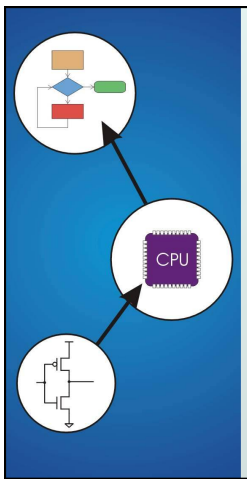



# Chapter 2
# Bits, Data Types, and Operations

Original slides from Gregory Byrd, North Carolina State University

Modified slides by Chris Wilcox, Colorado State University



## How do we represent data in a computer?

- At the lowest level, a computer is an electronic machine.
  - works by controlling the flow of electrons
- Easy to recognize two conditions:
  1. presence of a voltage – we'll call this state "1"
  2. absence of a voltage – we'll call this state "0"
- Could base state on *value* of voltage, but control and detection circuits more complex.
  - compare turning on a light switch to measuring or regulating voltage





## Computer is a binary digital system.

Digital system:
- finite number of symbols

Binary (base two) system:
- has two states: 0 and 1


- Basic unit of information is the *binary digit*, or *bit*.
- Values with >2 states require multiple bits.
  - A collection of **two** bits has **four** possible states: **00, 01, 10, 11**
  - A collection of **three** bits has **eight** possible states: **000, 001, 010, 011, 100, 101, 110, 111**
  - *A collection of n bits has $2^n$ possible states.*





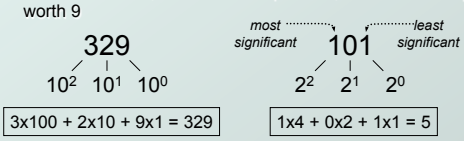
## What kinds of data do we need to represent?

- **Numbers** – signed, unsigned, integers, floating point, complex, rational, irrational, …
- **Text** – characters, strings, …
- **Logical** – true, false
- **Images** – pixels, colors, shapes, …
- **Sound** – wave forms
- **Instructions**
- …

- Data type:
  - *representation* and *operations* within the computer
- We'll start with numbers…

## Unsigned Integers

- Non-positional notation
  - could represent a number ("5") with a string of ones ("11111")
  - problems?
- Weighted positional notation
  - like decimal numbers: "329"
  - "3" is worth 300, because of its position, while "9" is only worth 9








## Unsigned Integers (cont.)

- An *n*-bit unsigned integer represents $2^n$ values: from 0 to $2^n$-1.

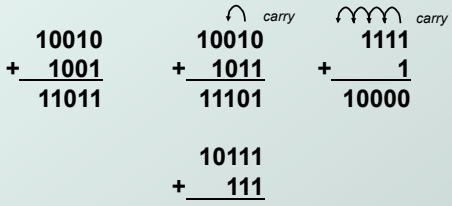
| $2^2$ | $2^1$ | $2^0$ | |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 2 |
| 0 | 1 | 1 | 3 |
| 1 | 0 | 0 | 4 |
| 1 | 0 | 1 | 5 |
| 1 | 1 | 0 | 6 |
| 1 | 1 | 1 | 7 |





## Unsigned Binary Arithmetic

- Base-2 addition – just like base-10!
  - add from right to left, propagating carry




Subtraction, multiplication, division: remember integer math!





## Signed Integers

- With n bits, we have $2^n$ distinct values.
  - assign about half to positive integers (1 through $2^{n-1}$)
  - assign about half to negative (- $2^{n-1}$ through -1)
  - that leaves two values: one for 0, and one extra
- Positive integers
  - just like unsigned – zero in *most significant* (MS) bit
    **00101 = 5**
- Negative integers
  - sign-magnitude – set sign bit to show negative
    **10101 = -5**
  - One's complement – flip every bit to represent negative
    **11010 = -5**
  - in either case, MS bit indicates sign: 0=pos., 1=neg.

## Two's Complement

- Problems with sign-magnitude, 1's complement
  - two representations of zero (+0 and –0)
  - arithmetic circuits are complex
    - How to add two sign-magnitude numbers?
      - e.g., try 2 + (-3)
    - How to add to one's complement numbers?
      - e.g., try 4 + (-3)





## Two's Complement

- ***Two's complement*** representation developed to make circuits easy for arithmetic.
  - for each positive number (X), assign value to its negative (-X), such that X + (-X) = 0 with "normal" addition, ignoring carry out

| | | | | | |
|---|---|---|---|---|---|
| **00101** | (5) | | **01001** | (9) | |
| + **11011** | (-5) | | + | | (-9) |
| **00000** | (0) | | **00000** | (0) | |





