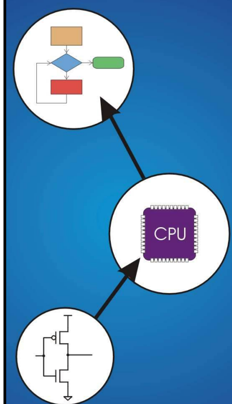
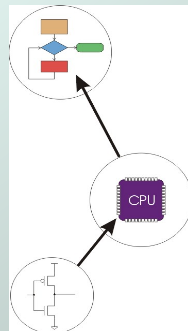


Chapter 3
Digital Logic Structures

Original slides from Gregory Byrd, North Carolina State University

Modified slides by Chris Wilcox, Colorado State University

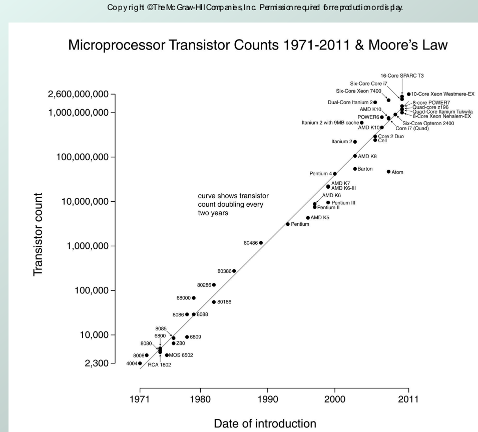
Computing Layers



- Problems
- Algorithms
- Language
- Instruction Set Architecture
- Microarchitecture
- Circuits
- Devices

Transistor: Building Block of Computers

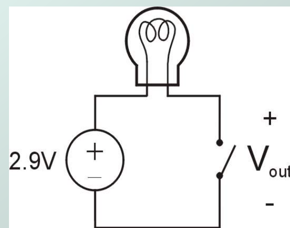
- Microprocessors contain lots of transistors
  - **Intel 8086 (1978):** 29 thousand
  - **Intel 80186 (1982):** 55 thousand
  - **Intel 80386 (1985):** 275 thousand
  - **Intel 80486 (1989):** 1.1 million
  - **Intel Pentium (1993):** 3.1 million
  - **Intel Pentium II (1998):** 7.5 million
  - **Intel Pentium III (2001):** 45 million
  - **Intel Pentium 4 (2006):** 184 million
  - **Intel Core 2 Duo (2006):** 291 million
  - **Intel Quad Core i7 (2011):** 1.1 billion
  - **Intel 8-core Xeon (2012):** 2.3 billion



Transistor: Building Block of Computers

- Logically, each transistor acts as a switch
- Combined to implement logic functions (gates)
  - AND, OR, NOT
- Combined to build higher-level structures
  - Multiplexer, decoder, register, memory …
  - Adder, multiplier …
- Combined to build simple processor
  - LC-3

Simple Switch Circuit

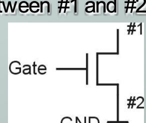


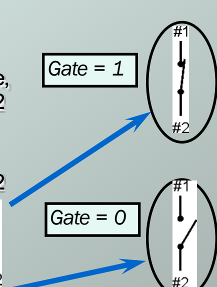
- Switch open:
  - Open circuit, no current
  - Light is off
  - V_{out} is +2.9V
- Switch closed:
  - Short circuit across switch, current flows
  - Light is on
  - V_{out} is 0V

Switch-based circuits can easily represent two states: on/off, open/closed, voltage/no voltage.

n-type MOS Transistor

- MOS = Metal Oxide Semiconductor
  - two types: n-type and p-type
- n-type
  - when Gate has positive voltage, short circuit between #1 and #2 (switch closed)
  - when Gate has zero voltage, open circuit between #1 and #2 (switch open)

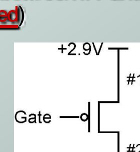


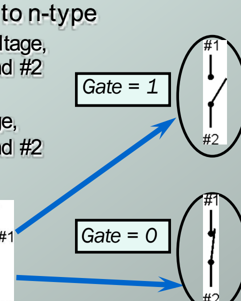


Terminal #2 must be connected to GND (0V).

p-type MOS Transistor

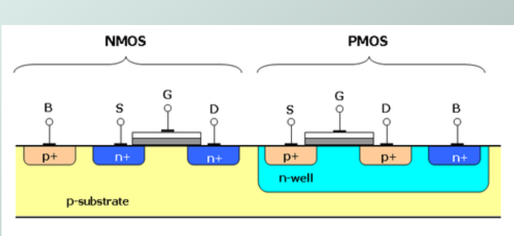
- p-type is *complementary* to n-type
  - when Gate has positive voltage, open circuit between #1 and #2 (switch open)
  - when Gate has zero voltage, short circuit between #1 and #2 (switch closed)





