Lecture 24: Bicubic Surfaces & Splines

December 5, 2019

Parametric Bicubic Surfaces

- The goal is to go from curves in space to curved surfaces in space.
- To do this, we will parameterize a surface in terms of two free parameters, s & t
- We will extend the Bezier curve in detail.
- Other surfaces are similar in concept.

Building a Bezier Surface Patch

1) Imagine a Bezier curve Gb1(t) in space.

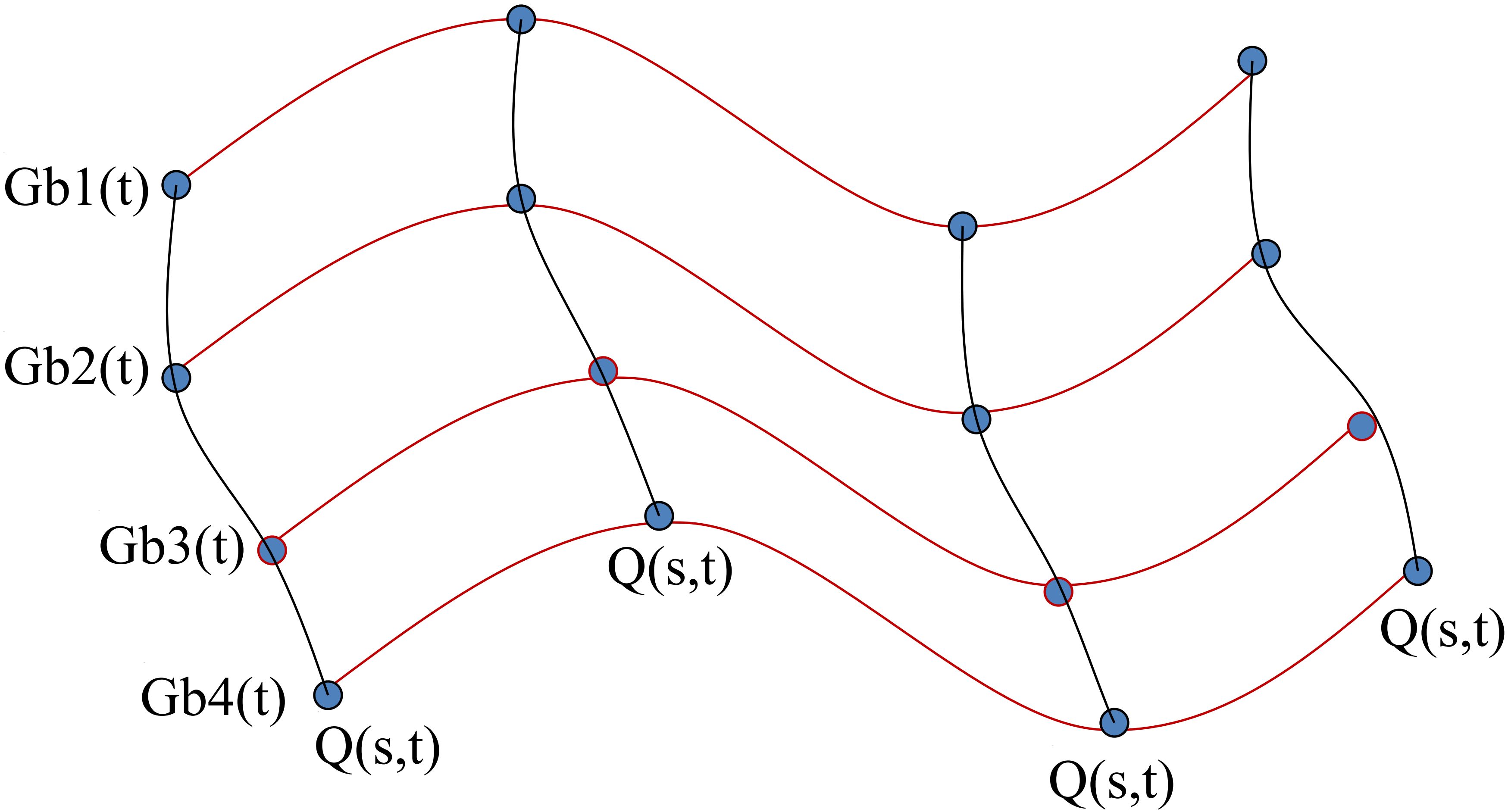
2) Imagine three more Bezier curves, Gb2(t) Gb3(t) and Gb4(t)

3) Let all four curves be parameterized by a single t

4) For t=0, we have four points: Gb1(0), Gb2(0), Gb3(0) and Gb4(0). Use these as the control points for another Bezier curve.

5) Repeat step #4 for all values of t

Bicubic Surfaces



Mapping from (s,t) to (x,y,z)

Basic Math – Version 1

$$Q(s,t) = S\,Mb\,Gb(t)$$

$$S = \begin{bmatrix} s^3 & s^2 & s & 1 \end{bmatrix}, T = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix}$$

$$Mb = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}, Gb(t) = \begin{bmatrix} Gb1(t) \\ Gb2(t) \\ Gb3(t) \\ Gb4(t) \end{bmatrix}$$

$$Gbi(t) = T\,Mb\,Gi$$

… and the Geometry

Specify four 4x3 geometry matrices, one for per curve.

$$G1 = \begin{bmatrix} x_{1,1} & y_{1,1} & z_{1,1} \\ x_{1,2} & y_{1,2} & z_{1,2} \\ x_{1,3} & y_{1,3} & z_{1,3} \\ x_{1,4} & y_{1,4} & z_{1,4} \end{bmatrix} G2 = \begin{bmatrix} x_{2,1} & y_{2,1} & z_{2,1} \\ x_{2,2} & y_{2,2} & z_{2,2} \\ x_{2,3} & y_{2,3} & z_{2,3} \\ x_{2,4} & y_{2,4} & z_{2,4} \end{bmatrix} G3 = \begin{bmatrix} x_{3,1} & y_{3,1} & z_{3,1} \\ x_{3,2} & y_{3,2} & z_{3,2} \\ x_{3,3} & y_{3,3} & z_{3,3} \\ x_{3,4} & y_{3,4} & z_{3,4} \end{bmatrix} G4 = \begin{bmatrix} x_{4,1} & y_{4,1} & z_{4,1} \\ x_{4,2} & y_{4,2} & z_{4,2} \\ x_{4,3} & y_{4,3} & z_{4,3} \\ x_{4,4} & y_{4,4} & z_{4,4} \end{bmatrix}$$