## Two's Complement Representation

- If number is positive or zero,
  - normal binary representation, zeroes in upper bit(s)
- If number is negative,
  - start with positive number
  - flip every bit (i.e., take the one's complement)
  - then add one

| | | | | | |
|---|---|---|---|---|---|
| **00101** | (5) | | **01001** | (9) | |
| **11010** | (1's comp) | | | (1's comp) | |
| + **1** | | | + **1** | | |
| **11011** | (-5) | | | (-9) | |

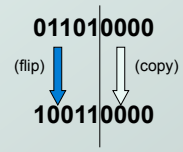




## Two's Complement Shortcut

- To take the two's complement of a number:
  - copy bits from right to left until (and including) first "1"
  - flip remaining bits to the left

| | |
|---|---|
| **011010000** | |
| **100101111** | (1's comp) |
| + **1** | |
| **100110000** | |



## Two's Complement Signed Integers

- MS bit is sign bit – it has weight $-2^{n-1}$.
- Range of an n-bit number: $-2^{n-1}$ through $2^{n-1} - 1$.
  - The most negative number has no positive counterpart.

| $-2^3$ | $2^2$ | $2^1$ | $2^0$ | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 2 |
| 0 | 0 | 1 | 1 | 3 |
| 0 | 1 | 0 | 0 | 4 |
| 0 | 1 | 0 | 1 | 5 |
| 0 | 1 | 1 | 0 | 6 |
| 0 | 1 | 1 | 1 | 7 |

| $-2^3$ | $2^2$ | $2^1$ | $2^0$ | |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | -8 |
| 1 | 0 | 0 | 1 | -7 |
| 1 | 0 | 1 | 0 | -6 |
| 1 | 0 | 1 | 1 | -5 |
| 1 | 1 | 0 | 0 | -4 |
| 1 | 1 | 0 | 1 | -3 |
| 1 | 1 | 1 | 0 | -2 |
| 1 | 1 | 1 | 1 | -1 |





## Converting Binary (2's C) to Decimal

1. If leading bit is one, take two's complement to get a positive number.
2. Add powers of 2 that have "1" in the corresponding bit positions.
3. If original number was negative, add a minus sign.

$$X = 01101000_{two}$$
$$= 2^6+2^5+2^3 = 64+32+8$$
$$= 104_{ten}$$

*Assuming 8-bit 2's complement numbers.*

| $n$ | $2^n$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |





## More Examples

$$X = 00100111_{two}$$
$$= 2^5+2^2+2^1+2^0 = 32+4+2+1$$
$$= 39_{ten}$$

$$X = 11100110_{two}$$
$$-X = 00011010$$
$$= 2^4+2^3+2^1 = 16+8+2$$
$$= 26_{ten}$$
$$X = -26_{ten}$$

*Assuming 8-bit 2's complement numbers.*

| $n$ | $2^n$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |





## Converting Decimal to Binary (2's C)

- First Method: ***Division***

1. Find magnitude of decimal number
2. Divide by two – remainder is least significant bit.
3. Keep dividing by two until answer is zero, writing remainders from right to left.
4. Append a zero as the MS bit; for negative, take two's complement.

$X = 104_{ten}$

| | | |
|---|---|---|
| 104 - 64 = | 40 | *bit 6* |
| 40 - 32 = | 8 | *bit 5* |
| 8 - 8 = | 0 | *bit 3* |

$X = 01101000_{two}$

| $n$ | $2^n$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |

## Converting Decimal to Binary (2's C)

- Second Method: ***Subtract Powers of Two***

1. Find magnitude of decimal number.
2. Subtract largest power of two less than or equal to number.
3. Put a one in the corresponding bit position.
4. Keep subtracting until result is zero.
5. Append a zero as MS bit; if original was negative, take two's complement.

$X = 104_{ten}$

104 - 64 = 40 *bit 6*
40 - 32 = 8 *bit 5*
8 - 8 = 0 *bit 3*

$X = 01101000_{two}$

| *n* | $2^n$ |
|---|---|
| 0 | 1 |
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 16 |
| 5 | 32 |
| 6 | 64 |
| 7 | 128 |
| 8 | 256 |
| 9 | 512 |
| 10 | 1024 |





## Operations: Arithmetic and Logical

- Recall: data types include *representation* and *operations*.
- 2's complement is a good representation for signed integers, now we need arithmetic operations:
  - **Addition (including overflow)**
  - **Subtraction**
  - **Sign Extension**
- Multiplication and division can be built from these basic operations.
- Logical operations are also useful:
  - **AND**
  - **OR**
  - **NOT**





## Addition

- As we've discussed, 2's comp. addition is just binary addition.
  - assume all integers have the same number of bits
  - ignore carry out
  - for now, assume that sum fits in n-bit 2's comp. representation

| | | | | | |
|---|---|---|---|---|---|
| | **01101000** | (104) | | **11110110** | (-10) |
| + | **11110000** | (-16) | + | _____ | (-9) |
| | **01011000** | (98) | | | (-19) |

