing RunningSum



High-Level Example: Binary Search

- Given a sorted set of exams, in alphabetical order, find the exam for a particular student.

1. Look at the exam **halfway** through the pile.
2. If it matches the name, we're done;
   if it does not match, then...

3a. If the name is greater (alphabetically), then **search the upper half** of the stack.

3b. If the name is less than the halfway point, then **search the lower half** of the stack.

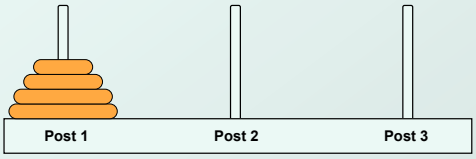
Binary Search: Pseudocode

- Pseudocode is a way to describe algorithms without completely coding them in C.

```
FindExam(studentName, start, end) {
  halfwayPoint = (end + start)/2;
  if (end < start)
    ExamNotFound();  /* exam not in stack */
  else if (studentName == NameOfExam(halfwayPoint))
    ExamFound(halfwayPoint); /* found exam! */
  else if (studentName < NameOfExam(halfwayPoint))
    /* search lower half */
    FindExam(studentName, start, halfwayPoint-1)
  else
    /* search upper half */
    FindExam(studentName, halfwayPoint + 1, end);
}
```


High-Level Example: Towers of Hanoi

- **Task:** Move all disks from one post to another post.



Rules:

(1) Can only move one disk at a time.

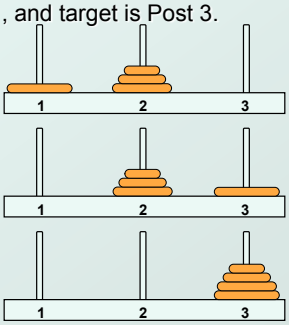
(2) Cannot put larger disk on top of a smaller disk.

(3) May use third post for temporary storage.

Task Decomposition

- Disks start on Post 1, and target is Post 3.

1. Move top n-1 disks to Post 2.
2. Move largest disk to Post 3.
3. Move n-1 disks from Post 2 to Post 3.



Task Decomposition (cont.)

- Task 1 is really the **same problem**, with fewer disks and a different target post.
  - "Move n-1 disks from Post 1 to Post 2."
- And Task 3 is also the **same problem**, with fewer disks and different starting and target posts.
  - "Move n-1 disks from Post 2 to Post 3."
- So this is a **recursive** algorithm.
  - The terminal case is moving the smallest disk -- can move directly without using third post.
  - Number disks from 1 (smallest) to n (largest).

Towers of Hanoi: Pseudocode

```
MoveDisk(diskNumber, startPost, endPost, midPost) {
  if (diskNumber > 1) {
    /* Move top n-1 disks to mid post */
    MoveDisk(diskNumber-1, startPost, midPost, endPost)
    printf("Move disk number %d from %d to %d.\n",
             diskNumber, startPost, endPost);

    /* Move n-1 disks from mid post to end post */
    MoveDisk(diskNumber-1, midPost, endPost, startPost);
  }
  else
    printf("Move disk number 1 from %d to %d.\n",
             startPost, endPost);
}
```


Detailed Example: Fibonacci Numbers

- Mathematical Definition:

$$f(n) = f(n-1) + f(n-2)$$
$$f(1) = 1$$
$$f(0) = 1$$

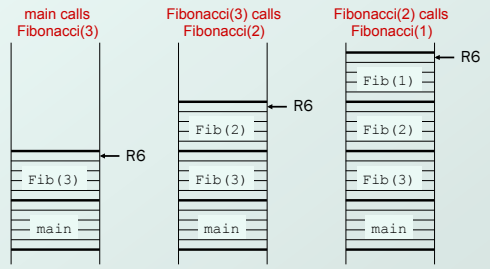
- In other words, the n-th Fibonacci number is the sum of the previous two Fibonacci numbers.