Here is the first Bezier curve of the four.

$$Gb1\,(t)^T = \begin{bmatrix} \left(-t^3+3\,t^2-3\,t+1\right) x_{1,1} + \left(3\,t^3-6\,t^2+3\,t\right) x_{1,2} + \left(-3\,t^3+3\,t^2\right) x_{1,3} + t^3 x_{1,4} \\ \left(-t^3+3\,t^2-3\,t+1\right) y_{1,1} + \left(3\,t^3-6\,t^2+3\,t\right) y_{1,2} + \left(-3\,t^3+3\,t^2\right) y_{1,3} + t^3 y_{1,4} \\ \left(-t^3+3\,t^2-3\,t+1\right) z_{1,1} + \left(3\,t^3-6\,t^2+3\,t\right) z_{1,2} + \left(-3\,t^3+3\,t^2\right) z_{1,3} + t^3 z_{1,4} \end{bmatrix}$$

The surface …

Recall we are building three functions of two variables.

$$Q(s,t)^T = \begin{bmatrix} x(s,t) \\ y(s,t) \\ z(s,t) \end{bmatrix}$$

Look just at the first – the x coordinate - function …

$$\begin{aligned} x(s,t) &= \underline{\left(-s^3+3s^2-3s+1\right)}\left(\left(-t^3+3t^2-3t+1\right)x_{1,1}+\left(3t^3-6t^2+3t\right)x_{1,2}+\left(-3t^3+3t^2\right)x_{1,3}+t^3\right. \\ &+ \underline{\left(3s^3-6s^2+3s\right)}\left(\left(-t^3+3t^2-3t+1\right)x_{2,1}+\left(3t^3-6t^2+3t\right)x_{2,2}+\left(-3t^3+3t^2\right)x_{2,3}+t^3x_{2,4}\right. \\ &+ \underline{\left(-3s^3+3s^2\right)}\left(\left(-t^3+3t^2-3t+1\right)x_{3,1}+\left(3t^3-6t^2+3t\right)x_{3,2}+\left(-3t^3+3t^2\right)x_{3,3}+t^3x_{3,4}\right) \\ &+ \underline{s^3}\left(\left(-t^3+3t^2-3t+1\right)x_{4,1}+\left(3t^3-6t^2+3t\right)x_{4,2}+\left(-3t^3+3t^2\right)x_{4,3}+t^3x_{4,4}\right) \end{aligned}$$

Alternative Decomposition

Break apart the x, y and z parts of the surface patch Q

$$Q_x(u,v) = \begin{vmatrix} v^3 & v^2 & v & 1 \end{vmatrix} \begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{vmatrix} \begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} u^3 \\ u^2 \\ u \\ 1 \end{vmatrix}$$

$$Q_y(u,v) = \begin{vmatrix} v^3 & v^2 & v & 1 \end{vmatrix} \begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{vmatrix} \begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} u^3 \\ u^2 \\ u \\ 1 \end{vmatrix}$$

$$Q_z(u,v) = \begin{vmatrix} v^3 & v^2 & v & 1 \end{vmatrix} \begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{vmatrix} \begin{vmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix} \begin{vmatrix} u^3 \\ u^2 \\ u \\ 1 \end{vmatrix}$$

Example Geometry

$$Gx = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$$Gy = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 \\ 3 & 3 & 3 & 3 \end{bmatrix}$$

$$Gz = \begin{bmatrix} 5 & 6 & 3 & 5 \\ 3 & 5 & 6 & 3 \\ 4 & 3 & 5 & 6 \\ 5 & 4 & 3 & 5 \end{bmatrix}$$

$$Points = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} & \begin{bmatrix} 1 \\ 0 \\ 6 \end{bmatrix} & \begin{bmatrix} 2 \\ 0 \\ 3 \end{bmatrix} & \begin{bmatrix} 3 \\ 0 \\ 5 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix} & \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix} & \begin{bmatrix} 3 \\ 1 \\ 3 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix} & \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} & \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} & \begin{bmatrix} 3 \\ 2 \\ 6 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 3 \\ 5 \end{bmatrix} & \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} & \begin{bmatrix} 2 \\ 3 \\ 3 \end{bmatrix} & \begin{bmatrix} 3 \\ 3 \\ 5 \end{bmatrix} \end{bmatrix}$$

… here it is in algebra

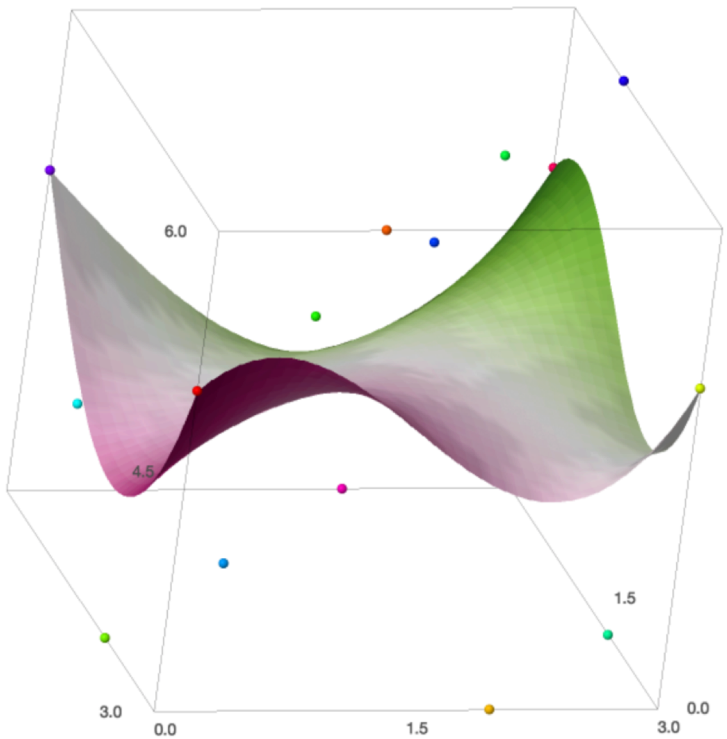
Note the very simple form of the x and y components.