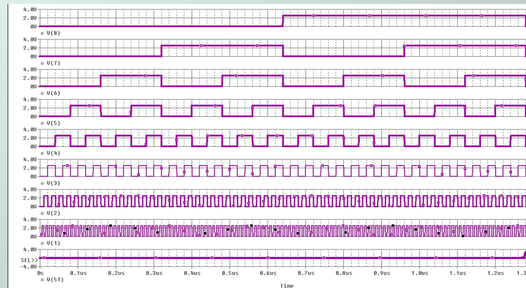
Terminal #1 must be connected to +2.9V.

Physical Transistor



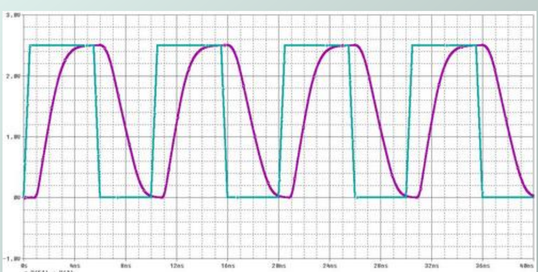
http://en.wikipedia.org/wiki/CMOS

Transistor Output (Ideal)



Logic analyzer view of waveforms

Transistor Output (Actual)



Actual waveform is not ideal!

Propagation Delay

- Each gate has a propagation delay, typically fraction of a nanosecond (10^{-9} sec).
- Delays accumulate depending on the chain of gates the signals have to go through.
- Clock frequency of a processor is determined by the delay of the longest combinational path between storage elements, i.e. cycle time.

Logic Gates

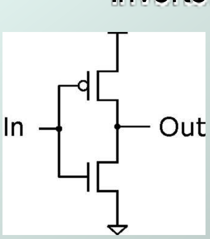
- Use switch behavior of MOS transistors to implement logical functions: AND, OR, NOT.
- Digital symbols:
  - recall that we assign a range of analog voltages to each digital (logic) symbol

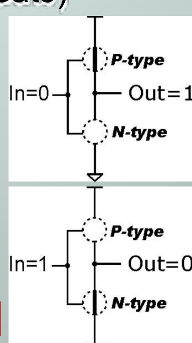
  - assignment of voltage ranges depends on electrical properties of transistors being used
    - typical values for "1": +5V, +3.3V, +2.9V
    - from now on we'll use +2.9V

CMOS Circuit

- **Complementary** MOS
- Uses both **n-type** and **p-type** MOS transistors
  - p-type
    - Attached to + voltage
    - Pulls output voltage UP when input is zero
  - n-type
    - Attached to GND
    - Pulls output voltage DOWN when input is one
- **For all inputs, make sure that output is either connected to GND or to +, but not both!**

Inverter (NOT Gate)





In	Out
0 V	2.9 V
2.9 V	0 V

In	Out
0	1
1	0

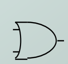
Truth table

Logical Operation: OR and NOR

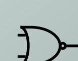
A	B	OR
0	0	0
0	1	1
1	0	1
1	1	1

A	B	NOR
0	0	1
0	1	0
1	0	0
1	1	0

Inputs: *2 or more*



Output=A+B



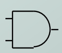
Output=$\overline{A+B}$

AND and NAND

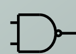
A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

A	B	NAND
0	0	1
0	1	1
1	0	1
1	1	0

Inputs: 2 or more

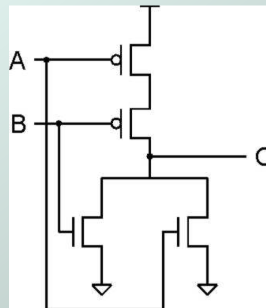


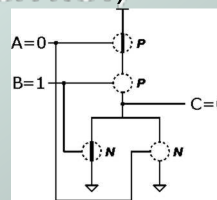
Output = A•B



Output = $\overline{A \bullet B}$

NOR Gate (OR-NOT)



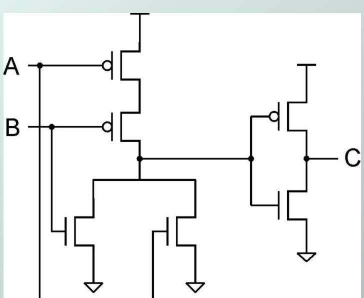


A	B	C
0	0	1
0	1	0
1	0	0
1	1	0

Truth table

Note: Serial structure on top, parallel on bottom.

