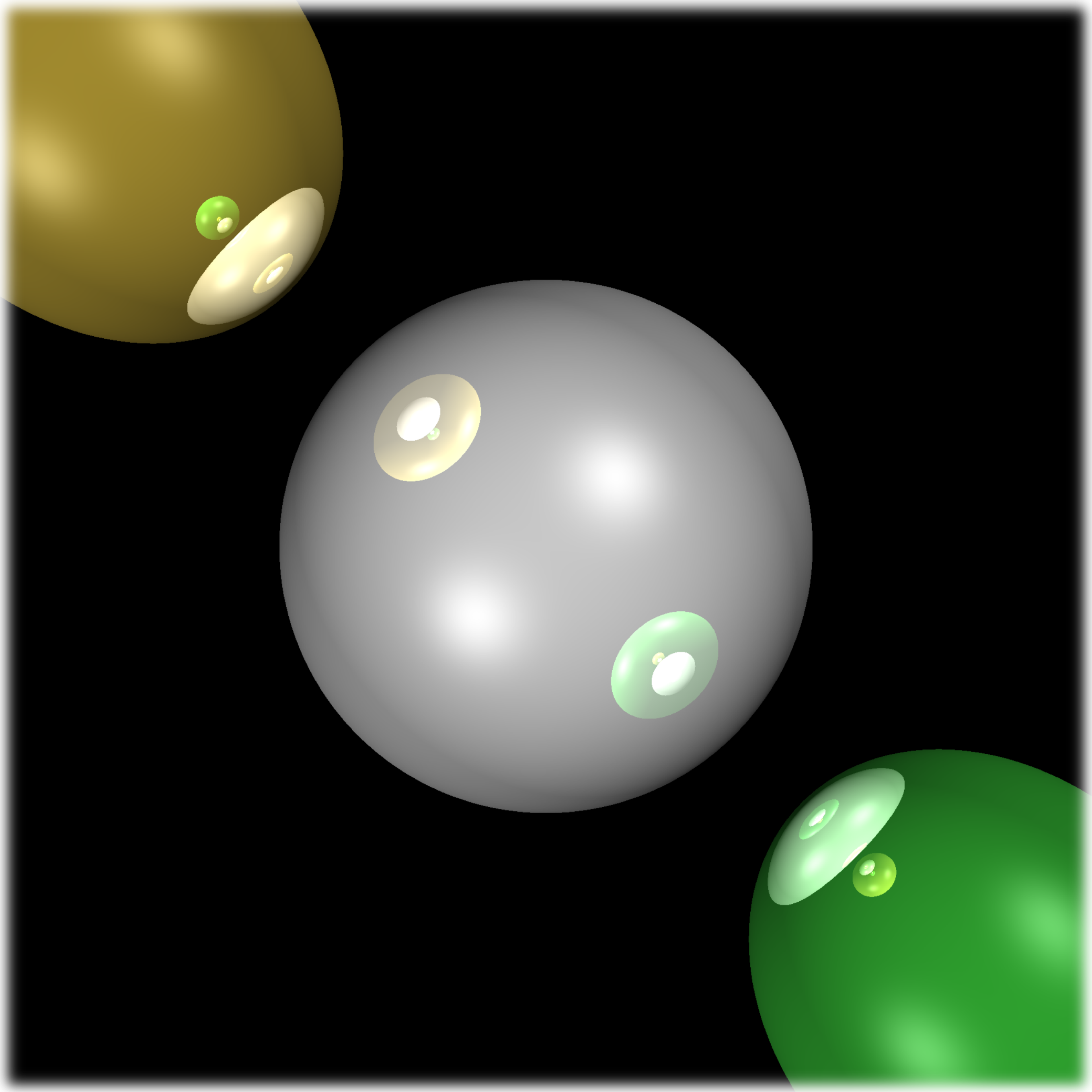
Lecture 15:
Phong Reflection
Recursive Reflection

October 10, 2019

Today's Goal



Specular Reflection

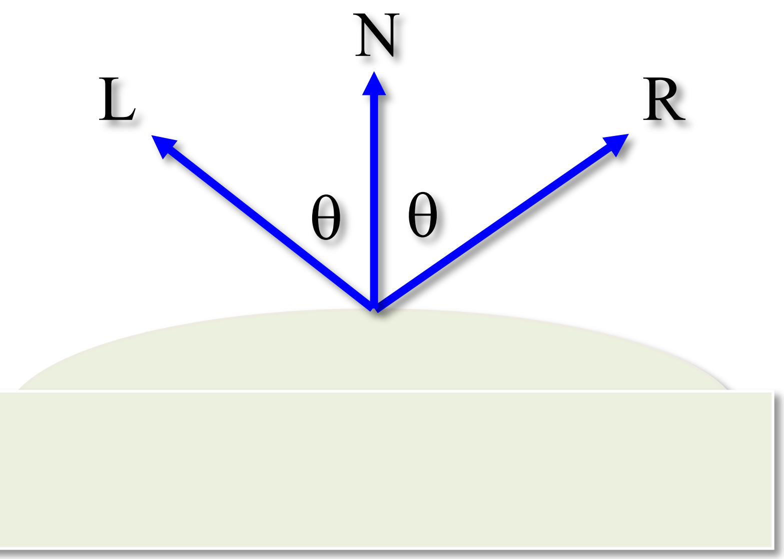
- Think about reflection and consider
- Two extremes:
  - 1: a diffuse surface,
  - 2: a mirror.
- Specular reflection component
  - Mirrors are extreme (2^{nd} half of this lecture)
  - Mirrors reflect about a single angle
- Metals, for example, combine
  - Diffuse and specular components.