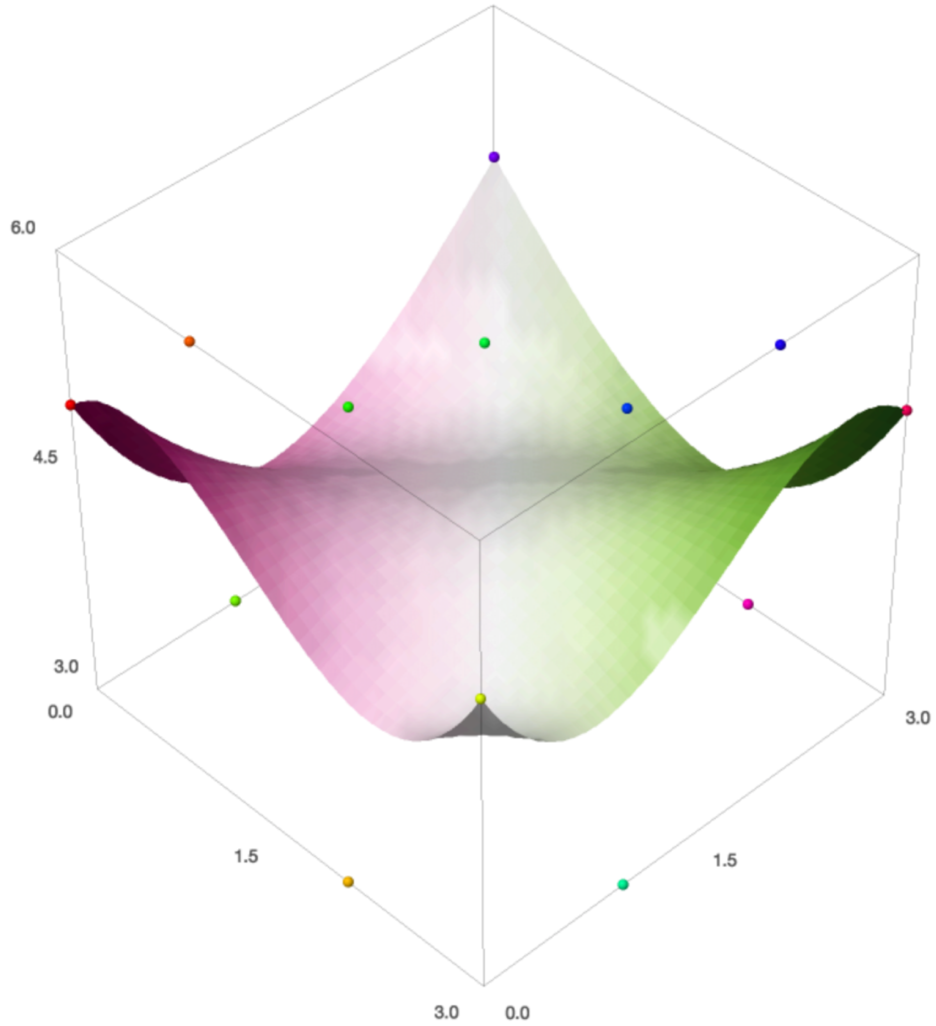
$$
\begin{aligned}
x(s,t) &= 3t \\
y(s,t) &= 3s \\
z(s,t) &= \left(-3t^3 - 24t^2 - 3 + 21t\right)s^3 + \left(33t^3 + 9t^2 + 9 - 36t\right)s^2 \\
&+ \left(-36t^3 + 27t^2 - 6 + 9t\right)s + 9t^3 + 5 - 12t^2 + 3t
\end{aligned}
$$

Can you relate the x and y forms back to the geometry?

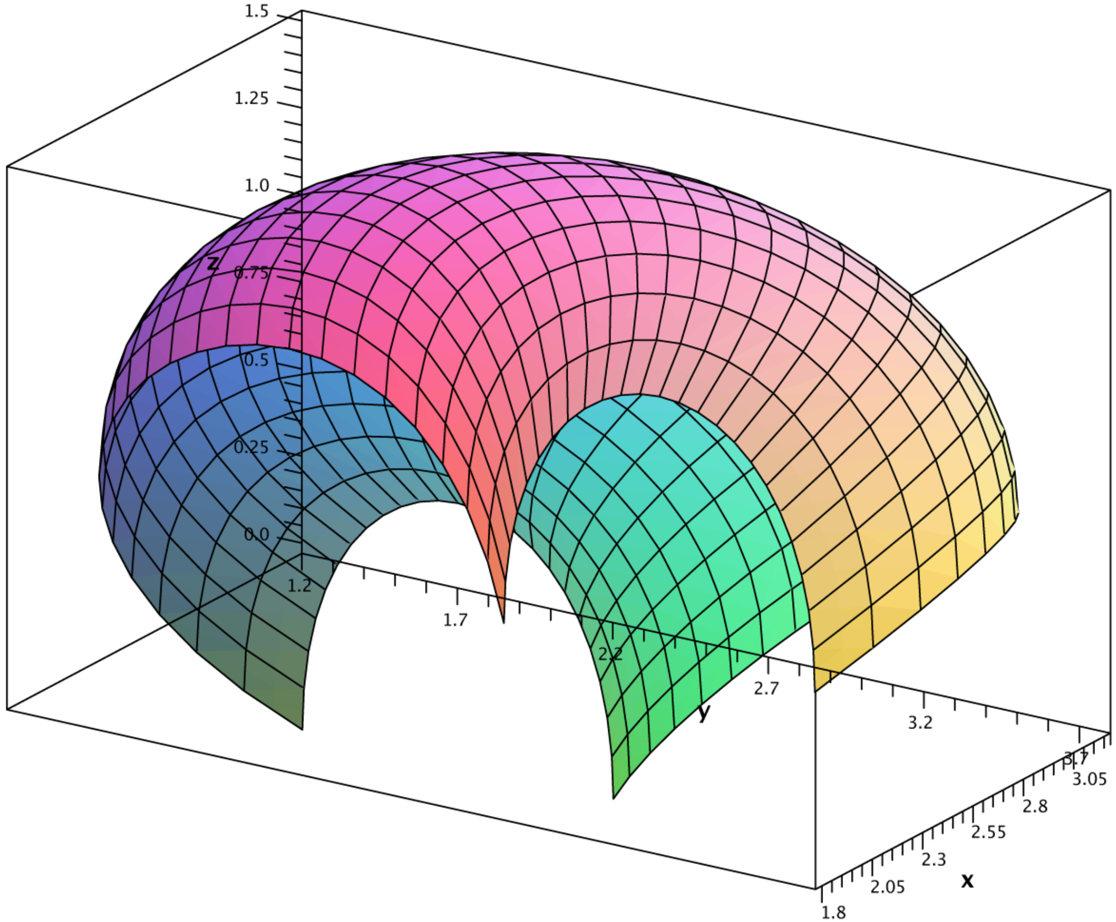
What is the height (z) of the surface at Q(0,0)?