Fibonacci: C Code

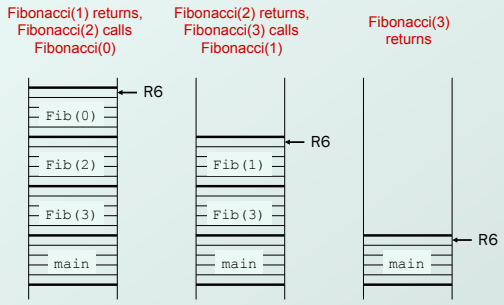
```
int Fibonacci(int n)
{
  if ((n == 0) || (n == 1))
    return 1;
  else
    return Fibonacci(n-1) + Fibonacci(n-2);
}
```


Activation Records

- Whenever Fibonacci is invoked, a new activation record is pushed onto the stack.



Activation Records (cont.)



Tracing the Function Calls

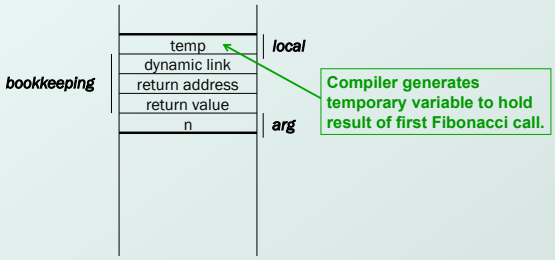
- If we are debugging this program, we might want to trace all the calls of Fibonacci.
  - Note: A trace will also contain the arguments passed into the function.
- For Fibonacci(3), a trace looks like:

```
Fibonacci(3)
Fibonacci(2)
Fibonacci(1)
Fibonacci(0)
Fibonacci(1)
```

- What would trace of Fibonacci(4) look like?

Fibonacci: LC-3 Code

Activation Record



LC-2 Code (part 1 of 3)

```
Fibonacci

    ADD  R6, R6, #-2 ; skip ret val, push ret addr
    STR  R7, R6, #0
    ADD  R6, R6, #-1  ; push dynamic link
    STR  R5, R6, #0
    ADD  R5, R6, #-1  ; set frame pointer
    ADD  R6, R6, #-2  ; space for locals and temps

    LDR  R0, R5, #4   ; load n
    BRz  FIB_BASE     ; check for terminal cases
    ADD  R0, R0, #-1
    BRz  FIB_BASE
```

LC-3 Code (part 2 of 3)

```
LDR  R0, R5, #4  ; read parameter n
ADD  R0, R0, #-1 ; calculate n-1
ADD  R6, R6, #-1 ; push n-1
STR  R0, R6, #0
JSR  Fibonacci   ; call self

LDR  R0, R6, #0  ; pop return value
ADD  R6, R6, #1
STR  R0, R5, #-1 ; store in temp
LDR  R0, R5, #4  ; read parameter n
ADD  R0, R0, #-2 ; calculate n-2
ADD  R6, R6, #-1 ; push n-2
STR  R0, R6, #0
JSR  Fibonacci   ; call self
```


LC-3 Code (part 3 of 3)

```
          LDR  R0, R6, #0  ; pop return value
          ADD  R6, R6, #1
          LDR  R1, R5, #-1 ; read temp
          ADD  R0, R0, R1  ; Fib(n-1) + Fib(n-2)
          BRnzp  FIB_END   ; all done

FIB_BASE  AND  R0, R0, #0  ; base case - return 1
          ADD  R0, R0, #1

FIB_END   STR  R0, R5, #3  ; write return value (R0)
          ADD  R6, R5, #1  ; pop local variables
          LDR  R5, R6, #0  ; pop dynamic link
          ADD  R6, R6, #1
          LDR  R7, R6, #0  ; pop return address
          ADD  R6, R6, #1
          RET
```


A Final C Example: Printing an Integer

- Recursively converts an unsigned integer as a string of ASCII characters.
  - If integer <10, convert to char and print.
  - else, call self on first (n-1) digits and then print last digit.

```
void IntToAscii(int num) {
  int prefix, currDigit;
  if (num < 10)
    putchar(num + '0');    /* print number */
  else {
    prefix = num / 10;     /* previous digits */
    digit = num % 10;      /* current digit */
    IntToAscii(prefix);    /* recursive call */
    putchar(digit + '0'); /* print digit */
  }
}
```


Trace of IntToAscii

- Calling IntToAscii with parameter 12345:

```
IntToAscii(12345)
 IntToAscii(1234)
  IntToAscii(123)
   IntToAscii(12)
    IntToAscii(1)
    putchar('1')
   putchar('2')
  putchar('3')
 putchar('4')
putchar('5')
```