Only Skin Deep

- Diffuse models light "deep" reflection
  - Enters through micro-holes in the surface
  - Bounced from facet to facet,
    - Possibly changing color
  - Exits at a random angle
- Specular reflection is "surface" reflection
  - Light hits a single surface facet, skips off.
  - The color of the light is unchanged.
  - The exit angle depends on the entry angle.
    - Catchy Phrase – ***equal and opposite***.

Limiting Case - Mirror

- Angle of incidence = Angle of reflection.
- Idealized form only applies to mirrors.



Reflection – Mirror Like Surface

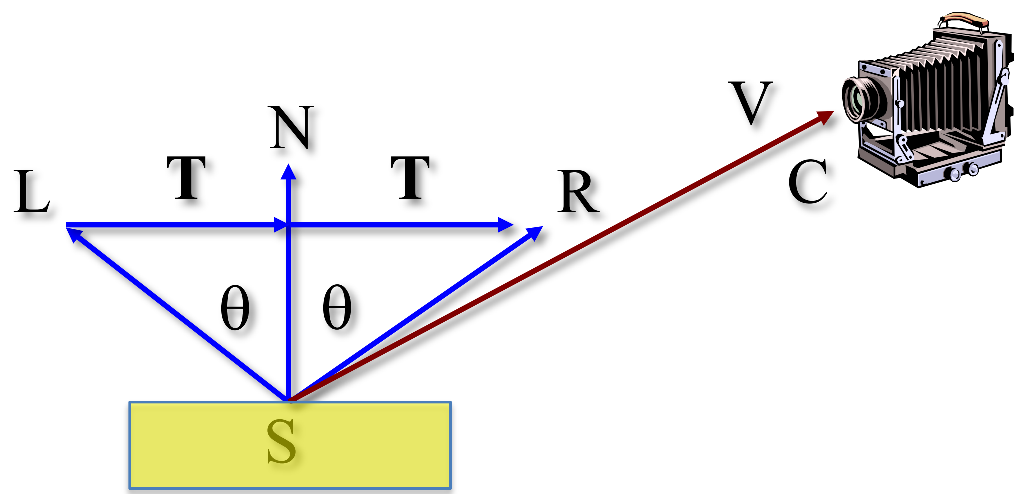
Light Source Reflecting

Reflection and Viewer

- V is the unit vector from the surface point to the camera:

  V = (C – S)/|C – S|

- Calculating R is more involved



Reflection Vector

- Let NL be the projection of L onto N
  - Assume N is a unit vector

    $NL = (L \cdot N)N$
- T is defined as

    $T = NL - L = (L \cdot N)N - L$
- R is L + 2T

$$R = L + 2\big((L \cdot N)N - L\big)$$

$$= 2(L \cdot N)N - L$$

Back to Perfect (for now)

- If V $\neq$ R, there is no (pure) specular reflection.
- If V = R, then ….

- Note that k_s is a scalar, measuring the percent of light reflected.
  – Color reflected is that of the light source.

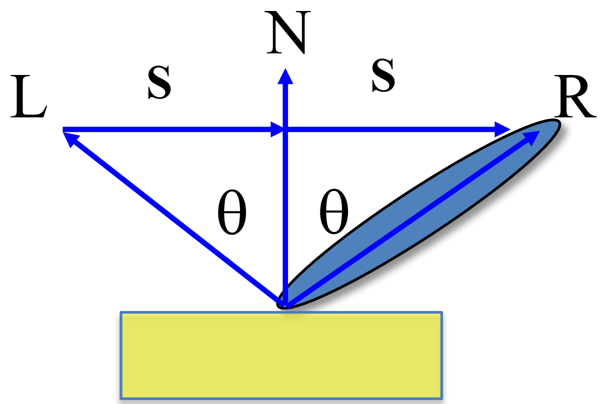
$$I = k_s B$$

$$= k_s \begin{vmatrix} b_r \\ b_g \\ b_b \end{vmatrix}$$

Warning: This is the correct mental model, it is not useful in practice.