… and here it is in 3D



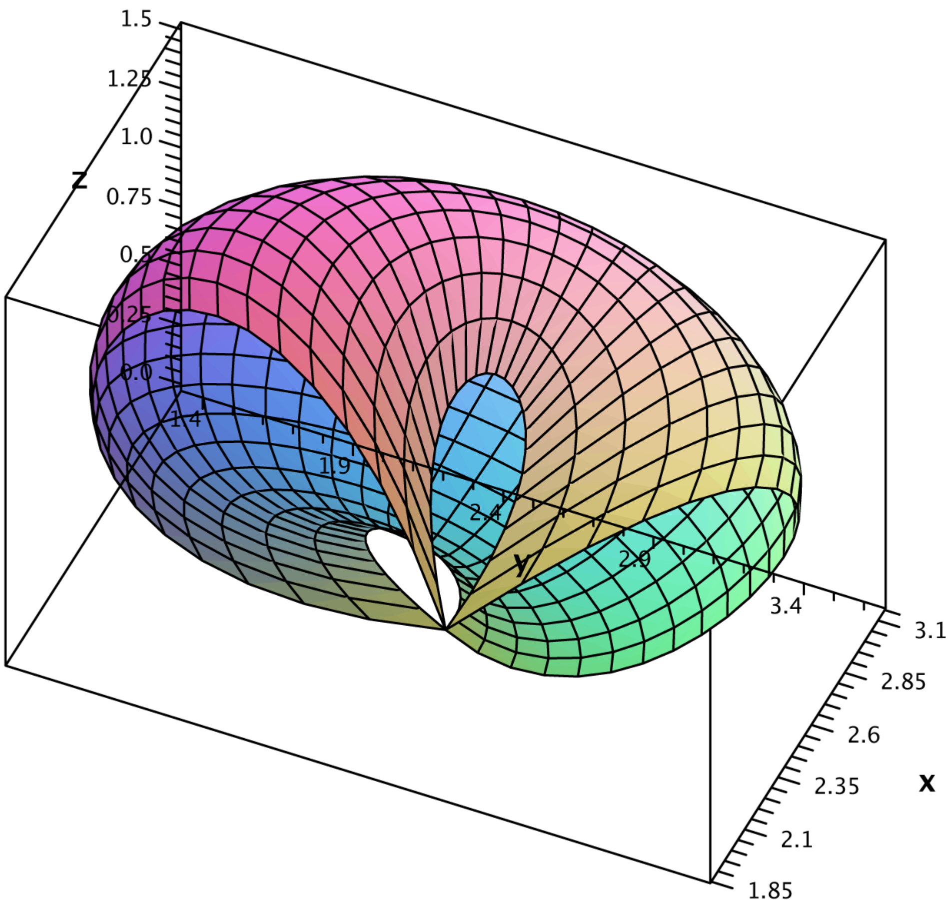


Another Example



$$\begin{bmatrix} \begin{bmatrix}2\\2\\0\end{bmatrix} & \begin{bmatrix}1\\1\\1\end{bmatrix} & \begin{bmatrix}4\\1\\1\end{bmatrix} & \begin{bmatrix}3\\2\\0\end{bmatrix} \\ \begin{bmatrix}2\\2\\1\end{bmatrix} & \begin{bmatrix}1\\1\\2\end{bmatrix} & \begin{bmatrix}4\\1\\2\end{bmatrix} & \begin{bmatrix}3\\2\\1\end{bmatrix} \\ \begin{bmatrix}2\\3\\1\end{bmatrix} & \begin{bmatrix}1\\4\\2\end{bmatrix} & \begin{bmatrix}4\\4\\2\end{bmatrix} & \begin{bmatrix}3\\3\\1\end{bmatrix} \\ \begin{bmatrix}2\\3\\0\end{bmatrix} & \begin{bmatrix}1\\4\\1\end{bmatrix} & \begin{bmatrix}4\\4\\1\end{bmatrix} & \begin{bmatrix}3\\3\\0\end{bmatrix} \end{bmatrix}$$

And another example



$$\begin{bmatrix} \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} & \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} & \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix} & \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} & \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} & \begin{bmatrix} 4 \\ 4 \\ 2 \end{bmatrix} & \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix} \\ \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} & \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix} & \begin{bmatrix} 4 \\ 4 \\ 1 \end{bmatrix} & \begin{bmatrix} 2.5 \\ 2.5 \\ 0 \end{bmatrix} \end{bmatrix}$$

And Now in SageMath

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Run Code

In [34]:
```
var('x11,x12,x13,x14,x21,x22,x23,x24,x31,x32,x33,x34,x41,x42,x43,x44')
var('y11,y12,y13,y14,y21,y22,y23,y24,y31,y32,y33,y34,y41,y42,y43,y44')
var('z11,z12,z13,z14,z21,z22,z23,z24,z31,z32,z33,z34,z41,z42,z43,z44')
GX = Matrix(SR, 4,4, (x11,x12,x13,x14,x21,x22,x23,x24,x31,x32,x33,x34,x41,x42,x43,x44))
GY = Matrix(SR, 4,4, (y11,y12,y13,y14,y21,y22,y23,y24,y31,y32,y33,y34,y41,y42,y43,y44))
GZ = Matrix(SR, 4,4, (z11,z12,z13,z14,z21,z22,z23,z24,z31,z32,z33,z34,z41,z42,z43,z44))
pretty_print(LatexExpr('Q_{x}(u,v) = '),SV.transpose(), MB.transpose(), GX, MB, TV)
pretty_print(LatexExpr('Q_{y}(u,v) = '),SV.transpose(), MB.transpose(), GY, MB, TV)
pretty_print(LatexExpr('Q_{z}(u,v) = '),SV.transpose(), MB.transpose(), GZ, MB, TV)
```

$$Q_x(u,v) = \begin{pmatrix} v^3 & v^2 & v & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{pmatrix} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u^3 \\ u^2 \\ u \\ 1 \end{pmatrix}$$

$$Q_y(u,v) = \begin{pmatrix} v^3 & v^2 & v & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} y_{11} & y_{12} & y_{13} & y_{14} \\ y_{21} & y_{22} & y_{23} & y_{24} \\ y_{31} & y_{32} & y_{33} & y_{34} \\ y_{41} & y_{42} & y_{43} & y_{44} \end{pmatrix} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u^3 \\ u^2 \\ u \\ 1 \end{pmatrix}$$

$$Q_z(u,v) = \begin{pmatrix} v^3 & v^2 & v & 1 \end{pmatrix} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{pmatrix} \begin{pmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u^3 \\ u^2 \\ u \\ 1 \end{pmatrix}$$

The next three equations are correct by the nature of how SageMath is constructing them. Quickly can them to gain a general impression for what is taking place. Then, recognized that while symbolic math packages allow us to see equations in the full

Back to Curves - New Requirements

Note that these 3rd-order segments are neither exactly Hermite nor Bezier curves:

1) The curve from P_i to P_{i+3} is only drawn between P_{i+1} and P_{i+2} (otherwise segments would overlap)

2) The curve is not constrained to pass through either P_{i+1} or P_{i+2}.

