Function Ratios & Bounds Relations
CS 320, Fall 2017
Function Ratios

Problem: We have 2 functions, \( f(n) \) and \( g(n) \) and we want to determine their bounds relation: \( \Theta, \omega, \Omega, o, O \)
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In other words, what is their relation to each other as their input size gets bigger and bigger?
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In other words, what is their relation to each other as their input size gets bigger and bigger?

A lower bound will always be smaller, and upper bound will always be larger, and a $\Theta$ bound will roughly “be the same”
What does this tell us about the relative growth of $g(n)$ compared to $f(n)$?
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$g(n)$ grows faster!
\[
\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0
\]

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So what bound is this? Lower? Upper?
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So what bound is this? Lower? Upper?
We can say that \( f(n) \) is not faster growing than \( g(n) \), but it could be growing the same at some points:

\[
f(n) = o(g(n)) \text{ which implies that } f(n) = O(g(n))
\]
What does this tell us about the relative growth of $g(n)$ compared to $f(n)$?

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\lim_{n \to \infty} \frac{f(n)}{g(n)} = \infty
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\( g(n) \) grows slower!
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\lim_{{n \to \infty}} \frac{f(n)}{g(n)} = \infty
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So what bound is this? Lower? Upper?
What does this tell us about the relative growth of $g(n)$ compared to $f(n)$?

$g(n)$ grows slower!

So what bound is this? **Lower?**  **Upper?**
We can say that $f(n)$ is not slower growing than $g(n)$, but it could be growing the same at some points:

$$f(n) = \omega(g(n))$$
which means that $f(n) = \Omega(g(n))$
\[ \lim_{n \to \infty} \frac{f(n)}{g(n)} = C \text{ and } C > 0 \]

What does this tell us about the relative growth of \( g(n) \) compared to \( f(n) \)?
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\lim_{n \to \infty} \frac{f(n)}{g(n)} = C \text{ and } C > 0
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What does this tell us about the relative growth of \(g(n)\) compared to \(f(n)\)?

They grow the same way with a constant difference between them!
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What does this tell us about the relative growth of \( g(n) \) compared to \( f(n) \)?

They grow the same way with a constant difference between them!

So what bound is this? Lower? Upper?
Here we have a clear case where \( g(n) \) is both the upper and lower bound for \( f(n) \):

\[ f(n) = \Theta(g(n)) \]