OR Gate

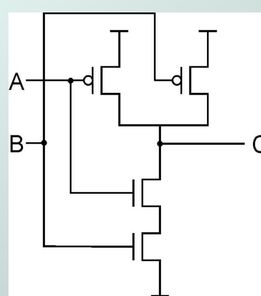


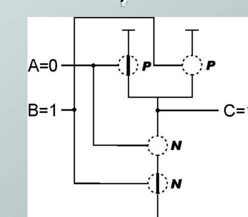
A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

Add inverter to NOR.

NAND Gate (AND-NOT)



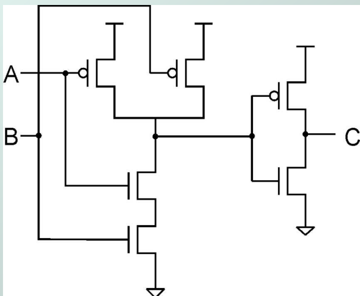


Truth table

A	B	C
0	0	1
0	1	1
1	0	1
1	1	0

Note: Parallel structure on top, serial on bottom.

AND Gate

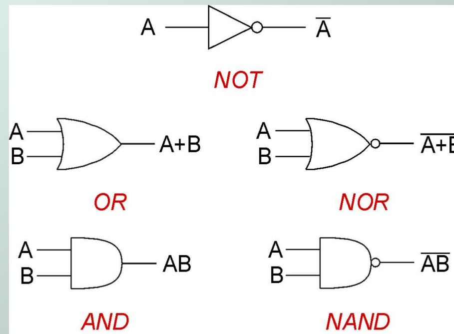


A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

Truth table

Add inverter to NAND.

Basic Logic Gates



Boolean Algebra

$x \cdot 0 = 0$ $x \cdot 1 = x$ $x \cdot \bar{x} = 0$

$x+0 = x$ $x+1 =$ $x+\bar{x} =$

Remember Identify, Domination, Negation Laws from Logic!

Boolean Algebra Laws

- Commutative
  - A+B = B+A
  - A•B = B•A
- Associative
  - A+(B+C)=(A+B)+C = A+B+C
  - A•(B•C)=(A•B)•C = ABC
- Distributive
  - A•(B+C)=A•B+A•C
  - A+(B•C)=(A+B)•(A+C)

Some Useful Identities for simplification

- $A\overline{B}+AB = A$

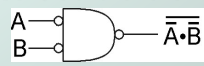
  Proof: $A\overline{B}+AB$ = $A(\overline{B}+B)$ **// Distributive Law**
  = A(T) **// Negation Law**
  = A **// Identity Law**

- A+AB = A

  Proof: A+AB = A(1+B) **// Distributive Law**
  = A(1) **// Domination Law**
  = A **// Identity Law**

DeMorgan's Law

- Converting AND to OR (with some help from NOT)
- Consider the following gate:



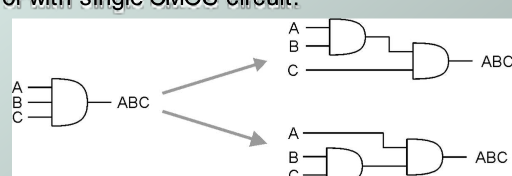
A	B	$\overline{A}$	$\overline{B}$	$\overline{A}\cdot\overline{B}$	$\overline{\overline{A}\cdot\overline{B}}$
0	0	1	1	1	0
0	1	1	0	0	1
1	0	0	1	0	1
1	1	0	0	0	1

Same as A OR B!

To convert AND to OR (or vice versa), invert inputs and output.

More than 2 Inputs?

- AND/OR can take any number of inputs.
  - AND = 1 if all inputs are 1.
  - OR = 1 if any input is 1.
  - Similar for NAND/NOR.
- Can implement with multiple two-input gates, or with single CMOS circuit.



Summary

- MOS transistors are used as switches to implement logic functions.
  - n-type: connect to GND, turn on (1) to pull down to 0
  - p-type: connect to +2.9V, turn on (0) to pull up to 1
- Basic gates: NOT, NOR, NAND
  - Logic functions are usually expressed with AND, OR, and NOT
- DeMorgan's Law
  - Convert AND to OR (and vice versa) by inverting inputs and output

Building Functions from Logic Gates

- ***Combinational Logic Circuit***
  - output depends only on the current inputs
  - stateless
- ***Sequential Logic Circuit***
  - output depends on the sequence of inputs (past and present)
  - stores information (state) from past inputs
- We'll first look at some useful combinational circuits, then show how to use sequential circuits to store information.