Therefore, what is their equation?

B-Splines

By convention, the geometry matrix and basis matrix for B-Splines are:

$$G_B = \begin{bmatrix} P_i \\ P_{i+1} \\ P_{i+2} \\ P_{i+3} \end{bmatrix} \qquad M_{Bs} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

B-Spline Blending Functions

$$\text{Blend} = \text{T M}_{\text{Bs}}$$

$$Blend = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \cdot \frac{1}{6} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

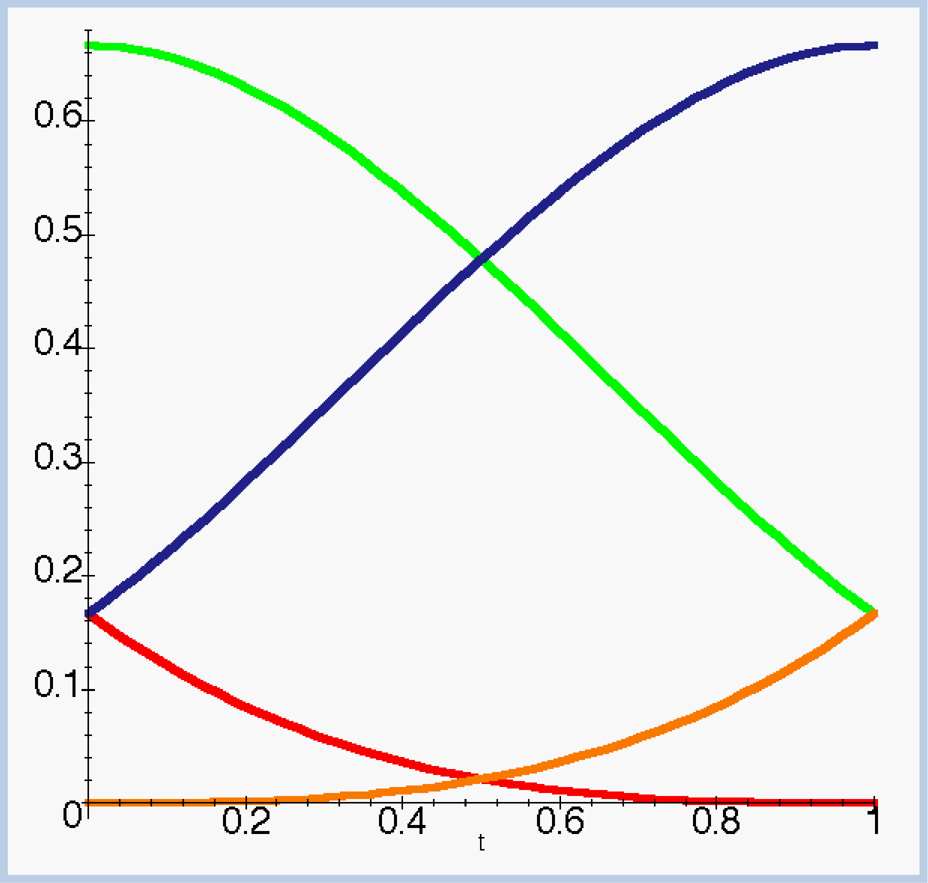
$$B_o = \tfrac{1}{6}\left(-t^3 + 3t^2 - 3t + 1\right) P_i = -\tfrac{1}{6}\left(t - 1\right)^3 P_i$$

$$B_1 = \tfrac{1}{6}\left(3t^3 - 6t^2 + 4\right) P_{i+1}$$

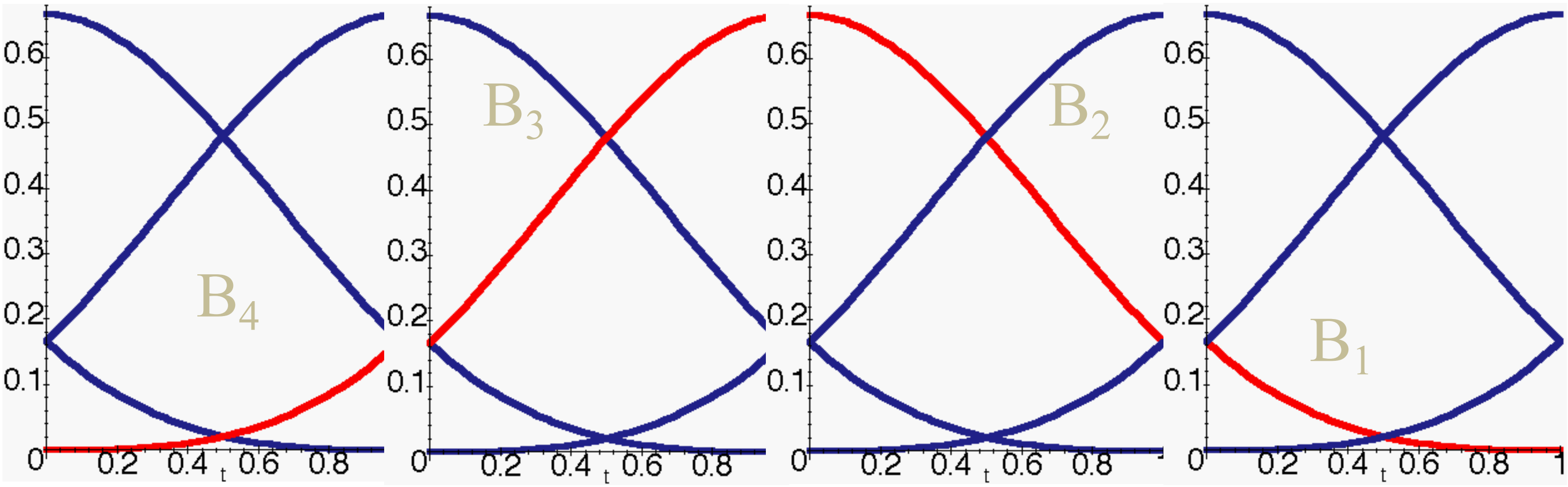
$$B_2 = \tfrac{1}{6}\left(-3t^3 + 3t^2 + 3t + 1\right) P_{i+2}$$