Close Counts – Phong Reflection

- In Phong's model, reflection is strongest in the direction of the angle of reflection
- Drops off with the cosine of the deviation from the angle of reflection



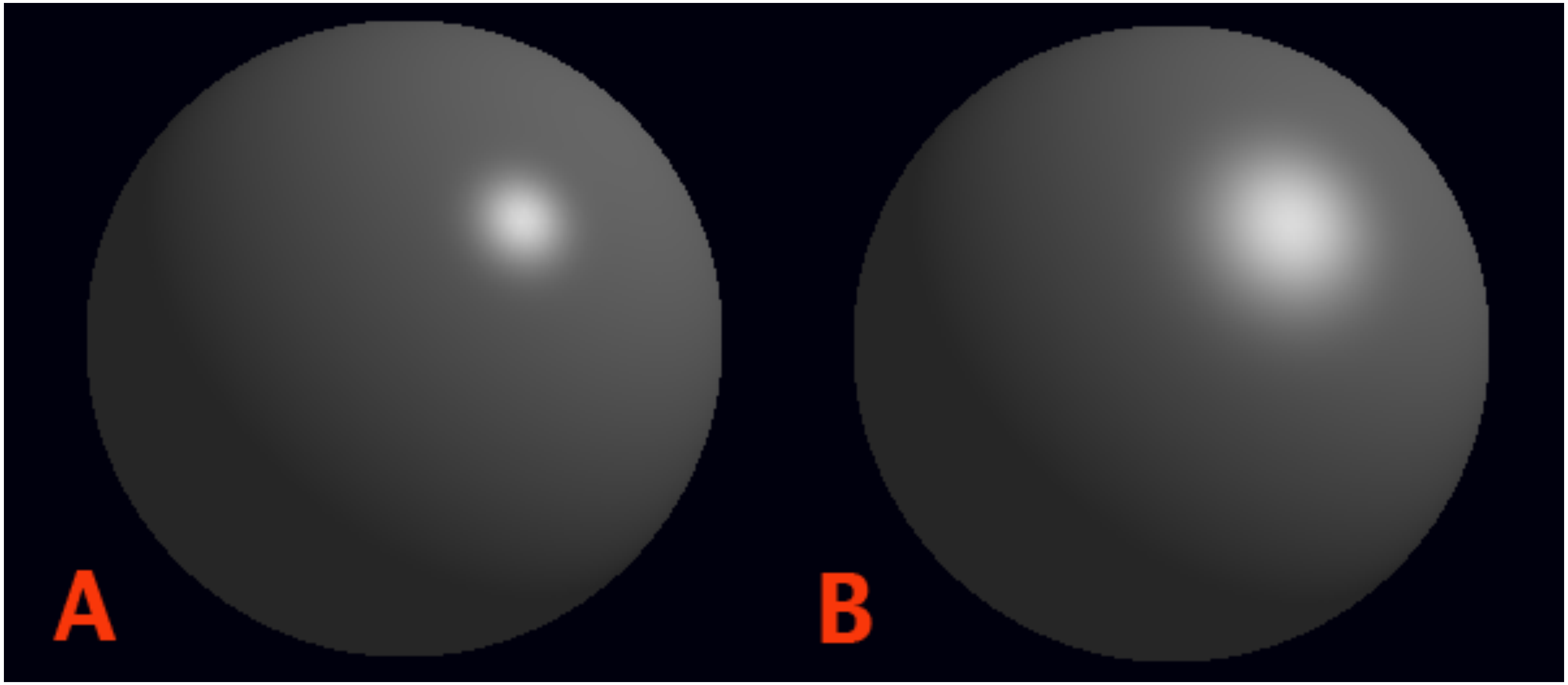
Phong Specular Highlights

$$I = k_s B(\cos\Phi)^\alpha$$

- Φ is the angle between the viewing ray and the angle of reflection.
- α is the so-called Phong constant, expressing how "shiny" an object is
  - Mirrors: α = 200
  - Dull objects: α = 5 … 50

Appearance with Changing α

- Consider which has the larger alpha?



Phong - The Algebra

- $\cos(\Phi) = V \cdot R = V \cdot (2(L \cdot N)N - L)$

$$I = k_s B \left(V \cdot R \right)^{\alpha}$$

$$I = k_s B \left(V \cdot \left(2 \left(L \cdot N \right) N - L \right) \right)^{\alpha}$$

Total Reflectance

- Reflectance off a surface point is the sum of:
- Reflection from the ambient light
- Diffuse reflection off of every point light
- Specular reflection off of every point light