*Assuming 8-bit 2's complement numbers.*





## Subtraction

- Negate second operand, then add.
  - assume all integers have the same number of bits
  - ignore carry out
  - for now, assume that difference fits in n-bit 2's comp. representation

| | | | | | |
|---|---|---|---|---|---|
| | **01101000** | (104) | | **11110110** | (-10) |
| - | **00010000** | (16) | - | _____ | (-9) |
| | **01101000** | (104) | | **11110110** | (-10) |
| + | **11110000** | (-16) | + | _____ | (9) |
| | **01011000** | (88) | | | (-1) |

*Assuming 8-bit 2's complement numbers.*

## Sign Extension

- To add two numbers, we must represent them with the same number of bits.
- If we just pad with zeroes on the left:

| 4-bit | | 8-bit | |
|---|---|---|---|
| **0100** | (4) | **00000100** | (still 4) |
| **1100** | (-4) | **00001100** | (12, not -4) |

- Instead, replicate the MS bit -- the sign bit:

| 4-bit | | 8-bit | |
|---|---|---|---|
| **0100** | (4) | **00000100** | (still 4) |
| **1100** | (-4) | **11111100** | (still -4) |





## Overflow

- If operands are too big, then sum cannot be represented as an *n*-bit 2's comp number.

| | | | |
|---|---|---|---|
| **01000** | (8) | **11000** | (-8) |
| **+ 01001** | (9) | **+ 10111** | (-9) |
| **10001** | (-15) | **01111** | (+15) |

- We have overflow if:
  - signs of both operands are the same, and
  - sign of sum is different.
- Another test -- easy for hardware:
  - carry into MS bit does not equal carry out





## Logical Operations

- Operations on logical TRUE or FALSE
  - two states -- takes one bit to represent: TRUE=1, FALSE=0

| A | B | A AND B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

| A | B | A OR B |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |

| A | NOT A |
|---|---|
| 0 | 1 |
| 1 | 0 |

- View *n*-bit number as a collection of *n* logical values
  - operation applied to each bit independently





## Examples of Logical Operations

- AND
  - useful for clearing bits
    - AND with zero = 0
    - AND with one = no change

```
  11000101
AND 00001111
  00000101
```

- OR
  - useful for setting bits
    - OR with zero = no change
    - OR with one = 1

```
  11000101
OR 00001111
  11001111
```

- NOT
  - unary operation -- one argument
  - flips every bit

```
NOT 11000101
    00111010
```

## Hexadecimal Notation

- It is often convenient to write binary (base-2) numbers in hexadecimal (base-16) instead.
  - fewer digits - four bits per hex digit
  - less error prone - no long string of 1's and 0's

| Binary | Hex | Decimal | Binary | Hex | Decimal |
|---|---|---|---|---|---|
| 0000 | 0 | 0 | 1000 | 8 | 8 |
| 0001 | 1 | 1 | 1001 | 9 | 9 |
| 0010 | 2 | 2 | 1010 | A | 10 |
| 0011 | 3 | 3 | 1011 | B | 11 |
| 0100 | 4 | 4 | 1100 | C | 12 |
| 0101 | 5 | 5 | 1101 | D | 13 |
| 0110 | 6 | 6 | 1110 | E | 14 |
| 0111 | 7 | 7 | 1111 | F | 15 |





## Converting from Binary to Hexadecimal

- Every four bits is a hex digit.
  - start grouping from right-hand side

**011 1010 1000 1111 0100 1101 0111**

⇩ ⇩ ⇩ ⇩ ⇩ ⇩ ⇩

**3 A 8 F 4 D 7**

*This is not a new machine representation, just a convenient way to write the number.*





## Fractions: Fixed-Point

- How can we represent fractions?
  - Use a "binary point" to separate positive from negative powers of two -- just like "decimal point."
  - 2's comp addition and subtraction still work (if binary points are aligned)

$2^{-1} = 0.5$
$2^{-2} = 0.25$
$2^{-3} = 0.125$

**00101000.101** (40.625)
\+ **11111110.110** (-1.25)
**00100111.011** (39.375)

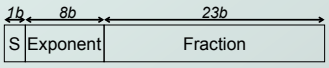
*No new operations -- same as integer arithmetic.*





## Very Large and Very Small: Floating-Point

- Large values: $6.023 \times 10^{23}$ -- requires 79 bits
- Small values: $6.626 \times 10^{-34}$ -- requires >110 bits
- Use equivalent of "scientific notation": $F \times 2^E$
- Must have F (*fraction*), E (*exponent*), and sign.
- IEEE 754 Floating-Point Standard (32-bits):




$N = (-1)^S \times 1.\text{fraction} \times 2^{\text{exponent}-127}, \quad 1 \le \text{exponent} \le 254$

$N = (-1)^S \times 0.\text{fraction} \times 2^{-126}, \quad \text{exponent} = 0$

## Floating Point Example

- Single-precision IEEE floating point number:
- 1 01111110 10000000000000000000000

  sign exponent fraction

  - Sign is 1 – number is negative.
  - Exponent field is 01111110 = 126 (decimal).
  - Fraction is 1.100000000000… = 1.5 (decimal).