$$B_3 = \tfrac{1}{6}\left(t^3\right) P_{i+3}$$

Plot of Blending Functions

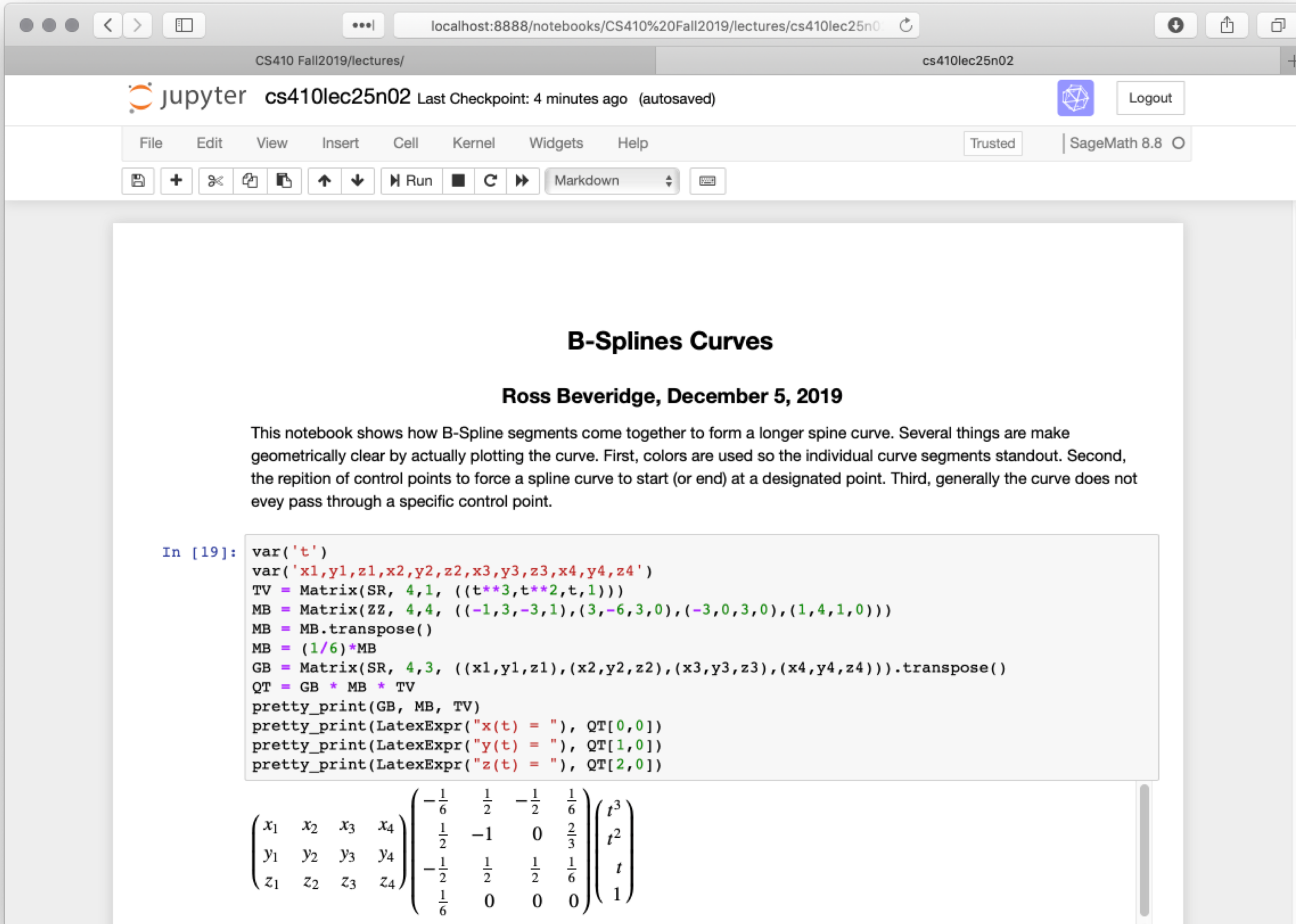


See Cycle in Blending Functions



As the Spline Sequences from one segment to the next, control points are passed from B_4, to B_3, to B_2 and finally to B_1. Consequently, the weight exerted on the curve rises then falls as indicated by the red curve above.

SageMath Notebook



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Run Markdown

B-Splines Curves

Ross Beveridge, December 5, 2019

This notebook shows how B-Spline segments come together to form a longer spine curve. Several things are make geometrically clear by actually plotting the curve. First, colors are used so the individual curve segments standout. Second, the repition of control points to force a spline curve to start (or end) at a designated point. Third, generally the curve does not evey pass through a specific control point.

In [19]:

```
var('t')
var('x1,y1,z1,x2,y2,z2,x3,y3,z3,x4,y4,z4')
TV = Matrix(SR, 4,1, ((t**3,t**2,t,1)))
MB = Matrix(ZZ, 4,4, ((-1,3,-3,1),(3,-6,3,0),(-3,0,3,0),(1,4,1,0)))
MB = MB.transpose()
MB = (1/6)*MB
GB = Matrix(SR, 4,3, ((x1,y1,z1),(x2,y2,z2),(x3,y3,z3),(x4,y4,z4))).transpose()
QT = GB * MB * TV
pretty_print(GB, MB, TV)
pretty_print(LatexExpr("x(t) = "), QT[0,0])
pretty_print(LatexExpr("y(t) = "), QT[1,0])
pretty_print(LatexExpr("z(t) = "), QT[2,0])
```

$$\begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{pmatrix} \begin{pmatrix} -\frac{1}{6} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{6} \\ \frac{1}{2} & -1 & 0 & \frac{2}{3} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{6} \\ \frac{1}{6} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} t^3 \\ t^2 \\ t \\ 1 \end{pmatrix}$$