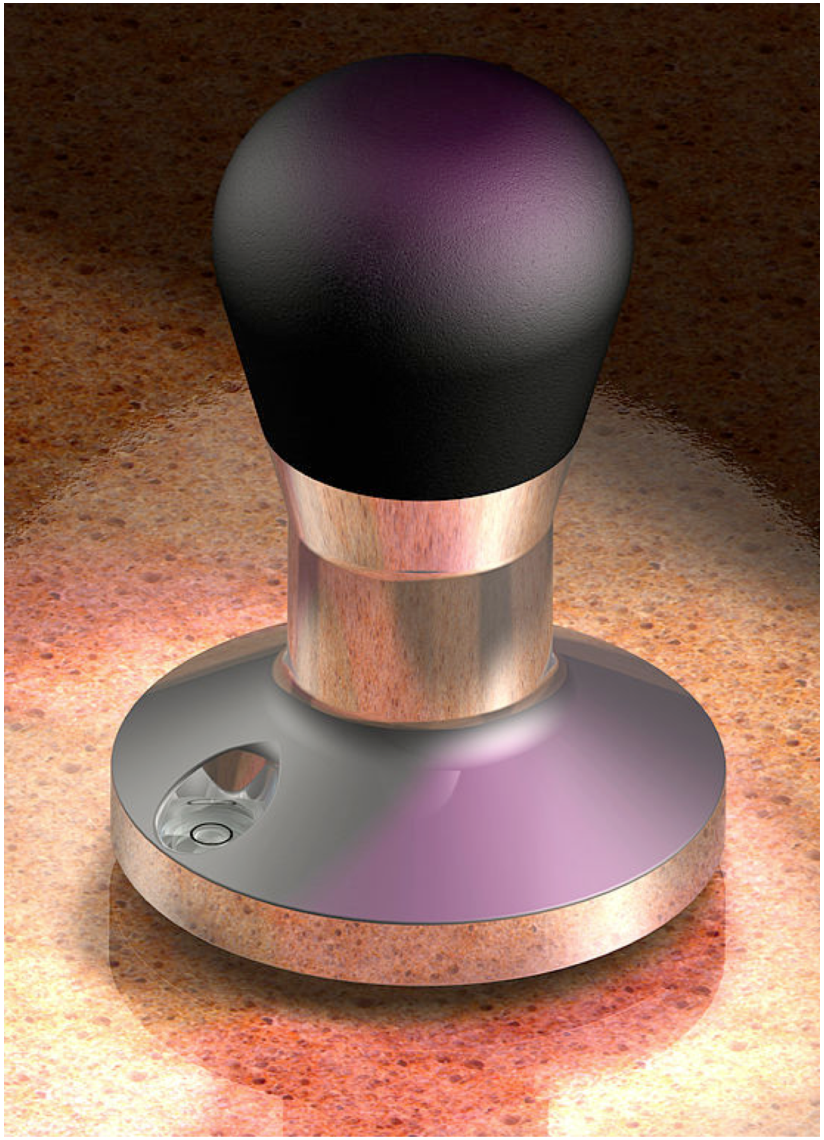
$$I = K_a B_a + \sum_{i \in lights} \left(K_d B_i \left(L_{ip} \cdot N \right) + k_s B_i \left(V \cdot R_p \right)^{\alpha} \right)$$

$$R = 2(L \cdot N)N - L$$

How about Reflections?

- Note reflections
- Granite tabletop
- Visible on base
- Also on handle

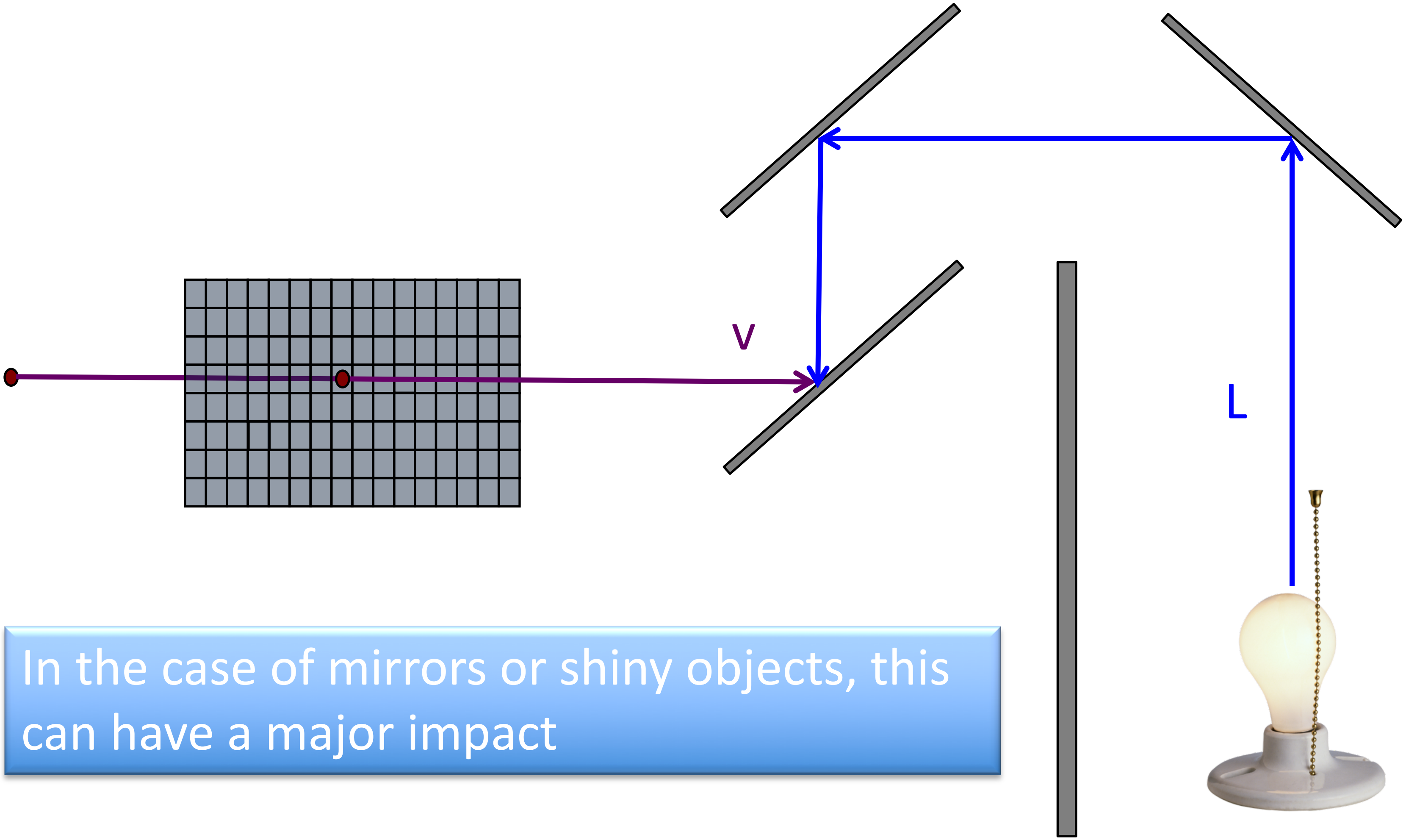
This is a featured picture on the English language Wikipedia (Featured pictures) and is considered one of the finest images. (October 2012)