- Value = $-1.5 \times 2^{(126-127)} = -1.5 \times 2^{-1} =$ **-0.75**





## Floating-Point Operations

- Will regular 2's complement arithmetic work for Floating Point numbers?
- (*Hint*: In decimal, how do we compute $3.07 \times 10^{12} + 9.11 \times 10^{8}$?)





## Text: ASCII Characters

- ASCII: Maps 128 characters to 7-bit code.
  - printable and non-printable (ESC, DEL, …) characters

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 00 | nul | 10 | dle | 20 | sp | 30 | 0 | 40 | @ | 50 | P | 60 | ` | 70 | p |
| 01 | soh | 11 | dc1 | 21 | ! | 31 | 1 | 41 | A | 51 | Q | 61 | a | 71 | q |
| 02 | stx | 12 | dc2 | 22 | " | 32 | 2 | 42 | B | 52 | R | 62 | b | 72 | r |
| 03 | etx | 13 | dc3 | 23 | # | 33 | 3 | 43 | C | 53 | S | 63 | c | 73 | s |
| 04 | eot | 14 | dc4 | 24 | $ | 34 | 4 | 44 | D | 54 | T | 64 | d | 74 | t |
| 05 | enq | 15 | nak | 25 | % | 35 | 5 | 45 | E | 55 | U | 65 | e | 75 | u |
| 06 | ack | 16 | syn | 26 | & | 36 | 6 | 46 | F | 56 | V | 66 | f | 76 | v |
| 07 | bel | 17 | etb | 27 | ' | 37 | 7 | 47 | G | 57 | W | 67 | g | 77 | w |
| 08 | bs | 18 | can | 28 | ( | 38 | 8 | 48 | H | 58 | X | 68 | h | 78 | x |
| 09 | ht | 19 | em | 29 | ) | 39 | 9 | 49 | I | 59 | Y | 69 | i | 79 | y |
| 0a | nl | 1a | sub | 2a | * | 3a | : | 4a | J | 5a | Z | 6a | j | 7a | z |
| 0b | vt | 1b | esc | 2b | + | 3b | ; | 4b | K | 5b | [ | 6b | k | 7b | { |
| 0c | np | 1c | fs | 2c | , | 3c | < | 4c | L | 5c | \ | 6c | l | 7c | \| |
| 0d | cr | 1d | gs | 2d | - | 3d | = | 4d | M | 5d | ] | 6d | m | 7d | } |
| 0e | so | 1e | rs | 2e | . | 3e | > | 4e | N | 5e | ^ | 6e | n | 7e | ~ |
| 0f | si | 1f | us | 2f | / | 3f | ? | 4f | O | 5f | _ | 6f | o | 7f | del |





## Text: ASCII Characters

- ASCII is a seven-bit code. "Eight-bit ASCII" makes as sense as a square circle.
- There is no need to memorize the ASCII chart.
- There is no need to insert ASCII values into a program.
  - if (c >= 65 && c <= 90) … // just showing off
  - if (c >= 'A' && c <= 'Z') … // easy to understand
  - if ('A' <= c && c <= 'Z') … // I like this even more

## Interesting Properties of ASCII Code

- What is relationship between a decimal digit ('0', '1', …) and its ASCII code?
- What is the difference between an upper-case letter ('A', 'B', …) and its lower-case equivalent ('a', 'b', …)?
- Given two ASCII characters, how do we tell which comes first in alphabetical order?
- Are 128 characters enough? (http://www.unicode.org/)

*No new operations needed for ASCII codes – integer arithmetic and logic are sufficient.*





## Other Data Types

- Text strings
  - array of characters, terminated with null character ('\0')
  - typically, no hardware support
- Image
  - array of pixels
    - monochrome: one bit (0/1 = black/white)
    - color: red, green, blue (RGB) components
    - other properties: transparency
  - hardware support:
    - typically none, in general-purpose processors
    - MMX -- multiple 8-bit operations on 32-bit word
- Sound
  - sequence of fixed-point numbers





## LC-3 Data Types

- Some data types are supported directly by the instruction set architecture.
- For LC-3, there is only one hardware-supported data type:
  - 16-bit 2's complement signed integer
  - Operations: ADD, AND, NOT
- Other data types are supported by interpreting 16-bit values as logical, text, fixed-point, floating-point, etc., in the software that we write.