Example of B-Spline

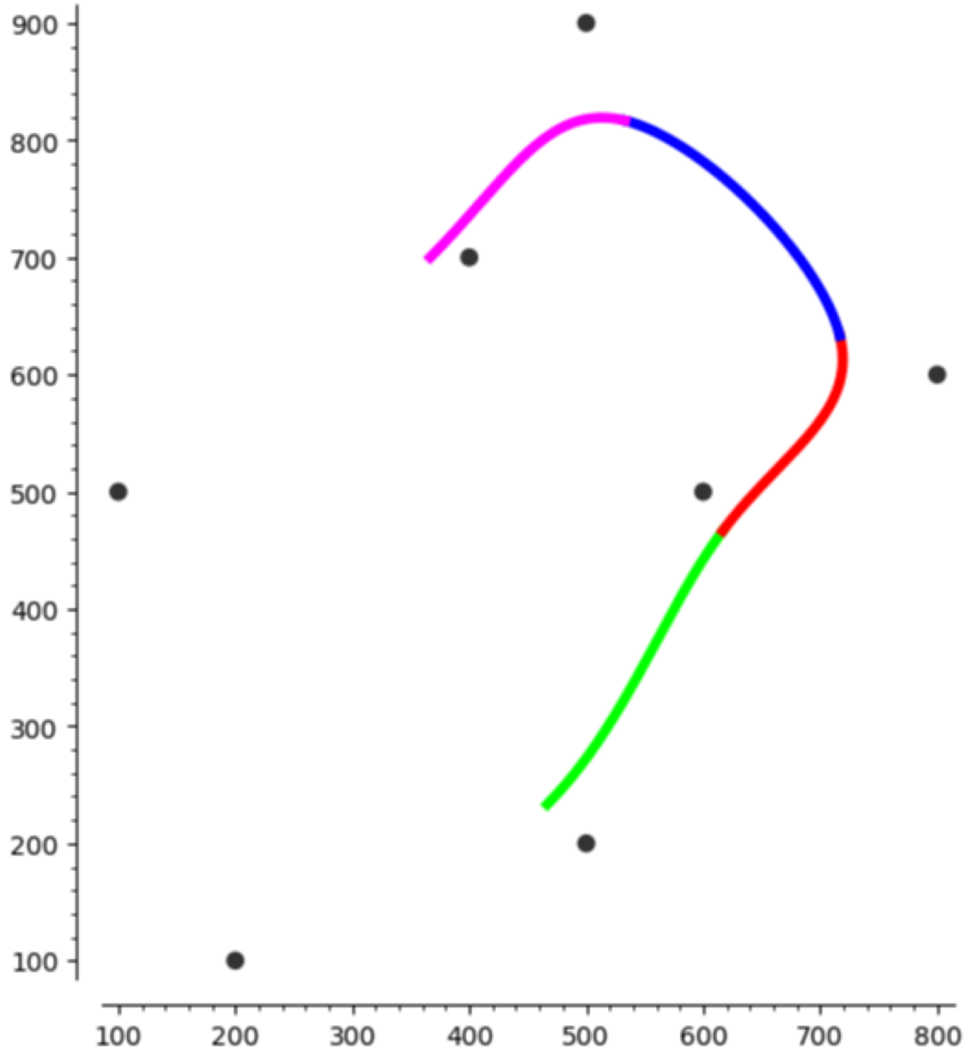
$$G_1 = \begin{bmatrix} 2 & 5 & 6 & 8 \\ 1 & 2 & 5 & 6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_2 = \begin{bmatrix} 5 & 6 & 8 & 5 \\ 2 & 5 & 6 & 9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$G_3 = \begin{bmatrix} 6 & 8 & 5 & 4 \\ 5 & 6 & 9 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

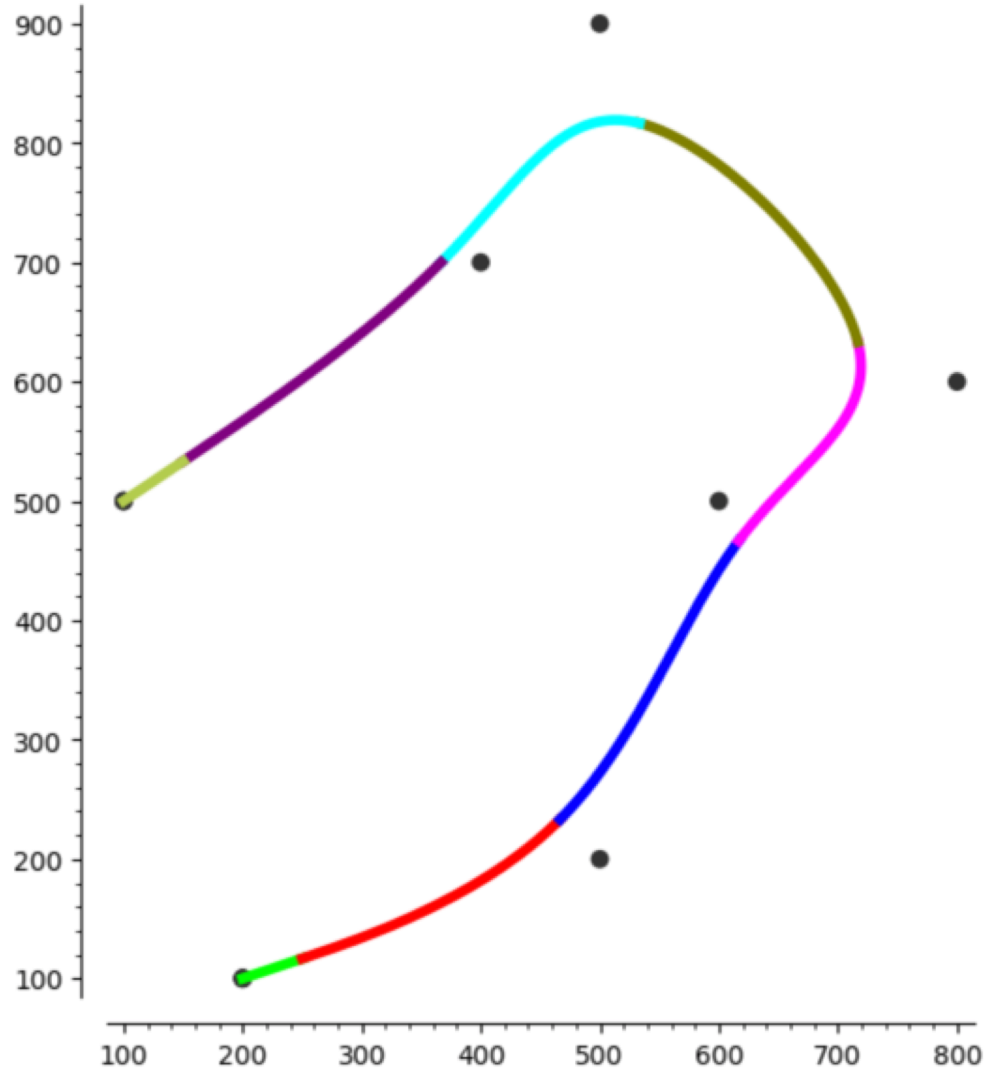
$$G_4 = \begin{bmatrix} 8 & 5 & 4 & 1 \\ 6 & 9 & 7 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$Q_j(t) = G_j\,\mathrm{B}(t)$$