Rationale for Interreflection

- Not all the light striking a surface comes directly from a light source.
- Some reflects from one surface onto another.
- We ignore diffuse reflected light:
  - Because its small, and we can get away with it
  - Because it is very expensive to compute
- Specular reflection much more sensitive
  - Just consider reflections in previous image.

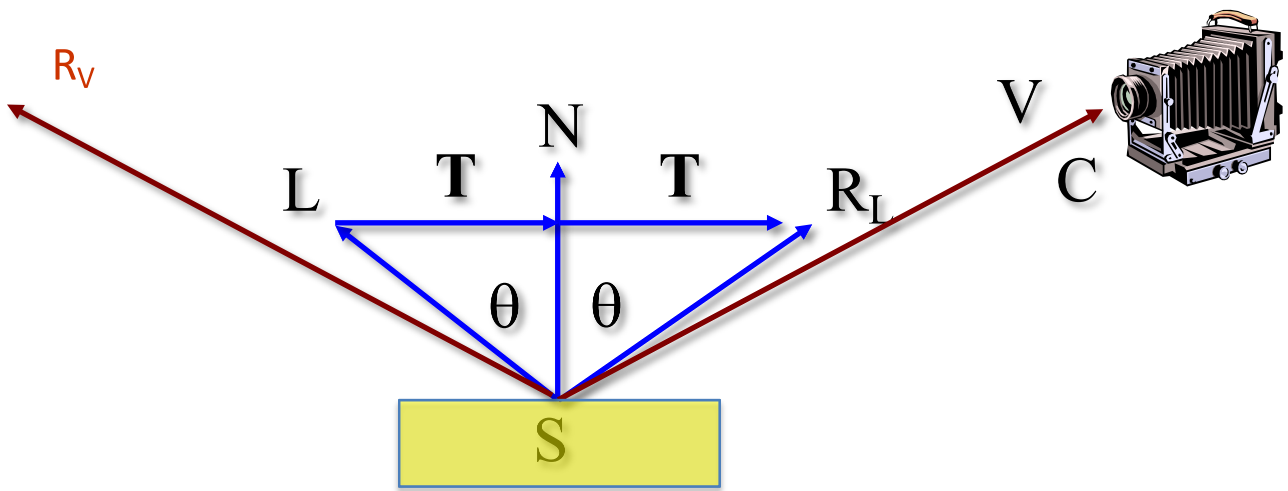
Interreflections



In the case of mirrors or shiny objects, this can have a major impact

Rays of Reflection

- To add interreflections, we need the light hitting the surface from the reflected viewing ray.



- Add to ambient, diffuse and specular

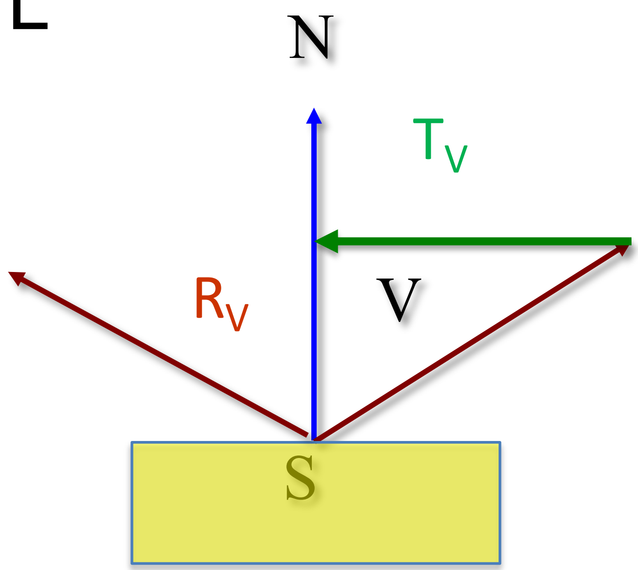
Computing V_R

- $R_V = 2(V \cdot N)N - V$
  - Just like R_L, but V replaces L
- To be more detailed…

