Making Change

Coin Changing

Goal. Given currency **integer** denominations: {100, 25, 10, 5, 1} devise a method to pay **integer** amount to customer using the **fewest number of coins.**

Example: 34¢.

Cashier's algorithm. At each iteration, add coin of the largest value that does not take us past the amount to be paid.

Example:
$2.89 = 289¢.

Coin-Changing: Greedy Algorithm

Cashier's algorithm. Use the maximal number of the largest denomination

```
x - amount to be changed
Sort coins denominations by value: c1 < c2 < … < cn.
S ← ϕ      ← coins selected
while (x > 0) {
   let k be largest integer such that ck ≤ x
   if (k = 0)
      return "no solution found"
   x ← x - ck
   S ← S ∪ {k}
}
return S
```

Does this algorithm always work?

Coin-Changing: Greedy doesn't always work

Greedy algorithm works for US coins
  Proof: number theory

Greedy fails changing **30 optimally** with coin set
  {25, 10, 1}

Greedy fails changing **30 at all** with coin set
  {25, 10}

Different problem: number of ways to pay

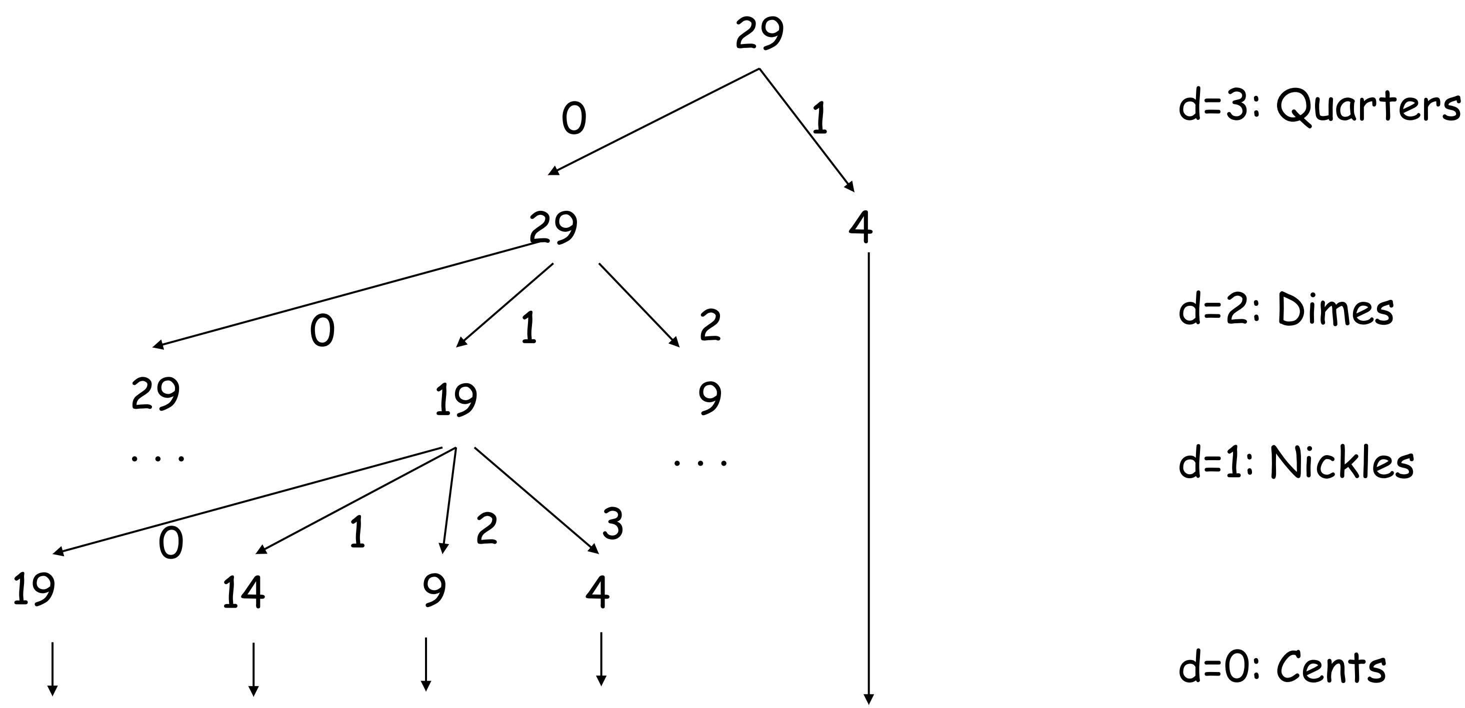
Given a coin set c = {c_0, c_1, ..., c_{d-1}} and an amount M, how many different ways can M be paid?
Recursive solution: is this a take / don't take type of problem?

e.g., for eg 56 cents I can use 0, 1, or 2 quarters

One possible (not the only) solution
Base:
if d == 0, how many ways? (is there always a way ?)
Step:
if d>0
at least how many c_d coins can be used
and which problem then remains to be solved?
...
at most how many c_d coins can be used
and which problem then remains to be solved?

Now turn Recursive into Dynamic Programming

Making Change Recursive



Compare it to Subset Sum Dynamic Programming

Go through the state space bottom-up: i=1 to n

- select coin type i,
  - first 1 coin type, then 2, ..., all coin types
  - what does the first column look like?
- use solutions of smaller sub-problems to efficiently compute solutions of larger ones
  - in sss / knapsack there are 2 sub-problems
  - in coins there are how many?