Replicating Control Points

```
P1  = (200,100,0)
P2  = (200,100,0)
P3  = (200,100,0)
P4  = (500,200,0)
P5  = (600,500,0)
P6  = (800,600,0)
P7  = (500,900,0)
P8  = (400,700,0)
P9  = (100,500,0)
P10 = (100,500,0)
P11 = (100,500,0)
```



More Variations

We have just described *uniform*, *non-rational* B-Splines

Uniform means that the control points are evenly spaced (in terms of the parameter t).

It is also possible to have non-uniform B-Splines. Why? because it is easier to interpolate starting and ending points, and it is possible to reduce the continuity at specific join points.

Non-uniform B-Splines

- Every control point must have a corresponding t-value
  - This is called a “knot vector”
- If the spacing (in *t*) between two control points is small, then a sharp curve will result.
- If the spacing (in *t*) is zero, the curve becomes discontinuous.

The Standard Knot Vector

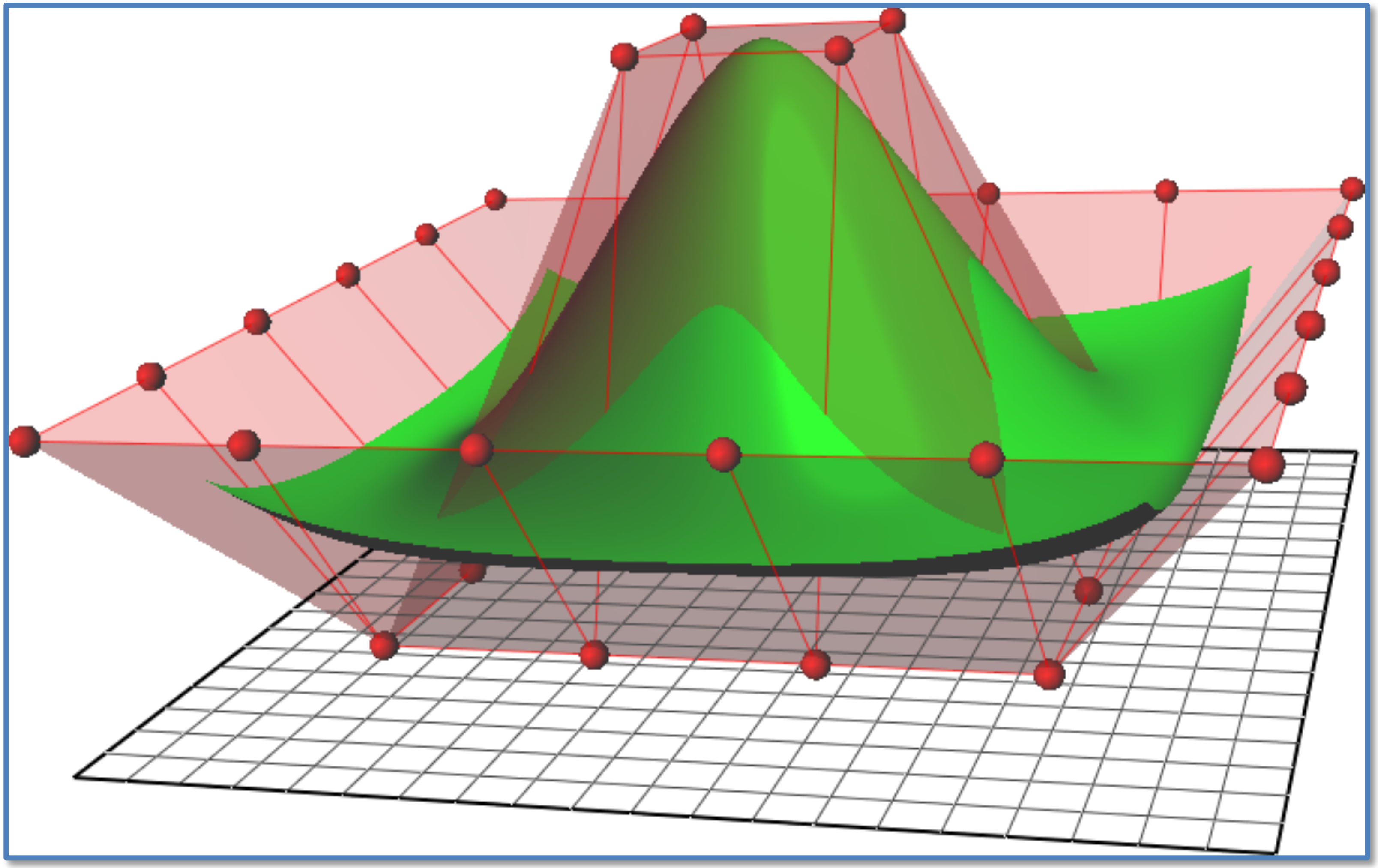
- The “standard knot vector” begins and ends with a four-fold knot:
  - e.g., for 5 control points T = (0, 0, 0, 0, 1, 2, 3, 4, 5, 5, 5, 5,)

- This means that the B-Spline will not loose the last point(s), and will behave correctly near the endpoints.

A Brief Comment on NURBS

- Naming: Non-Uniform Rational B-Spline
  - Non-Uniform -> knot vector.
  - Rational –> defined with x, y, z, w.
    - Points are rational, i.e. px = x / w
  - B-Spline -> uses B-Spline geometry
- NURBS Surface
  - Compose curves to generate surface
    - Recall Bezier Curve to Bezier Surface approach
- The inclusion of w means
  - Perspective projection does not distort.

Simple NURBS Illustration



This image from Wikimedia and part of the Wikipedia description of NURBS.