$N_V = (V \cdot N)N$

$T_V = N_V - V = (V \cdot N)N - V$

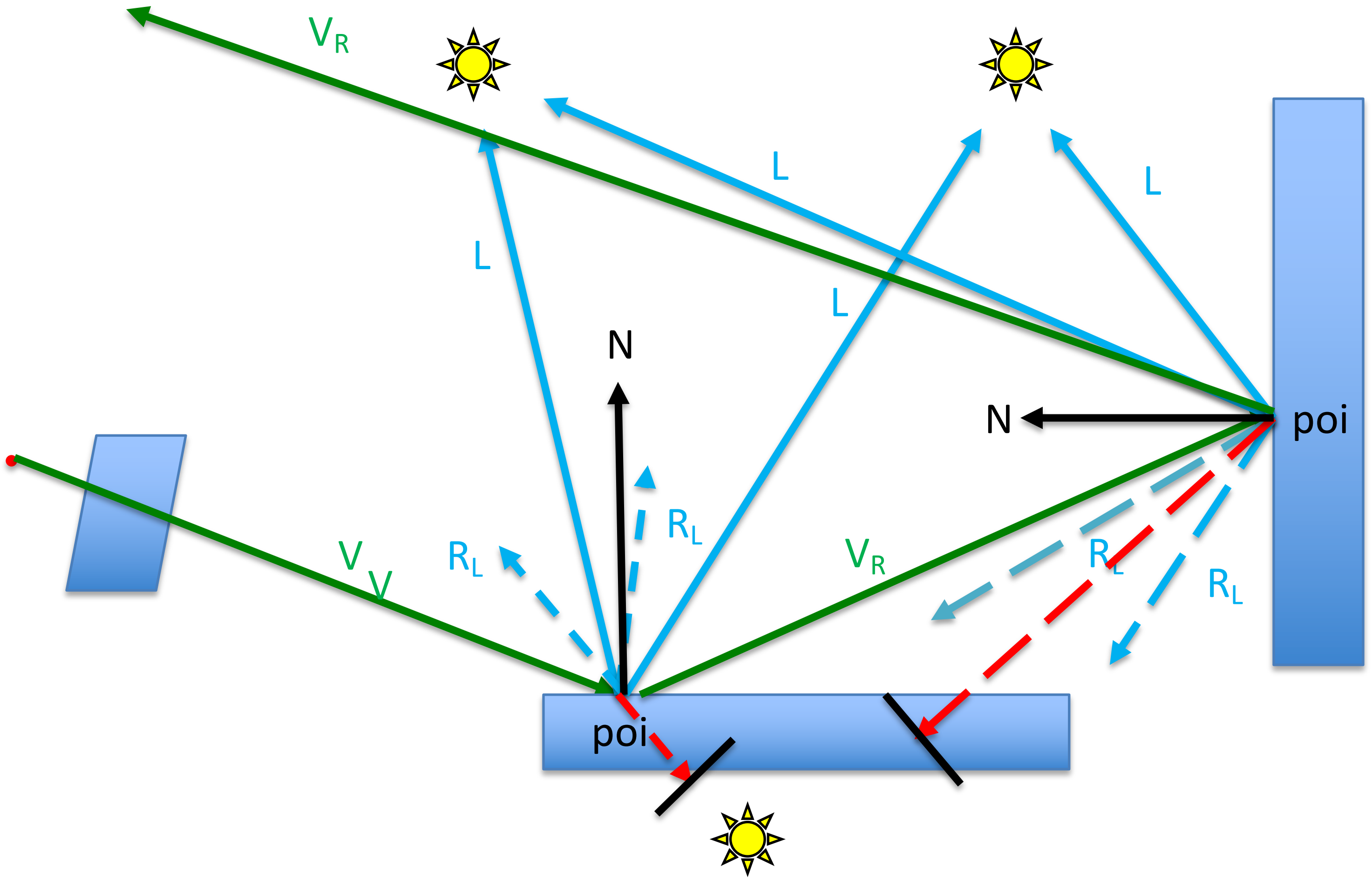
$R_V = V + 2T_V = 2(V \cdot N)N - V$



Recursive Ray Tracing

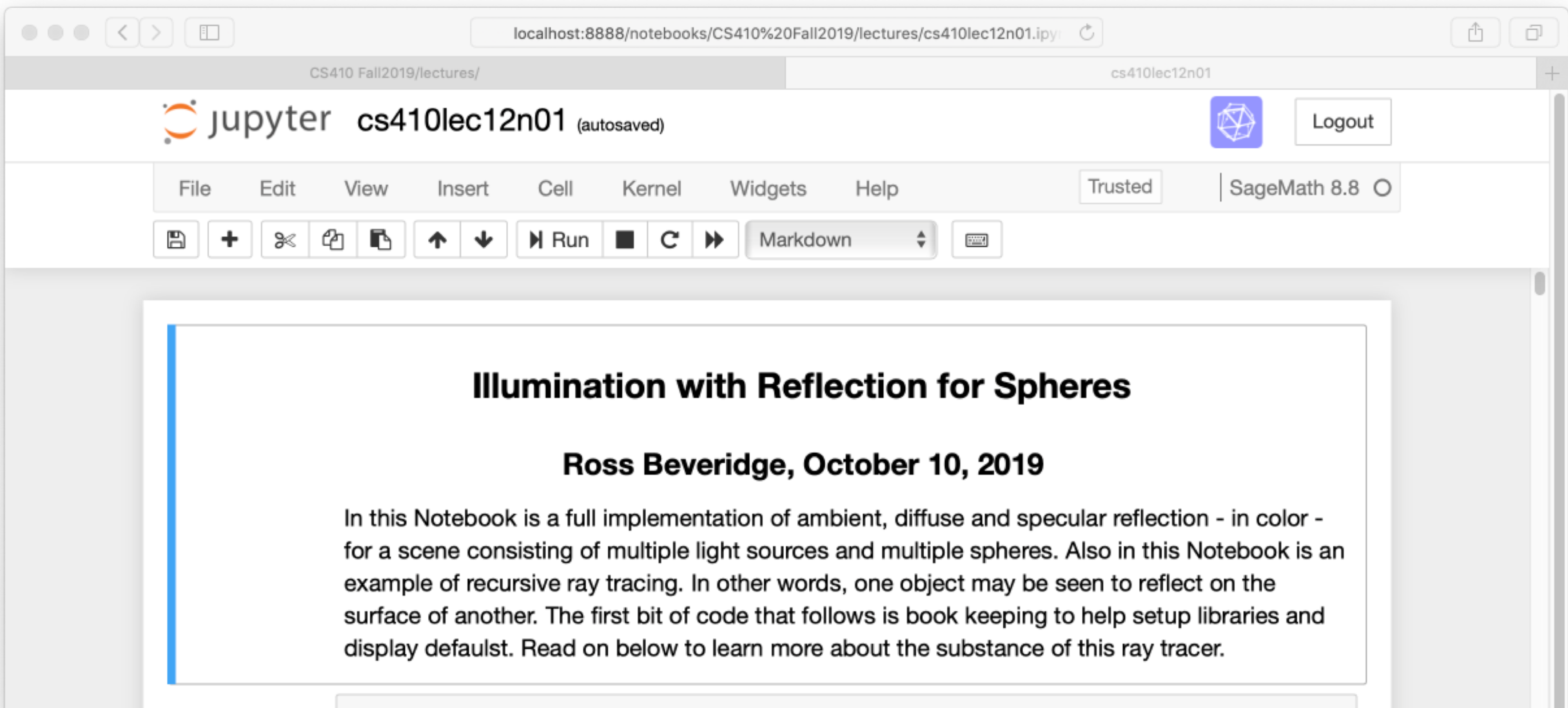
- Generalize ray trace – light from a ray
  - Ray leaves surface from point of intersection
  - In the direction of R_V
- And how does it compute illumination?
- Exactly as we did it before, when …
  - Starting at the pixel
  - In the direction of V
- So ray tracing is recursive!

Illustrating Recursive Ray Tracing

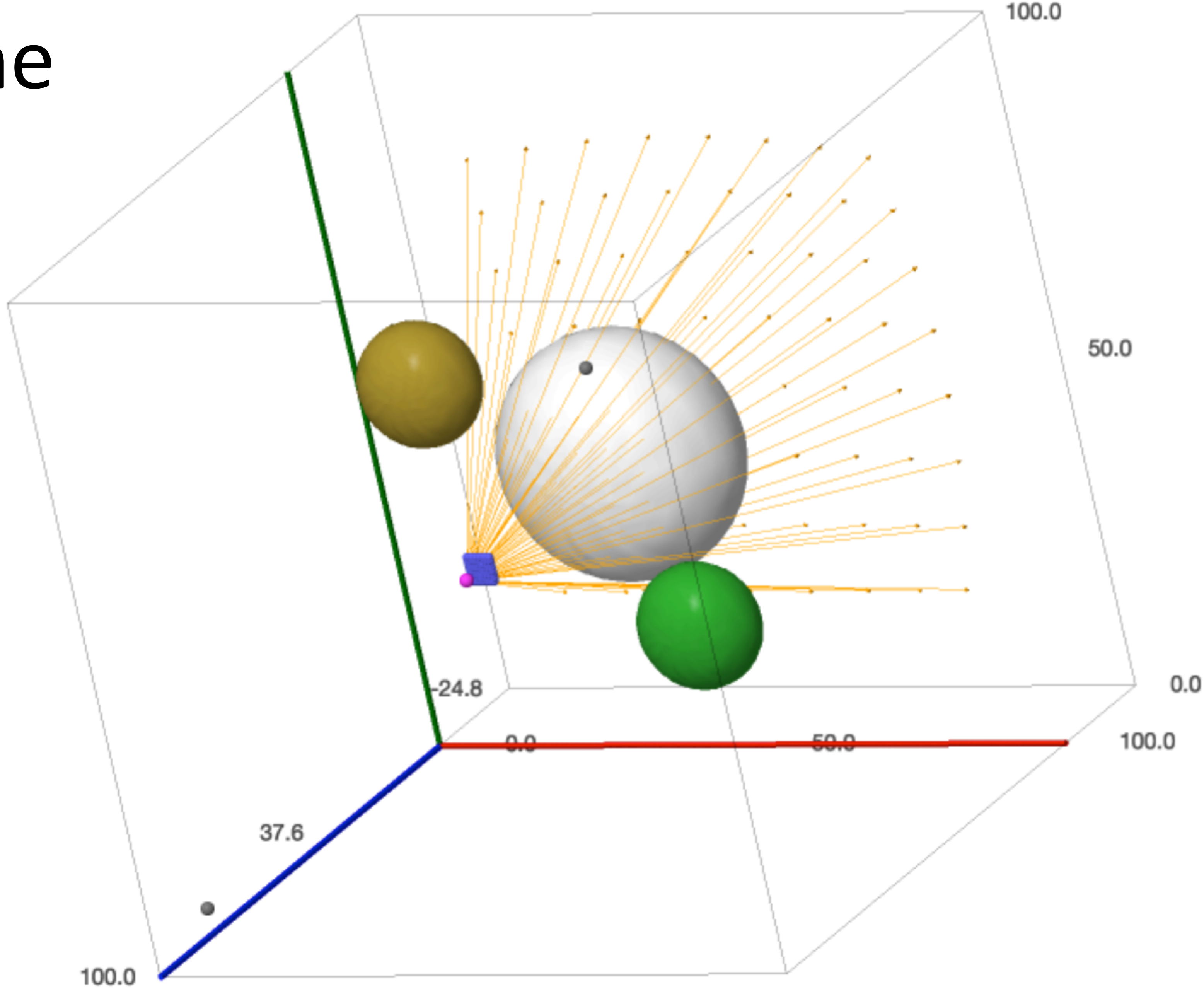


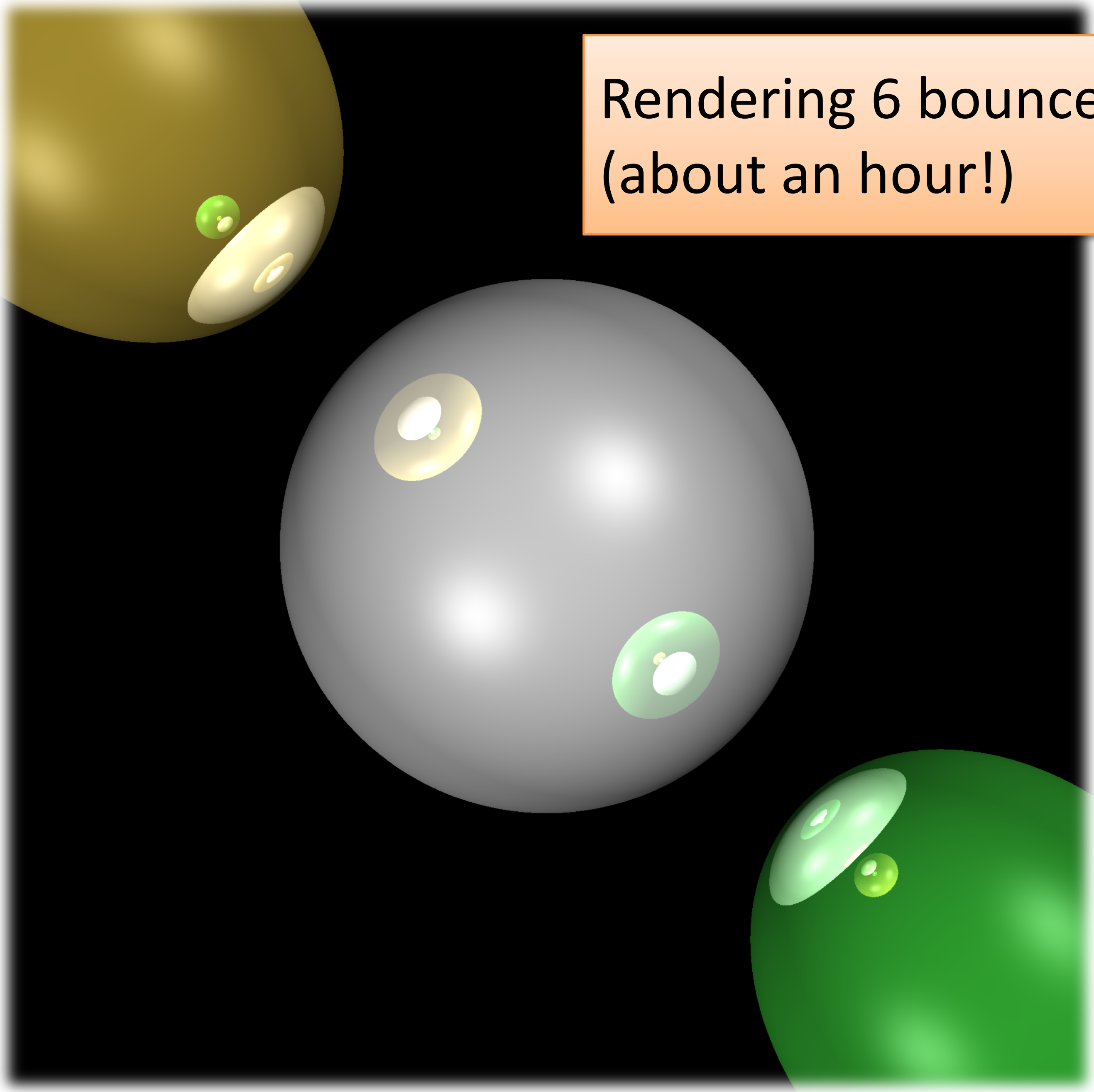
Now in SageMath

- Complete implementation in SageMath
- Scene with three spheres and interreflection
- Can modify recursion depth



Scene





Rendering 6 bounces at 2048 x 2048 (about an hour!)